Possibility of dark matter detection at future e^+e^- colliders

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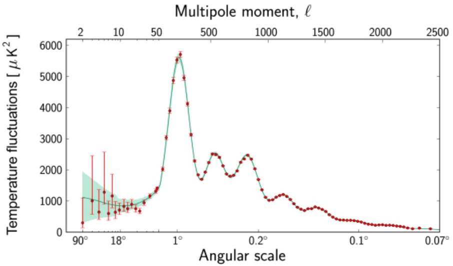
(work in progress)

Scalars 2019
Warsaw, 11-14 September 2019

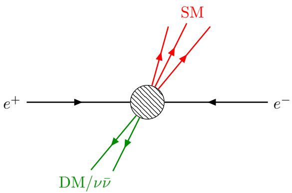
Introduction

Dark matter

- rotation curves
- CMB fluctuations $\Rightarrow$ ~~MOND~~
- gravitational lensing
- ...



source: http://sci.esa.int/planck



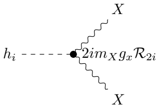
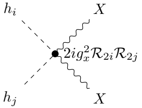
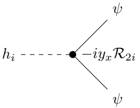
Collider search for dark matter (DM)

- Experimental approach: missing-energy analysis
- Signal more clear at e^+e^- than at hadron colliders
- Near-future plans: ILC (Japan), CLIC (Europe), FCC-ee (Europe), CEPC (China)...

Outline

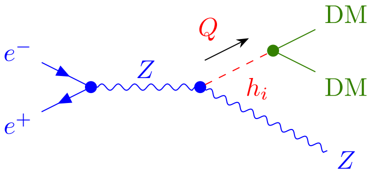
1. Simple vector-DM and fermion-DM models
2. DM production at e^+e^- colliders
3. Determination of mass and spin
4. Current experimental limits and constraints
5. Maximizing the cross section, detectability

Theoretical models

	VECTOR DM MODEL	FERMION DM MODEL
gauge group	$\mathcal{G} = \mathcal{G}_{\text{SM}} \times U(1)_X$	$\mathcal{G} = \mathcal{G}_{\text{SM}} \times Z_4$
new states ($\mathcal{G}_{\text{SM}}$-even)	gauge vector X_μ complex scalar S $\quad (q = 1)$	LH fermion χ $\quad (q = 1)$ $\psi \equiv \chi + \chi^c$ real scalar S $\quad (q = 2)$
Lagrangian	$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} - V(H,S)$	
scalar potential	$V(H,S) = -\mu_H^2 \lvert H\rvert^2 + \lambda_H \lvert H\rvert^4$ $-\mu_S^2 \lvert S\rvert^2 + \lambda_S \lvert S\rvert^4 + \boxed{\kappa \lvert H\rvert^2 \lvert S\rvert^2}$	$V(H,S) = -\mu_H^2 \lvert H\rvert^2 + \lambda_H \lvert H\rvert^4$ $-\frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \boxed{\frac{\kappa}{2} \lvert H\rvert^2 S^2}$
SSB	$H \to (\pi^+, v + i\pi^0 + h)^T/\sqrt{2}$ $S \to (v_s + \phi + i\sigma)/\sqrt{2}$	$H \to (\pi^+, v + i\pi^0 + h)^T/\sqrt{2}$ $S \to v_s + \phi$
Higgs sector mixing	$h = \cos\alpha \cdot h_1 - \sin\alpha \cdot h_2 \qquad \phi = \sin\alpha \cdot h_1 + \cos\alpha \cdot h_2$ $V(H,S) \longrightarrow \frac{1}{2} m_1^2 h_1^2 + \frac{1}{2} m_2^2 h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$	
Dark matter interactions	 	

input parameters: $\underbrace{v, m_1,}_{\text{assumed to be SM-like}}\ m_2, \sin\alpha, m_{\text{DM}}, v_s \longrightarrow \kappa \equiv \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2 v v_s}$

