Exact $SU(5)$ Yukawa matrix unification in the General Flavour Violating MSSM

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based on MI, K. Kowalska, JHEP 1504 (2015) 120

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Storyline

1. SU(5) Yukawa matrix unification
2. Minimal Supersymmetric Standard Model
3. chirally-enhanced SUSY threshold corrections
4. off-diagonal soft terms help $\rightarrow$ General Flavour Violating MSSM
5. Phenomenology of Yukawa unification in the GFV MSSM:
   - 2nd + 3rd generation
   - 1st + 2nd + 3rd generation

INNOVATIVE ECONOMY NATIONAL COHESION STRATEGY
FNP Foundation for Polish Science
EUROPEAN UNION EUROPEAN REGIONAL DEVELOPMENT FUND

Unification - SU(5) model: matter & Higgs sector

Georgi, Glashow, 1974

$$\underbrace{(\overline{\mathbf{3}},\mathbf{1},\tfrac{1}{3})}_{d_R^*}\oplus\underbrace{(\mathbf{1},\mathbf{2},-\tfrac{1}{2})}_{l}=\underbrace{\overline{\mathbf{5}}}_{\Psi_{\overline{5}}}$$

$$\underbrace{(\mathbf{3},\mathbf{2},\tfrac{1}{6})}_{q}\oplus\underbrace{(\overline{\mathbf{3}},\mathbf{1},-\tfrac{2}{3})}_{u_R^*}\oplus\underbrace{(\mathbf{1},\mathbf{1},1)}_{e_R^*}=\underbrace{\mathbf{10}}_{\Psi_{10}},$$

$$W\ni\Psi_{10}\mathbf{Y}^{de}\Psi_{\overline{5}}H_{\overline{5}}+\Psi_{10}\mathbf{Y}^{u}\Psi_{10}H_5$$

Unification - SU(5) model: matter & Higgs sector

Georgi, Glashow, 1974

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$$\underbrace{(\mathbf{3},\mathbf{2},\tfrac{1}{6})}_{q}\oplus\underbrace{(\bar{\mathbf{3}},\mathbf{1},-\tfrac{2}{3})}_{u_R^*}\oplus\underbrace{(\mathbf{1},\mathbf{1},1)}_{e_R^*}=\underbrace{\mathbf{10}}_{\Psi_{10}},$$

$$W \ni \Psi_{10}\mathbf{Y}^{de}\Psi_{\bar{5}}H_{\bar{5}}+\Psi_{10}\mathbf{Y}^{u}\Psi_{10}H_5$$

$$\boxed{Y_{ii}^{d,MSSM}=Y_{ii}^{e,MSSM}}$$

Gauge coupling unification

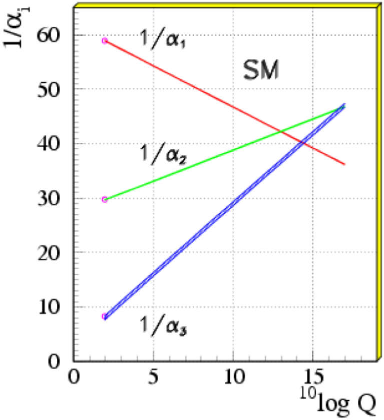
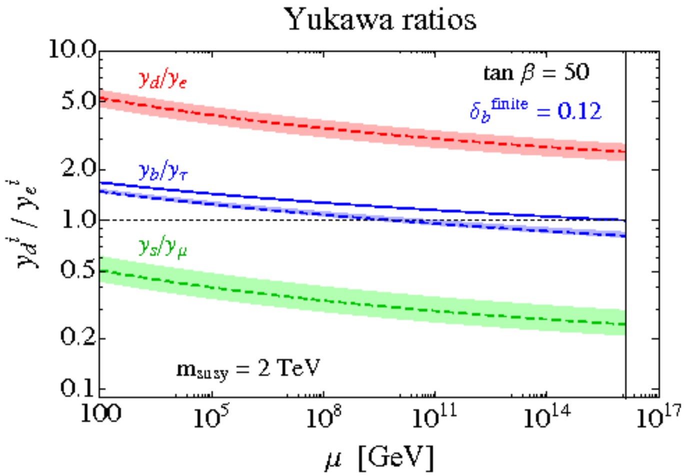


Figure : Gauge coupling unification in non-SUSY GUTs on the left vs. SUSY GUTs on the right using the LEP data (1991)
arXiv: hep-ph/0012288

Yukawa couplings at the GUT scale



Elor, Hall, Pinner, Ruderman, JHEP 1210 (2012) 111, arXiv:1206.5301

2nd generation: $Y_\mu(M_{\rm GUT}) \approx 3Y_s(M_{\rm GUT})$

1st generation: $Y_e(M_{\rm GUT}) \approx 1/3Y_d(M_{\rm GUT})$

Yukawa unification - Solution 1 - modify GUT structure

Change the boundary condition at the high scale

- additional Higgs fields, e.g.

  H. Georgi and C. Jarlskog, Phys. Lett. B86 (1979) 297

$$H_5, \quad H_{\bar{5}}, \quad H_{45} \rightarrow Y_\mu = 3Y_s, \quad Y_e = 1/3 Y_d$$

- correction O(1) from higher-dim. operators

  D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. B 661 (2003) 62
  S. Antusch and M. Spinrath, Phys. Rev. D 79 (2009) 095004
  S.Antusch, S.F.King and M. Spinrath, Phys. Rev. D 89 (2014) 055027

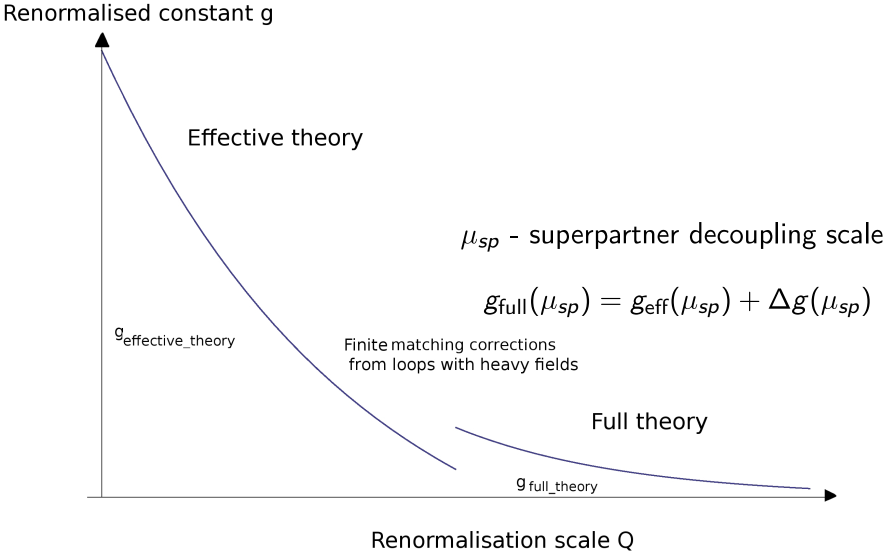
$$W = W_Y + W_{\rm HO} \rightarrow \qquad \begin{aligned} Y_d^{ij} &= Y_{de}^{ij} + \frac{1}{M_{\rm PL}} F^{ij} \\ Y_e^{ij} &= Y_{de}^{ij} + \frac{1}{M_{\rm PL}} G^{ij} \end{aligned}$$