Considered process



$$Q^2 = s - 2E_Z\sqrt{s} + m_Z^2$$
$$\equiv m_{\text{rec}}^2$$
$$\text{DM} = X_\mu,\ \psi$$

note: in principle $\Gamma_i^{(\text{VDM})} \neq \Gamma_i^{(\text{FDM})}$

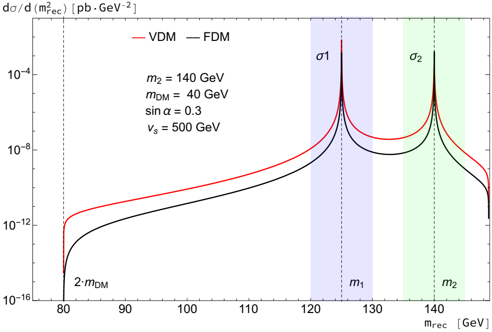
The differential cross-section:

$$\frac{d\sigma}{dm_{\text{rec}}^2} = \frac{\sigma_{\text{SM}}(m_{\text{rec}})}{\pi} \sin^2\alpha \cos^2\alpha \frac{m_{\text{rec}} \cdot (m_1^2 - m_2^2)^2}{[(m_{\text{rec}}^2 - m_1^2)^2 + (m_1\Gamma_1)^2]\,[(m_{\text{rec}}^2 - m_2^2)^2 + (m_2\Gamma_2)^2]} \cdot \frac{b\,(m_{\text{rec}}, m_{\text{DM}})}{v_s^2}$$

where

$$\sigma_{\text{SM}}(m_{\text{rec}}) \equiv \frac{g_V^2 + g_A^2}{24\pi}\left(\frac{g^2}{\cos\theta_W^2}\frac{1}{s - m_Z^2}\right)^2 \frac{\lambda^{1/2}(s, m_{\text{rec}}^2, m_Z^2)\left[12\, s\, m_Z^2 + \lambda(s, m_{\text{rec}}^2, m_Z^2)\right]}{8s^2}$$

$$b(m_{\text{rec}}, m_{\text{DM}}) \equiv \left[\frac{\mathcal{R}_{2i}}{v_s^2}\right]^{-1} \Gamma(h_i \to \text{DM}) = \frac{m_i^3}{32\pi}\sqrt{1 - \frac{4m_{\text{DM}}^2}{m_{\text{rec}}^2}} \cdot \begin{cases} 2\left[\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} - 4\left(\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}\right)^2\right] & \text{(FDM)} \\ 1 - 4\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} + 12\left(\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2}\right)^2 & \text{(VDM)} \end{cases}$$



Spin and mass of DM possible to determine

- total σ as a function of $\sqrt{s}$ (easy)
- shape fitting (not easy)

After integration

$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\mathrm{DM}}<m_1<\sqrt{s}-m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\mathrm{DM}}<m_2<\sqrt{s}-m_Z}$$

$$\sigma_1 \equiv \sigma_{\mathrm{SM}}(m_1) \cdot \cos^2\alpha \cdot \mathrm{BR}(h_1 \to \mathrm{DM}) \qquad \sigma_2 \equiv \sigma_{\mathrm{SM}}(m_2) \cdot \sin^2\alpha \cdot \mathrm{BR}(h_2 \to \mathrm{DM})$$

Current experimental limits and constraints

1. $|\sin\alpha| \lesssim 0.3$ — Robens, Stefaniak 1601.07880

experiment	channel	observed signal rate	observed mass [GeV]
ATLAS	$h \to WW \to \ell\nu\ell\nu$	$1.08^{+0.22}_{-0.20}$	–
ATLAS	$h \to ZZ \to 4\ell$	$1.44^{+0.40}_{-0.33}$	124.51 ± 0.52
ATLAS	$h \to \gamma\gamma$	$1.17^{+0.27}_{-0.27}$	125.98 ± 0.50
ATLAS	$h \to \tau\tau$	$1.42^{+0.43}_{-0.37}$	–
ATLAS	$Vh \to V(b\bar{b})$	$0.51^{+0.40}_{-0.37}$	–
CMS	$h \to WW \to \ell\nu\ell\nu$	$0.72^{+0.20}_{-0.18}$	–
CMS	$h \to ZZ \to 4\ell$	$0.93^{+0.29}_{-0.25}$	125.63 ± 0.45
CMS	$h \to \gamma\gamma$	$1.14^{+0.26}_{-0.23}$	124.70 ± 0.34
CMS	$h \to \tau\tau$	$0.78^{+0.27}_{-0.27}$	–
CMS	$Vh \to V(b\bar{b})$	$1.00^{+0.50}_{-0.50}$	–

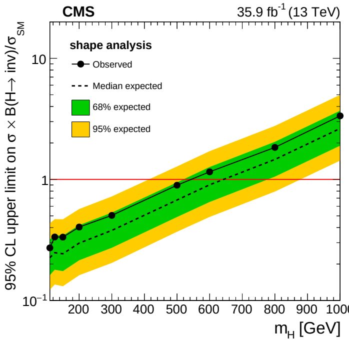
source: Robens, Stefaniak 1501.02234

2. LHC $\Rightarrow$ $\mathrm{BR}(h_1 \to \mathrm{DM}) < 19\%$ — CMS 1809.05937

3. DD experiments and ν telescopes $\Rightarrow$ limits on σ^{DD}_{SI} , σ^{DD}_{SD} — XENON1T 1805.12562, IceCube 1601.00653

4. ID experiments $\Rightarrow$ limit on $\langle\sigma v\rangle_{ann}$ — Fermi-LAT 1611.03184

5. $h^2\,\Omega_0^{DM} = 0.12 \pm 0.0012 \;\Rightarrow\;$ constraint on $\langle\sigma v\rangle_{ann}$ — Planck 1807.06209



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\,m_{\rm DM}<m_1<\sqrt{s}-m_Z} + \sigma_2 \cdot \mathbb{1}_{2\,m_{\rm DM}<m_2<\sqrt{s}-m_Z}$$

$$\sigma_1 \equiv \sigma_{\rm SM}(m_1) \cdot \cos^2\alpha \cdot {\rm BR}(h_1 \to {\rm DM})$$

$$\sigma_2 \equiv \sigma_{\rm SM}(m_2) \cdot \sin^2\alpha \cdot {\rm BR}(h_2 \to {\rm DM})$$

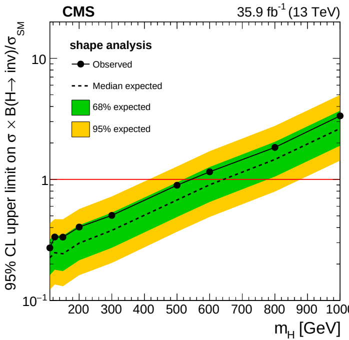
Conditions:

(1) $$\sigma_1 < 0.19\,\sigma_{\rm SM}(m_1)$$

(2) $$\log\left[\frac{\sigma_2}{\sigma_{\rm SM}(m_2)}\right] < 0.0011 \cdot \frac{m_2}{1\ {\rm GeV}} - 0.63 \qquad (\sqrt{s} = 13\ {\rm TeV})$$

For $\sin\alpha < 0.3$ condition (2) always satisfied:

$$\left.\frac{\sigma_2}{\sigma_{\rm SM}(m_2)}\right|_{\sqrt{s}=13\ {\rm TeV}} = \sin^2\alpha \cdot {\rm BR}(h_2 \to {\rm DM}) < 10^{-1} < 10\hat{\ }\left[0.0011 \cdot \frac{m_2}{1\ {\rm GeV}} - 0.63\right]$$



$$\sigma \approx \sigma_1 \cdot \mathbb{1}_{2\, m_{DM} < m_1 < \sqrt{s} - m_Z} + \sigma_2 \cdot \mathbb{1}_{2\, m_{DM} < m_2 < \sqrt{s} - m_Z}$$

$$\sigma_1 \equiv \sigma_{SM}(m_1) \cdot \cos^2\alpha \cdot \mathrm{BR}(h_1 \to \mathrm{DM})$$

$$\sigma_2 \equiv \sigma_{SM}(m_2) \cdot \sin^2\alpha \cdot \mathrm{BR}(h_2 \to \mathrm{DM})$$