Yukawa unification - Solution 2

Manipulate the boundary condition between SM and MSSM - play with threshold corrections

- Diaz-Cruz, Murayama, Pierce, Phys.Rev.D 65:075011, 2002
  (particular ansatz using A-terms for unification)
- Ts. Enkhbat, arXiv:0909.5597
  (general diagonal A-terms)
- MI, Eur.Phys.J. C75 (2015) 51
  (update - new exp results, broader $\tan\beta$ range, weaker impact on flavour observables)

Threshold corrections



SUSY threshold corrections to Yukawa couplings

A. Crivellin, L. Hofer, J. Rosiek, JHEP 1107 (2011) 017

$$v_f Y_{ii}^{f\ MSSM} = v_f Y_{ii}^{f\ SM} - \Sigma_{ii}^{f}(Y_j^{f'}, \ldots).$$

SUSY threshold corrections to Yukawa couplings

A. Crivellin, L. Hofer, J. Rosiek, JHEP 1107 (2011) 017

$$v_f Y_{ii}^{f\,MSSM} = v_f Y_{ii}^{f\,SM} - \Sigma_{ii}^{f}(Y_j^{f'}, \ldots).$$

$$m_i^{d(\ell)\,SM} - v_d Y_{ii}^{d(\ell)MSSM} = \Sigma_{ii\,\not{Y}}^{d(\ell)\,LR} + \epsilon_i^{d(\ell)}\, v_u\; Y_{ii}^{d(\ell)(0)} + O(\frac{v^2}{M_{SUSY}}),$$

SUSY threshold corrections to Yukawa couplings

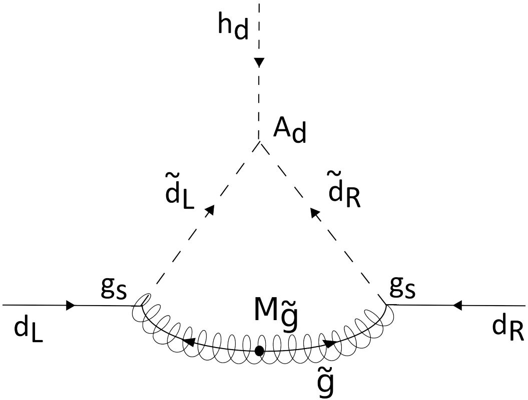
A. Crivellin, L. Hofer, J. Rosiek, JHEP 1107 (2011) 017

$$v_f\, Y_{ii}^{f\; MSSM} = v_f\, Y_{ii}^{f\; SM} - \Sigma_{ii}^{f}(Y_j^{f'}, \ldots).$$

$$m_i^{d(\ell)\; SM} - v_d\, Y_{ii}^{d(\ell) MSSM} \;=\; \Sigma_{ii\,\not{Y}}^{d(\ell)\; LR} \;+\; \epsilon_i^{d(\ell)}\, v_u \;\; Y_{ii}^{d(\ell)(0)} \;+\; O(\frac{v^2}{M_{SUSY}}),$$

$$Y_{ii}^{d(\ell) MSSM} = \frac{m_i^{d(\ell)\; SM} - \Sigma_{ii\,\not{Y}}^{d(\ell)\; LR}}{v_d(1 + \tan\beta \cdot \epsilon_i^{d(\ell)})}.$$

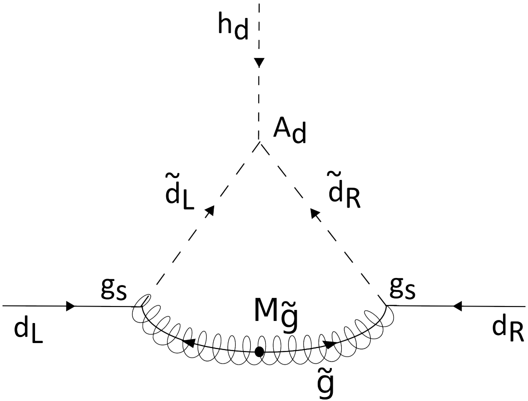
Threshold corrections - example diagrams



- Diaz-Cruz, Murayama, Pierce, Phys.Rev.D 65:075011, 2002
- Ts. Enkhbat, arXiv:0909.5597
- MI, Eur.Phys.J. C75 (2015) 51

$$(\Sigma^d_{ii})^{\tilde{g}} \sim \alpha_S m_{\tilde{g}}(v_d A^d_{ii} - v_d Y^d_{ii} \mu \tan\beta)$$

Threshold corrections - example diagrams



- Diaz-Cruz, Murayama, Pierce, Phys.Rev.D 65:075011, 2002
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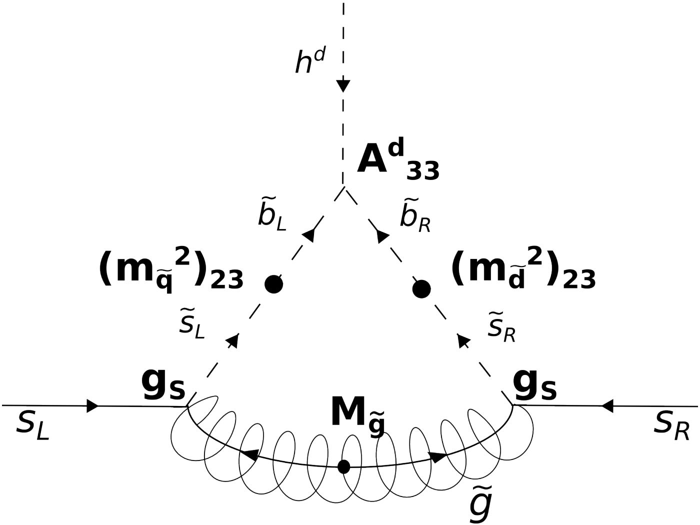
$A_s \sim m_{\tilde{s}}$ required for strange-muon unification
$\Rightarrow$ MSSM vacuum metastable

Threshold corrections - example diagrams



$$(\Sigma^d_{22})^{\tilde{g}} \sim \alpha_S M_{\tilde{g}} v_d (A^d_{33} - Y_b \mu \tan\beta)(m^2_{\tilde{q}})_{23}(m^2_{\tilde{d}})_{23}$$

SU(5) boundary conditions at M_{GUT}

$$(m^2_{\tilde{l}})_{ij} = (m^2_{\tilde{d}})_{ij} \equiv (\mathbf{m^2_{dl}})_{\mathbf{ij}}$$

$$(m^2_{\tilde{q}})_{ij} = (m^2_{\tilde{u}})_{ij} = (m^2_{\tilde{e}})_{ij} \equiv (\mathbf{m^2_{ue}})_{\mathbf{ij}}$$

$$A^d_{ij} = A^e_{ij} \equiv \mathbf{A^{de}_{ij}}$$

$$A^u_{ij}$$

$$M_1 = M_2 = M_3 \equiv \mathbf{M_{1/2}},$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$\mathbf{m^2_{H_u}}, \qquad \mathbf{m^2_{H_d}}$$

Tools

BayesFITSv.3.2 A. Fowlie, M. Kazana, K. Kowalska, S. Munir, L. Roszkowski, E. M. Sessolo, S. Trojanowski, Y. L. S. Tsai [arXiv:1206.0264], K. Kowalska [arXiv:1406.0710]

MultiNest v2.7
F. Feroz, M. P. Hobson and M. Bridges, [arXiv:0809.3437]

SUSY_Flavor v2.10
A. Crivellin, J. Rosiek, P. H. Chankowski, A. Dedes, S. Jaeger and P. Tanedo [arXiv:1203.5023]

SPheno v3.3.3
W. Porod and F. Staub [arXiv:1104.1573]

HIGGSBounds v4.0.0
HIGGSSIGNALS v1.0.0
P. Bechtle et al. [arXiv:0811.4169] [arXiv:1102.1898], [arXiv:1311.0055], [arXiv:1305.1933]

DarkSUSY v5.0.6
P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke, E. A. Baltz, [astro-ph/0406204]

Ranges of input parameters

$m^{dl}_{ii},\ i=1,2,3$	$[100,\ 7000]$ GeV
m^{dl}_{23}/m^{dl}_{33}	$[0,\ 1]$
m^{dl}_{13}/m^{dl}_{33}	$[0,\ 1]$
m^{dl}_{12}/m^{dl}_{33}	$[0,\ 1]$
$m^{ue}_{ii},\ i=1,2,3$	$[100,\ 7000]$ GeV

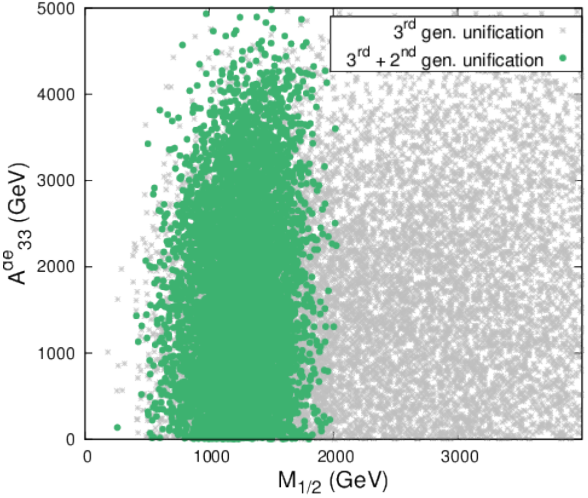
$M_{1/2}$	$[100,\ 4000]$ GeV
m_{H_u}	$[100,\ 8000]$ GeV
m_{H_d}	$[100,\ 8000]$ GeV
$\tan\beta$	$[3,\ 45]$

A^{de}_{33}	$[0,\ 5000]$ GeV
A^{u}_{33}	$[-9000,\ 9000]$ GeV
A^{de}_{11}/A^{de}_{33}	$[-0.00028,\ 0.00028]$
A^{de}_{22}/A^{de}_{33}	$[-0.065,\ 0.065]$
A^{u}_{22}/A^{u}_{33}	$[-0.005,\ 0.005]$
$A^{de}_{ij}/A^{de}_{33},\ i\neq j$	$[-0.5,\ 0.5]$

$$m^{dl}_{ij} \equiv \sqrt{(m^2_{dl})_{ij}}, \qquad m^{ue}_{ij} \equiv \sqrt{(m^2_{ue})_{ij}}.$$