Conditions:

(1) $$\boxed{\sigma_1 < 0.19\, \sigma_{SM}(m_1)}$$

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For $\sin\alpha < 0.3$ condition (2) always satisfied:

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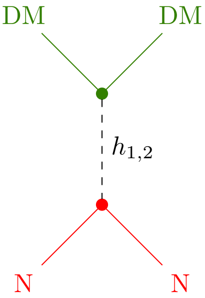
Direct detection limit

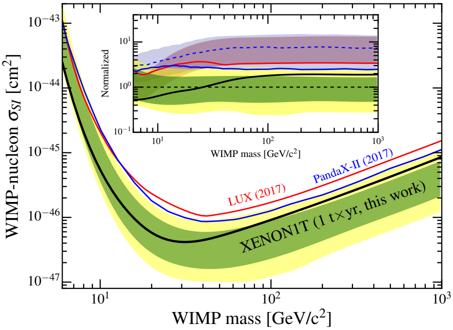
$\sigma_{SD} = 0$ (no axial couplings in our models)

$$\sigma_{SI} = \frac{\mu^2 m_{DM}^2}{\pi v_s^2} \cdot \frac{(m_1^2 - m_2^2)^2}{m_1^2\, m_2^2} \cdot \sin^2\alpha \cos^2\alpha \cdot \frac{m_N^2}{v^2} f_N^2$$

where

$$\mu = \frac{m_N m_{DM}}{m_N + m_{DM}} \approx m_N \approx 0.94 \text{ GeV}, \qquad f_N \approx 0.3$$





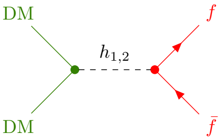
$$\frac{\sigma_{SI}}{1 \text{ cm}^2} \lesssim \frac{m_{DM}}{1 \text{ GeV}} \cdot 10^{-48.05}$$

$$\Downarrow$$

$$\boxed{\frac{\sin^2\alpha\cos^2\alpha}{v_s^2}(m_1^2 - m_2^2)^2 < \frac{m_2^4}{m_{DM}} \cdot 1.5 \cdot 10^{-6} \text{ GeV}^{-1}}$$

Relic density constraint on indirect detection

Steigman et al. 1204.3622



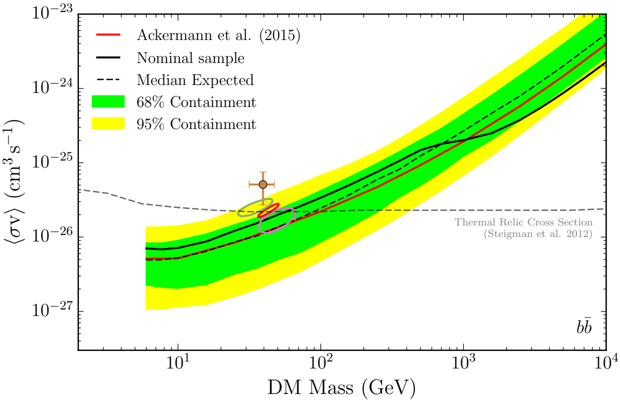
$$\langle \sigma v\rangle^{\mathrm{ID}}_{f\bar{f}} = \frac{1}{\pi}\frac{m_{\mathrm{DM}}\, m_f^2}{v^2}\frac{\sin^2\alpha\cos^2\alpha}{v_s^2}(m_1^2 - m_2^2)\times$$
$$\times \frac{(m_{\mathrm{DM}}^2 - m_f^2)^{3/2}}{[(m_1^2 - 4m_{\mathrm{DM}}^2)^2 + m_1^2\Gamma_1^2]\,[(m_2^2 - 4m_{\mathrm{DM}}^2)^2 + m_2^2\Gamma_2^2]}\times$$
$$\times \begin{cases} 1 & \text{VDM} \\ \frac{9}{8}\left(\frac{m_{\mathrm{DM}}}{T}\right)^{-1} & \text{FDM} \end{cases} + [\text{higher orders in } (m_{\mathrm{DM}}/T)^{-1}]$$

m_f^2 factor $\Rightarrow$ $b\bar{b}$ contribution dominates.