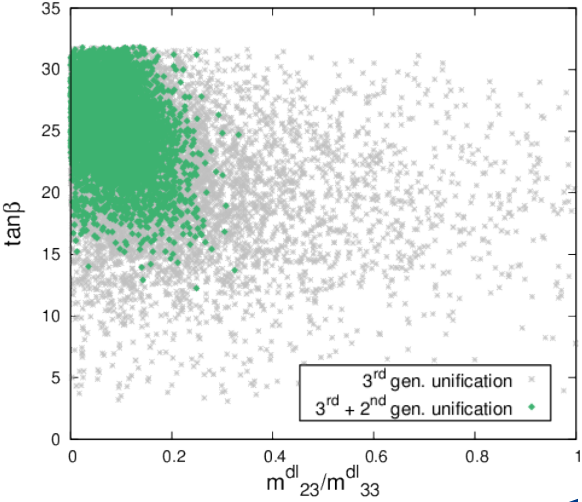
3rd + 2nd family Yukawa unification

relevant GFV parameter: m^{dl}_{23}



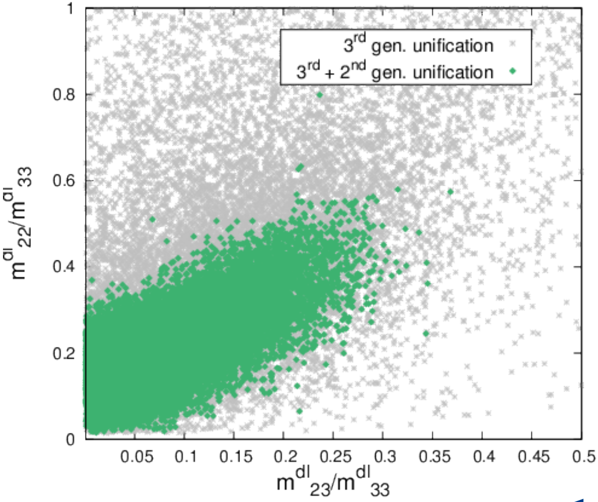
3rd + 2nd family Yukawa unification

relevant GFV parameter: m_{23}^{dl}



3rd + 2nd family Yukawa unification

relevant GFV parameter: m_{23}^{dl}



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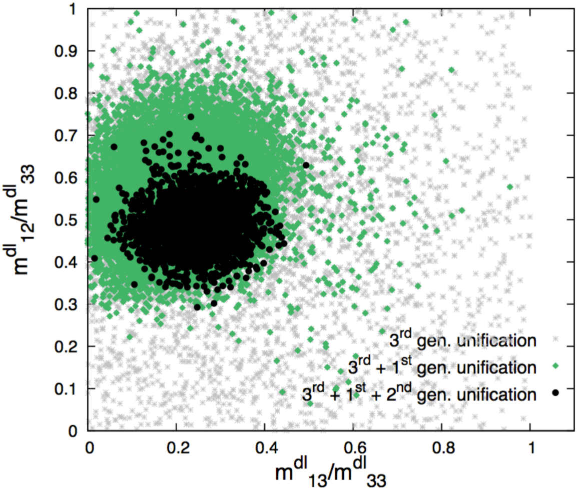
3rd + 2nd + 1st family Yukawa unification

relevant GFV parameters: m^{dl}_{23}, m^{dl}_{13}, m^{dl}_{12}, A^{de}_{12}



3rd + 2nd + 1st family Yukawa unification

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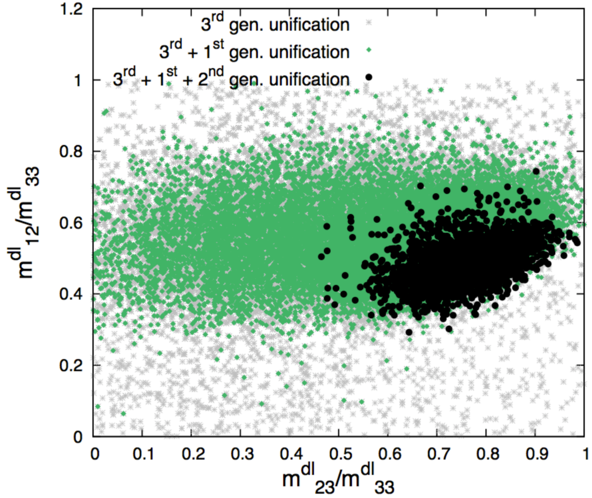

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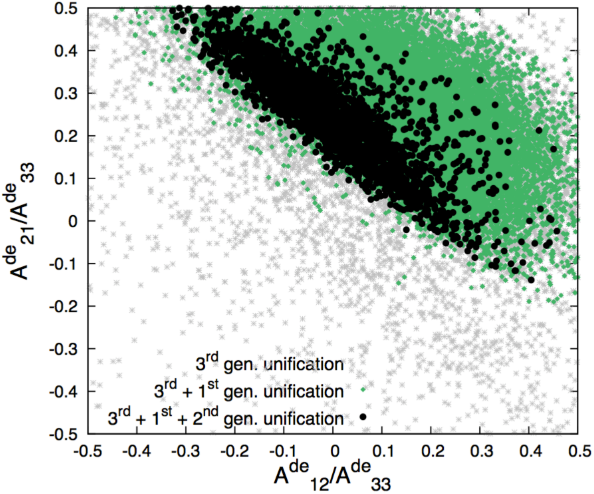
relevant GFV parameters: m^{dl}_{23}, m^{dl}_{13}, m^{dl}_{12}, A^{de}_{12}



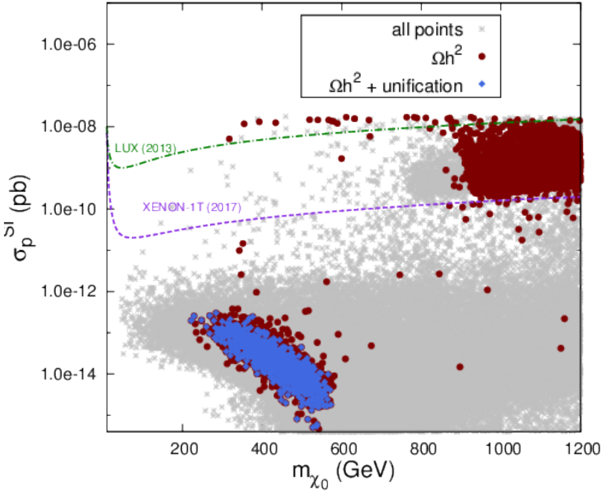
Experimental constraints

Measurement	Mean or range	Error [exp., th.]
$\Omega_\chi h^2$	0.1199	[0.0027, 10%]
m_h (by CMS)	125.7 GeV	[0.4, 3.0] GeV
$\sin^2\theta_{\rm eff}$	0.23155	[0.00012, 0.00015]
M_W	80.385 GeV	[0.015, 0.015] GeV
$\mathrm{BR}\left(\overline{\mathrm{B}} \to \mathrm{X_s}\gamma\right)\times 10^4$	3.43	[0.22, 0.23]
$\mathrm{BR}\left(\mathrm{B_s} \to \mu^+\mu^-\right) \times 10^9$	2.8	[0.7, 0.23]
$\mathrm{BR}\left(\mathrm{B_d} \to \mu^+\mu^-\right) \times 10^{10}$	3.9	[1.6, 0.2]
$\Delta M_{B_s} \times 10^{11}$	1.1691 GeV	[0.0014, 0.1580] GeV
$\Delta M_{B_d} \times 10^{13}$	3.357 GeV	[0.033, 0.340] GeV
$\Delta M_{B_d}/\Delta M_{B_s} \times 10^2$	2.87	[0.02, 0.14]
$\sin(2\beta)_{\rm exp}$	0.682	[0.019, 0.003]
$\mathrm{BR}\left(\mathrm{B_u} \to \tau\nu\right)\times 10^4$	1.14	[0.27, 0.07]
$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{10}$	1.73	[1.15, 0.04]
$\|d_n\| \times 10^{26}$	< 2.9 e cm	[0, 30%]
$\epsilon_K \times 10^3$	2.228	[0.011, 0.17]