- $\langle\sigma v\rangle^{\mathrm{ID}} = \sigma_0 \cdot (m_{\mathrm{DM}}/T)^{-n} + \ldots \longrightarrow h^2\Omega_0^{\mathrm{DM}} \sim (n+1)(m_{\mathrm{DM}}/T_f)^{n+1}/\sigma_0$
- correct $h^2\Omega_0^{\mathrm{DM}} \longleftrightarrow \langle\sigma v\rangle\big|_{\mathrm{now}} = (T_0/T_f)^n \cdot \underbrace{(n+1)\cdot 1.9\cdot 10^{-9}\ \mathrm{GeV}^{-2}}_{\langle\sigma v\rangle|_{\text{freeze out}}}$
- $T_f \sim m_{\mathrm{DM}}/25$, hence

$$\boxed{\frac{\sin^2\alpha\cos^2\alpha}{v_s^2}(m_1^2 - m_2^2)^2 = = 2.1\cdot 10^{-5}\ \mathrm{GeV}^{-2}\,\frac{[(m_1^2 - 4m_{\mathrm{DM}}^2)^2 + m_1^2\Gamma_1^2]\ [(m_2^2 - 4m_{\mathrm{DM}}^2)^2 + m_2^2\Gamma_2^2]}{m_{\mathrm{DM}}(m_{\mathrm{DM}}^2 - m_b^2)^{3/2}}\cdot k}$$

($k = 1$ for VDM, $k = \frac{16 m_\psi}{9 T_f} \approx 44$ for FDM)



value of $\langle \sigma v \rangle \big|_{\text{freeze out}}$ corresponding to correct Ω_0^{DM}