Experimental constraints - Lepton Flavour Violation

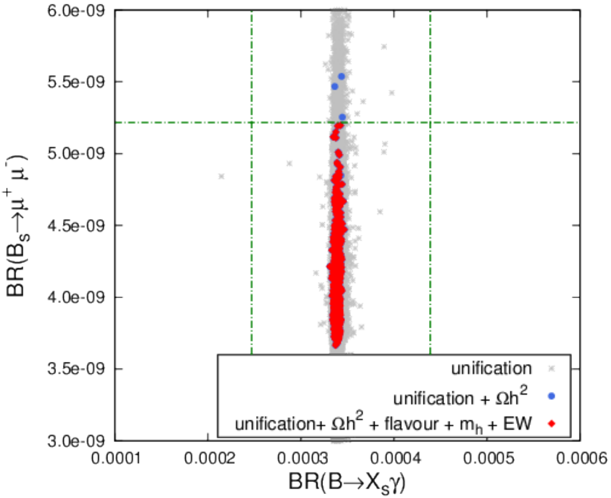
$\mathrm{BR}\,(\mu^+ \to \mathrm{e}^+\gamma)\times 10^{13}$	< 5.7
$\mathrm{BR}\,(\tau^\pm \to \mathrm{e}^+\gamma)\times 10^{8}$	< 3.3
$\mathrm{BR}\,(\tau^\pm \to \mu^+\gamma)\times 10^{8}$	< 4.4
$\mathrm{BR}\,(\mu^+ \to \mathrm{e}^+\mathrm{e}^+\mathrm{e}^-)\times 10^{12}$	< 1.0
$\mathrm{BR}\,(\tau^\pm \to \mathrm{e}^\pm\mathrm{e}^+\mathrm{e}^-)\times 10^{8}$	< 2.7
$\mathrm{BR}\,(\tau^\pm \to \mu^\pm\mu^+\mu^-)\times 10^{8}$	< 2.1

3rd + 2nd family unification: Dark matter



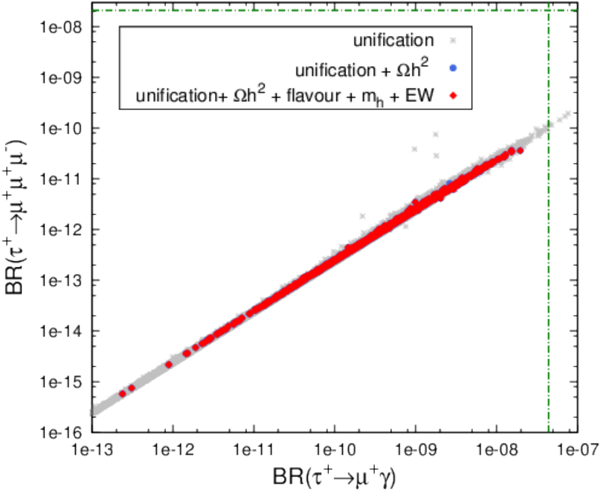
only bino DM

3rd + 2nd family unification: Flavour observables



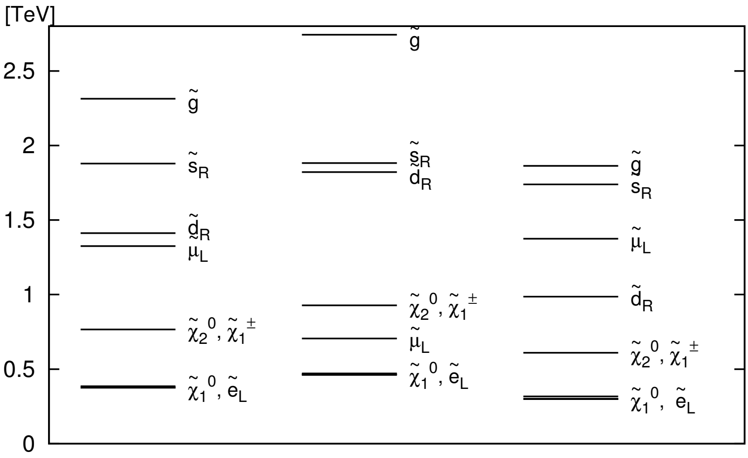
dashed lines - 3σ experimental limits

3rd + 2nd family unification: Flavour observables

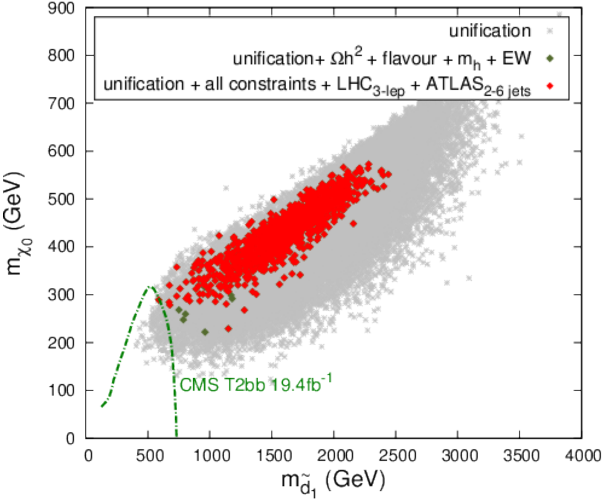


dashed lines - 3σ experimental limits

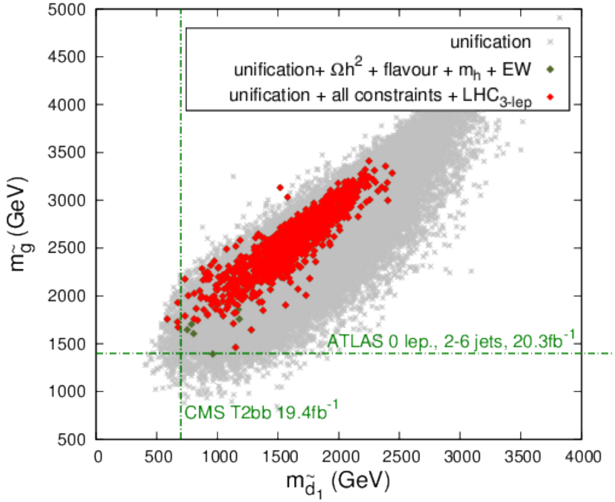
3rd + 2nd family unification: typical spectra



3rd + 2nd family unification: LHC SUSY searches

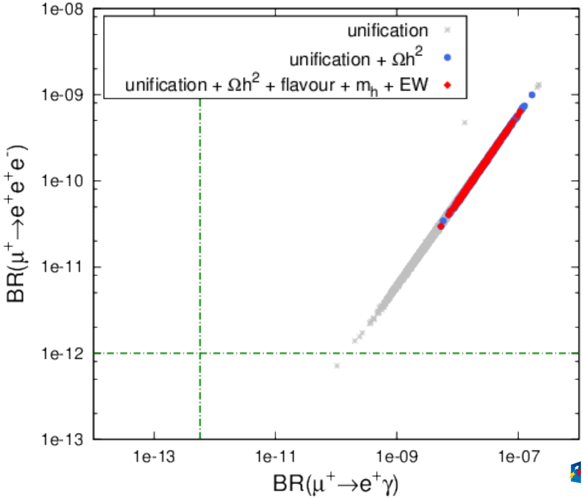


3rd + 2nd family unification: LHC SUSY searches



3rd + 2nd + 1st family unification: LFV

- consistent with quark flavour observables
- **strongly disfavoured** by the Lepton Flavour Violating observables



Open questions

- Are there other regions consistent with Yukawa unification?
- Could the exclusion of GFV_{123} Yukawa unification be avoided?
  e.g. much higher SUSY masses,
  an $SU(5)$ GUT scenario with $m_{\tilde{l}} \neq m_{\tilde{d}}$
- Could two-loop threshold corrections be any relevant?
- $Y_d = Y_e$ in a GFV_{23}-like scenario without vacuum metastability?