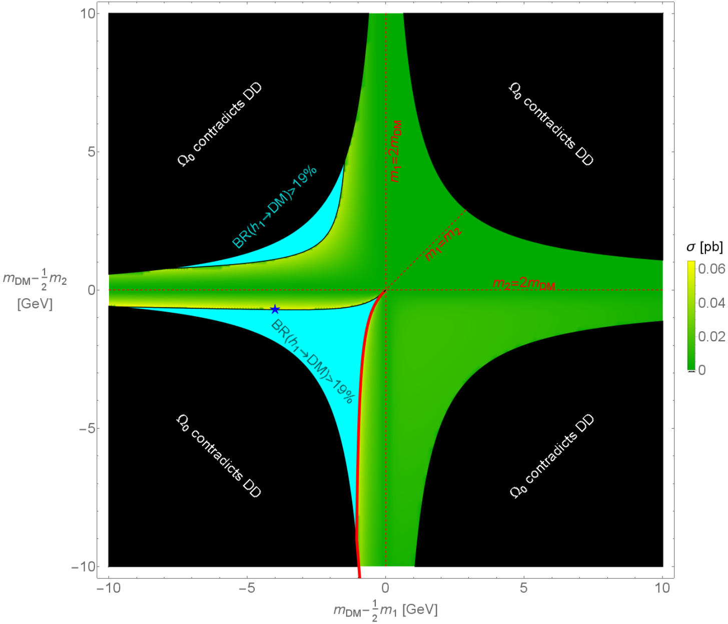
VDM: $m_X \gtrsim 30$ GeV

FDM: thermal relic cross section orders of magnitude lower than the ID limit

Results: methodology

- Constraints:
  - perturbativity: $\frac{m_{\mathrm{DM}}}{v_s} < 4\pi$
  - $|\sin\alpha| \lesssim 0.3$
  - $\mathrm{BR}(h_1 \to \mathrm{DM}) < 19\%$
  - $\Omega_0^{\mathrm{DM}}: \frac{\sin^2\alpha\cos^2\alpha}{v_s^2}(m_1^2 - m_2^2)^2 = \frac{\left[(m_1^2 - 4m_{\mathrm{DM}}^2)^2 + m_1^2\Gamma_1^2\right]\left[(m_2^2 - 4m_{\mathrm{DM}}^2)^2 + m_2^2\Gamma_2^2\right]}{m_{\mathrm{DM}}(m_{\mathrm{DM}}^2 - m_b^2)^{3/2}} \times$
    $\times\, 2.1 \cdot 10^{-5}\ \mathrm{GeV}^{-2} \cdot \begin{cases} 1 & (\mathrm{VDM}) \\ 44 & (\mathrm{FDM}) \end{cases}$
  - $\mathrm{DD}: \frac{\sin^2\alpha\cos^2\alpha}{v_s^2}(m_1^2 - m_2^2)^2 < \frac{m_2^4}{m_{\mathrm{DM}}} \cdot 1.5 \cdot 10^{-6}\ \mathrm{GeV}^{-1}$
- Free parameters: m_2, m_{DM}, $\sin\alpha$, v_s
- Value of v_s calculated from $\Omega_0^{\mathrm{DM}} \Rightarrow$ 3 parameters left
- Cross section maximized with respect to $\sin\alpha \Rightarrow$ free parameters: m_2, m_{DM}
- Low $\sqrt{s}$ gives high cross section (CEPC: $\sqrt{s} = 240$ GeV)

Results: maximal cross section for VDM



parameters of ★

$m_2 = 118.4$ GeV

$m_{DM} = 58.5$ GeV

$\sin\alpha = 0.30$

$v_s = 561$ GeV

$\Gamma_1 = 7.4 \cdot 10^{-3}$ GeV

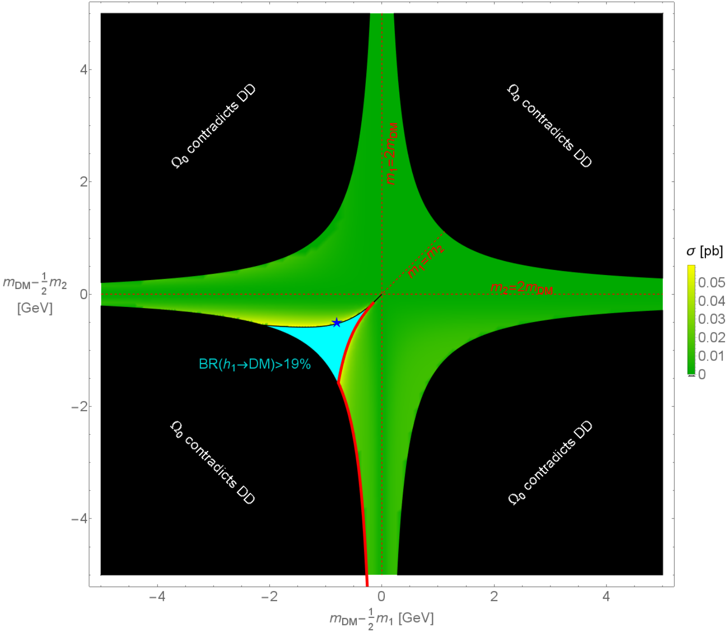
$\Gamma_2 = 6.4 \cdot 10^{-3}$ GeV

BR($h_1 \to$ DM) = 18%

BR($h_2 \to$ DM) = 92%

$\boxed{\sigma = 61 \text{ fb}}$

Results: maximal cross section for FDM



parameters of $\star$

$m_2 = 124.4$ GeV

$m_{\mathrm{DM}} = 61.7$ GeV

$\sin\alpha = 0.30$

$v_s = 57$ GeV

$\Gamma_1 = 7.1 \cdot 10^{-3}$ GeV

$\Gamma_2 = 6.5 \cdot 10^{-3}$ GeV

$\mathrm{BR}(h_1 \to \mathrm{DM}) = 15\%$

$\mathrm{BR}(h_2 \to \mathrm{DM}) = 92\%$

$\boxed{\sigma = 51 \text{ fb}}$

Results: detectability

- Number of events:

$$\# = \sigma \times \mathcal{L}_{\text{tot}} \qquad \mathcal{L}_{\text{tot}} \sim 5.6\ \text{ab}^{-1}\ \text{(7-years period)}$$