Conclusions

Non-trivial flavour structure of the MSSM

can facilitate the SU(5) Yukawa matrix unification

- Unification of the 2nd and 3rd generation phenomenologically allowed (relevant parameter: $(m^2_{dl})_{23}$)
- Full unification of all thee generations is strongly disfavoured by the limits on LFV (problems with: $(m^2_{dl})_{12}$, $A^{de}_{12/21}$)

Supplementary slides

EW vacuum stability

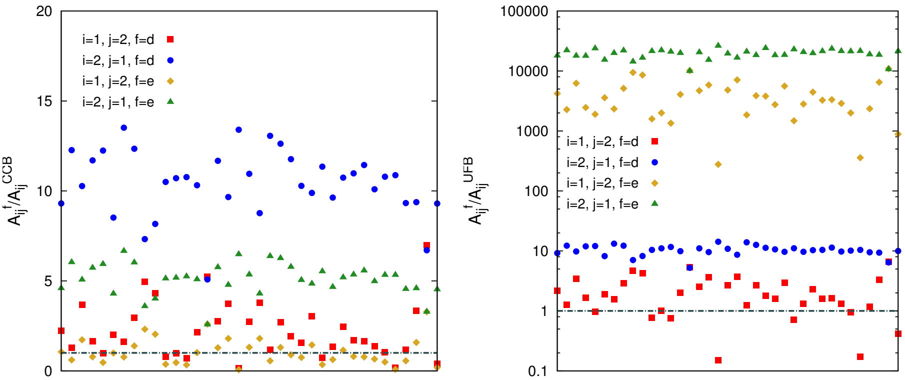
In the down-squark sector, Tree-level formulae for the CCB and UFB bounds in the down-squark sector:

$$(v_d/\sqrt{2})A^d_{ij} \leq m^d_k[(m^2_{\tilde{q}})_{ii} + (m^2_{\tilde{d}})_{jj} + m^2_{H_d} + \mu^2]^{1/2}, \quad k = \mathrm{Max}(i,j)$$

$$(v_d/\sqrt{2})A^d_{ij} \leq m^d_k[(m^2_{\tilde{q}})_{ii} + (m^2_{\tilde{d}})_{jj} + (m^2_{\tilde{l}})_{ii} + (m^2_{\tilde{e}})_{jj}]^{1/2}$$

J. A. Casas and S. Dimopoulos, [hep-ph/9606237]

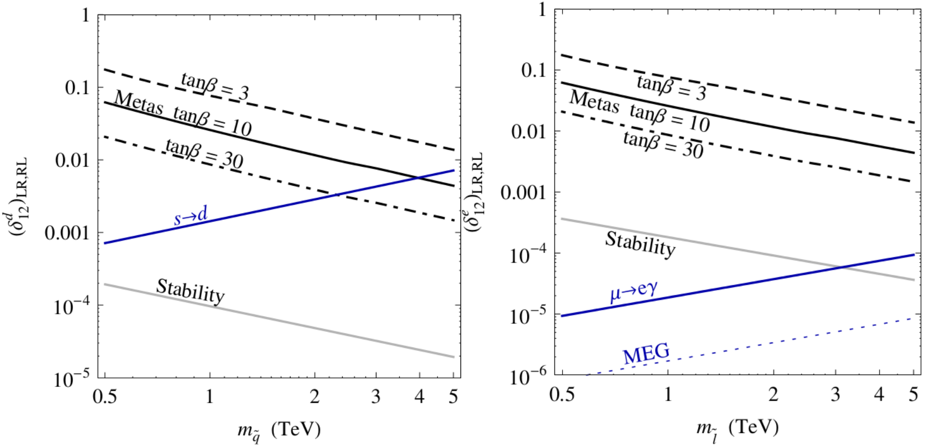
EW vacuum stability



EW vacuum CCB (a) and UFB (b) upper bounds (dashed) on the elements $A^{d,e}_{12/21}$

EW vacuum stability

J. h. Park, [arXiv:1011.4939]:



metastability bounds are 2-3 orders of magnitude weaker.

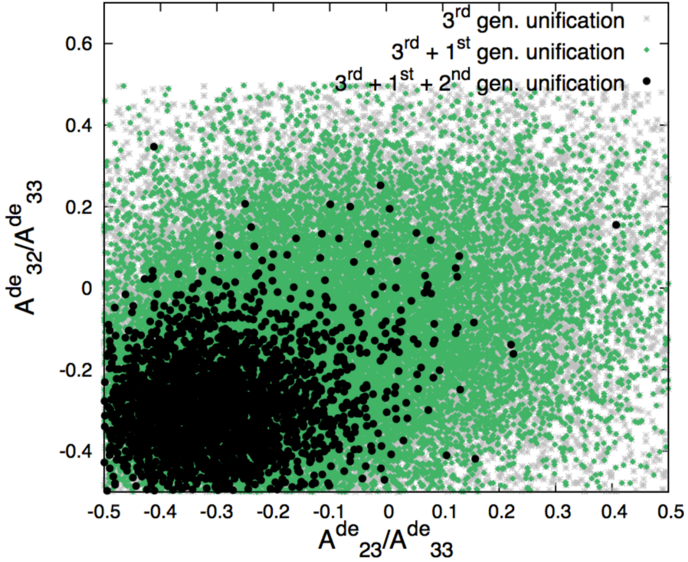
Constants values

we scanned over (m_t^{pole}, $m_b^{\overline{MS}}(m_b)$, $\alpha_{\text{em}}^{-1}(M_Z)$ and $\alpha_s^{\overline{MS}}(M_Z)$) ($\bar{\rho}$, $\bar{\eta}$, A, λ)