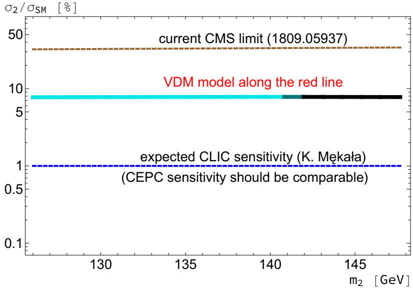
$\star \Rightarrow$ VDM: $3.4 \cdot 10^5$ events

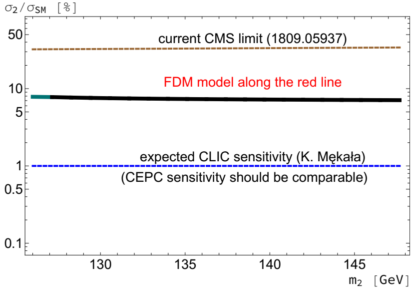
FDM: $2.9 \cdot 10^5$ events

expected SM background ($e^+e^- \to Z\nu\bar{\nu}$): $2.8 \cdot 10^6$ events

$\Rightarrow$ [signal] $\sim 10\% \times$ [background] (before cuts!)

- σ_1: BR($h_1 \to$ DM) detectable at the level of 0.3% (current CMS limit: 19%)
- σ_2: $\sin^2\alpha \cdot$ BR($h_2 \to$ DM)





Summary

Optimistic part

- Current experimental limits still allow DM to be detectable
- Signal-to-background ratio can be $\sim 10\%$
- Mass and spin in principle can be determined (but possibly not easily)

Pessimistic part

- If parameters far from optimal, DM hard (if possible) to be detected

Conclusion

- Let's build an e^+e^- collider and check!

BACKUP SLIDES

Vector DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times U(1)_X$.
- Vector X_μ ($U(1)_X$ g. f.) and complex scalar S introduced, $U(1) : S \to -S$
- Dark particle: X_μ
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM} - V(H,S)$$

$$\mathcal{L}_{\rm DM} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + (\mathcal{D}^\mu S)^\dagger(\mathcal{D}_\mu S)$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \underbrace{\kappa|H|^2|S|^2}_{\text{Higgs portal coupling}}$$

- SSB $\quad H \to (\pi^+, v + i\pi^0 + h)^T/\sqrt{2} \qquad S \to (v_s + \phi + i\sigma)/\sqrt{2}$
- We introduce h_1, h_2:

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \qquad \begin{aligned} m_1^2 &= \lambda_H v^2(1+\sec 2\alpha) + \lambda_S v_s^2(1-\sec 2\alpha) \\ m_2^2 &= \lambda_H v^2(1-\sec 2\alpha) + \lambda_S v_s^2(1+\sec 2\alpha) \end{aligned}$$

$$\mathcal{L}_{\rm DM} \longrightarrow -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi + \frac{g_x^2 v_s^2}{2}X^\mu X_\mu + \frac{g_x^2}{2}X^\mu X_\mu\phi^2 + g_x^2 v_s X^\mu X_\mu \phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + \text{const} + \mathcal{O}\left([\text{field}]^3\right)$$