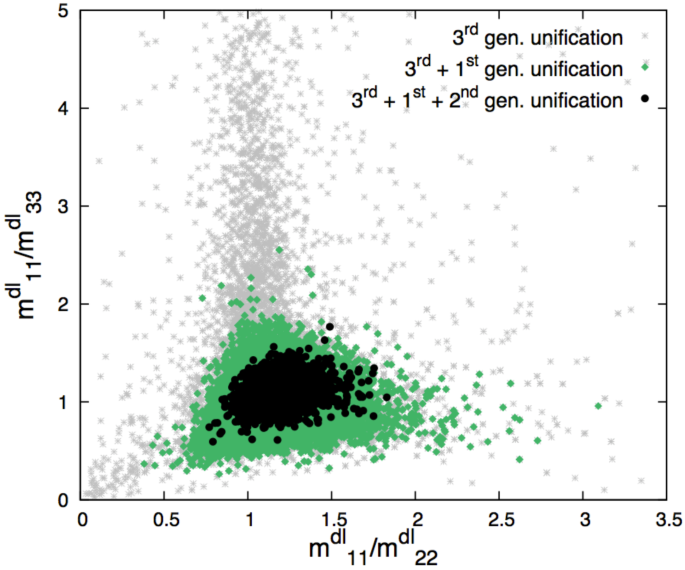
m_t^{pole} 173.34 $\pm$ 0.76 GeV		$m_b^{\overline{MS}}(m_b)$ 4.18 $\pm$ 0.03 GeV		$\alpha_s^{\overline{MS}}(M_Z)$ 0.1184 $\pm$ 0.0007		$\alpha_{\text{em}}^{-1}(M_Z)$ 127.944 $\pm$ 0.015	
$m_u^{\overline{MS}}$ 2.3 MeV	$m_d^{\overline{MS}}$ 4.8 MeV	$m_s^{\overline{MS}}$ 95 MeV	$m_c^{\overline{MS}}(m_c)$ 1.275 GeV	m_e^{pole} 511 keV	m_μ^{pole} 106 MeV	m_τ^{pole} 1.777 GeV	M_Z^{pole} 91.19
$\bar{\rho}$ 0.159 $\pm$ 0.045		$\bar{\eta}$ 0.363 $\pm$ 0.049		A 0.802 $\pm$ 0.020		λ 0.22535 $\pm$ 0.0006	

Table : Standard Model parameters (PDG 2014) used in our numerical calculations. The light (u, d, s) quark masses are $\overline{\text{MS}}$-renormalized at 2 GeV.

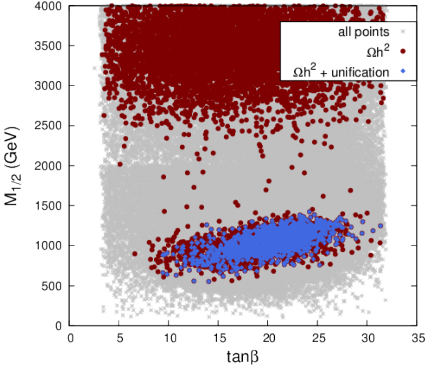
Yukawa unification

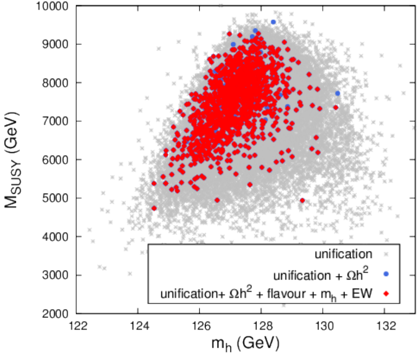


Yukawa unification

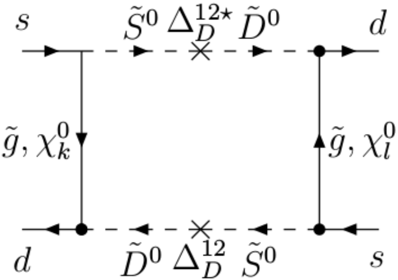


Dark matter & Higgs mass





Kaon and B mixing

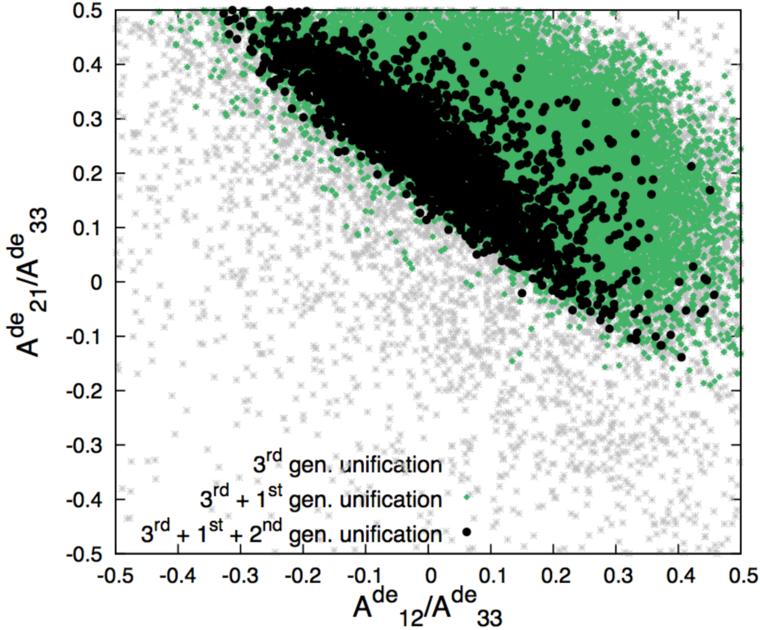


$$\Delta M_{B_{d(s)}} = 2 \left| \langle \bar{B}^0_{d(s)} | H_{\text{eff}}^{\Delta B=2} | B^0_{d(s)} \rangle \right|$$

$$\varepsilon_K = \frac{\exp(i\pi/4)}{\sqrt{2}\Delta M_K} \Im\mathrm{m} \langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle$$

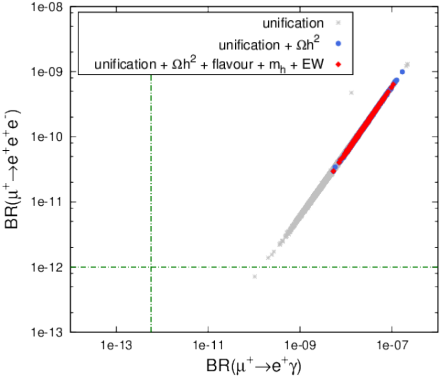
$\Delta_D^{12} = m_{12}^d$ in super-CKM basis
Misiak, Pokorski, Rosiek,
hep-ph/9703442

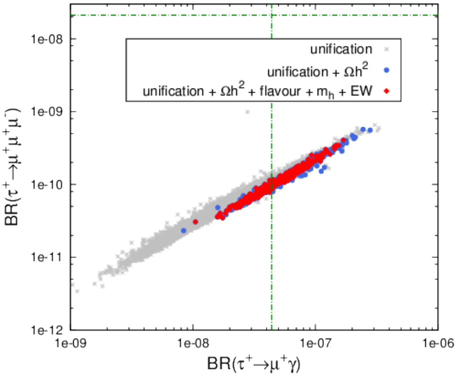
Ad12 Ad21


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5
A^{de}_{21}/A^{de}_{33}
A^{de}_{12}/A^{de}_{33}
3^{rd} gen. unification
3^{rd} + 1^{st} gen. unification
3^{rd} + 1^{st} + 2^{nd} gen. unification

3rd + 2nd + 1st family unification: LFV

- consistent with quark flavour observables
- **strongly disfavoured** by the Lepton Flavour Violating observables





Parameter	Scanning Range
$M_{1/2}$	$[100,\ 4000]$ GeV
m_{H_u}	$[100,\ 8000]$ GeV
m_{H_d}	$[100,\ 8000]$ GeV
$\tan\beta$	$[3,\ 45]$
$\mathrm{sgn}\,\mu$	-1
A^{de}_{33}	$[0,\ 5000]$ GeV
A^{u}_{33}	$[-9000,\ 9000]$ GeV
A^{de}_{11}/A^{de}_{33}	$[-0.00028,\ 0.00028]$
A^{de}_{22}/A^{de}_{33}	$[-0.065,\ 0.065]$
A^{u}_{22}/A^{u}_{33}	$[-0.005,\ 0.005]$
$A^{de}_{ij}/A^{de}_{33},\ i \neq j$	$[-0.5,\ 0.5]$

Parameter	Scanning Range
$m^{dl}_{ii},\ i = 1, 2, 3$	$[100,\ 7000]$ GeV
m^{dl}_{23}/m^{dl}_{33}	$[0,\ 1]$
m^{dl}_{13}/m^{dl}_{33}	$[0,\ 1]$
m^{dl}_{12}/m^{dl}_{33}	$[0,\ 1]$
$m^{ue}_{ii},\ i = 1, 2, 3$	$[100,\ 7000]$ GeV

Table : Ranges of the input SUSY parameters used in our initial scan. Several omitted soft SUSY-breaking parameters at the GUT scale (namely A^{u}_{11} as well as A^{u}_{ij} and m^{ue}_{ij} for $i \neq j$) have been set to zero.

Minimal Supersymmetric Standard Model

Superfields	Fermions	Scalars
$Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$	$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\tilde{q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$
U_R	u_R	$\tilde{u}_R$
D_R	d_R	$\tilde{d}_R$
$L = \begin{pmatrix} N \\ E_L \end{pmatrix}$	$l = \begin{pmatrix} \nu \\ e_L \end{pmatrix}$	$\tilde{l} = \begin{pmatrix} \tilde{\nu} \\ \tilde{e}_L \end{pmatrix}$
E_R	e_R	$\tilde{e}_R$
H_d	$\tilde{h}_d = \begin{pmatrix} \tilde{h}_d^0 \\ \tilde{h}_d^- \end{pmatrix}$	$h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$
H_u	$\tilde{h}_u = \begin{pmatrix} \tilde{h}_u^+ \\ \tilde{h}_u^0 \end{pmatrix}$	$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$

Yukawa unification - anatomy of the problem

Yukawa interactions in the superpotential of the minimal $SU(5)$ SUSY GUT:

$$\mathcal{W} \ni \psi_{10} \mathbf{Y^{de}} \psi_5 \overline{H}_5 + \psi_{10} \mathbf{Y^{u}} \psi_{10} H_5 \tag{0.1}$$

Here $\overline{H}_5$ and H_5 are two Higgs superfields that couple to model's matter fields. The masses of known fermions are thus given by only two independent 3×3 matrices $\mathbf{Y_{de}}$ and $\mathbf{Y_{u}}$

MSSM

the superpotential of MSSM:

$$\mathcal{W}_{MSSM} = Q\mathbf{Y^u}U_R H_u + Q\mathbf{Y^d}D_R H_d + L\mathbf{Y^e}E_R H_d + \mu H_d H_u.$$

MSSM

the superpotential of MSSM:

$$\mathcal{W}_{MSSM} = Q\mathbf{Y^u}U_R H_u + Q\mathbf{Y^d}D_R H_d + L\mathbf{Y^e}E_R H_d + \mu H_d H_u .$$

the soft supersymmetry-breaking terms:

$$\begin{aligned}\mathcal{L}^{MSSM}_{soft} &= -\tfrac{1}{2}[m_{\tilde{g}}(\tilde{G}^a)^T C\tilde{G}^a + m_{\tilde{W}}(\tilde{W}^I)^T C\tilde{W}^I + m_{\tilde{B}}\tilde{B}^T C\tilde{B} + h.c.] - m^2_{h_d} h^\dagger_d \\ &- \tilde{q}^\dagger(\mathbf{m}^2_{\tilde{q}})\tilde{q} - (\tilde{u}_R)^\dagger(\mathbf{m}^2_{\tilde{u}})(\tilde{u}_R) - (\tilde{d}_R)^\dagger(\mathbf{m}^2_{\tilde{d}})(\tilde{d}_R) - \tilde{l}^\dagger(\mathbf{m}^2_{\tilde{l}})\tilde{l} - (\tilde{e}_R)^\dagger(\mathbf{m}^2_{\tilde{e}} \\ &+ \tilde{q}\mathbf{A^u}\tilde{u}_R h_u + \tilde{q}\mathbf{A^d}\tilde{d}_R h_d + \tilde{l}\mathbf{A^e}\tilde{e}_R h_d + B\mu h_d h_u \quad + \quad \mathrm{h.c.}\end{aligned}$$

Problem's anatomy in SU(5)

In SM and MSSM the fermion masses are independent parameters and are given by 3 Yukawa matrices:

$$
\begin{aligned}
Y^u &\to m_u,\ m_c,\ m_t \\
Y^d &\to m_d,\ m_s,\ m_b \\
Y^e &\to m_e,\ m_\mu,\ m_\tau
\end{aligned}
$$

In the minimal SU(5) Grand Unified Theory the symmetry requires:

$$Y_d = Y_e,\ Y_s = Y_\mu,\ Y_b = Y_\tau$$

flavour mixing (CKM matrix can be included in) $\mathbf{Y_u}$