Fermion DM model

- Gauge group: $\mathcal{G} = \underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{Standard Model gauge group}} \times \mathbb{Z}_4$
- Left-handed fermion χ and real scalar S introduced, $\mathbb{Z}_4 : \chi \to i\chi\,,\ S \to -S$
- Dark particle: Majorana fermion $\psi \equiv \chi + \chi^c$
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{DM}} - V(H,S)$$

$$\mathcal{L}_{\mathrm{DM}} = i\bar{\chi}\partial\!\!\!/\chi + \frac{1}{2}\partial^\mu S\,\partial_\mu S - \frac{y_x}{2}(\bar{\chi}^c\chi + \bar{\chi}\chi^c)S$$

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \underbrace{\frac{\kappa}{2}|H|^2 S^2}_{\text{Higgs portal coupling}}$$

- SSB $\quad H \to (\pi^+, v + h + i\pi^0)^T/\sqrt{2} \qquad S \to v_s + \phi$
- We introduce $h_1,\ h_2$

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$m_1^2 = \lambda_H v^2(1 + \sec 2\alpha) + \lambda_S v_s^2(1 - \sec 2\alpha)$$
$$m_2^2 = \lambda_H v^2(1 - \sec 2\alpha) + \lambda_S v_s^2(1 + \sec 2\alpha)$$

$$\mathcal{L}_{\mathrm{DM}} \longrightarrow \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{y_x v_s}{2}\bar{\psi}\psi - \frac{y_x}{2}\bar{\psi}\psi\phi$$

$$V(H,S) \longrightarrow \frac{1}{2}m_1^2 h_1^2 + \frac{1}{2}m_2^2 h_2^2 + \mathrm{const} + \mathcal{O}\left([\mathrm{field}]^3\right)$$

Input parameters

$$V(H,S) = -\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4 + \frac{\kappa}{2}|H|^2S^2$$

- Input parameters: $\underbrace{v,\ m_1,}_{\text{assumed to be SM-like}}\ m_2,\ \sin\alpha,\ m_{\mathrm{DM}},\ v_s$
- Other parameters of the models in terms of the input parameters:

$$g_x = \frac{m_X}{v_s} \quad (\mathrm{VDM}) \qquad y_x = \frac{m_X}{v_s} \quad (\mathrm{FDM})$$

$$\lambda_H = \frac{m_1^2\cos^2\alpha + m_2^2\sin^2\alpha}{2v^2} \qquad \lambda_S = \frac{m_1^2\sin^2\alpha + m_2^2\cos^2\alpha}{2v_s^2}$$

$$\mu_H^2 = \frac{1}{2}m_1^2\cos^2\alpha + \frac{1}{2}m_2^2\sin^2\alpha + \frac{1}{4}\frac{v_s}{v}(m_1^2 - m_2^2)\sin 2\alpha$$

$$\mu_S^2 = \frac{1}{2}m_1^2\sin^2\alpha + \frac{1}{2}m_2^2\cos^2\alpha + \frac{1}{4}\frac{v}{v_s}(m_1^2 - m_2^2)\sin 2\alpha$$

$$\kappa = \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}$$

- $m_2 = m_1 \Rightarrow \kappa = 0 \Rightarrow$ no Higgs portal $\Rightarrow$ DM completely decoupled
- Tree-level stability condition always satisfied

$$\kappa > -2\sqrt{\lambda_H\lambda_S} \Leftrightarrow \frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s} > -\sqrt{\left[\frac{(m_1^2 - m_2^2)\sin 2\alpha}{2vv_s}\right]^2 + \frac{m_1^2m_2^2}{v^2v_s^2}}$$

- DM annihilation rate in the Sun: Γ_A
- Capture rate: C_C
- Change of ammount of DM in the Sun

$$\dot{N} = C_C - C_A N^2$$

where $C_A \equiv 2 \cdot \Gamma_A / N^2$

- Solution

$$N(t) = \sqrt{C_C / C_A} \cdot \tanh(t \sqrt{C_C C_A})$$

- For large t

$$N(t) \to \sqrt{C_C / C_A} \Rightarrow \Gamma_A = \frac{1}{2} C_C$$

- C_C expressible in terms of DM-nucleon cross section