# RG IMPROVEMENT OF MULTI-FIELD EFFECTIVE POTENTIALS

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*Based on JHEP 1803 (2018) 014, in collaboration with*
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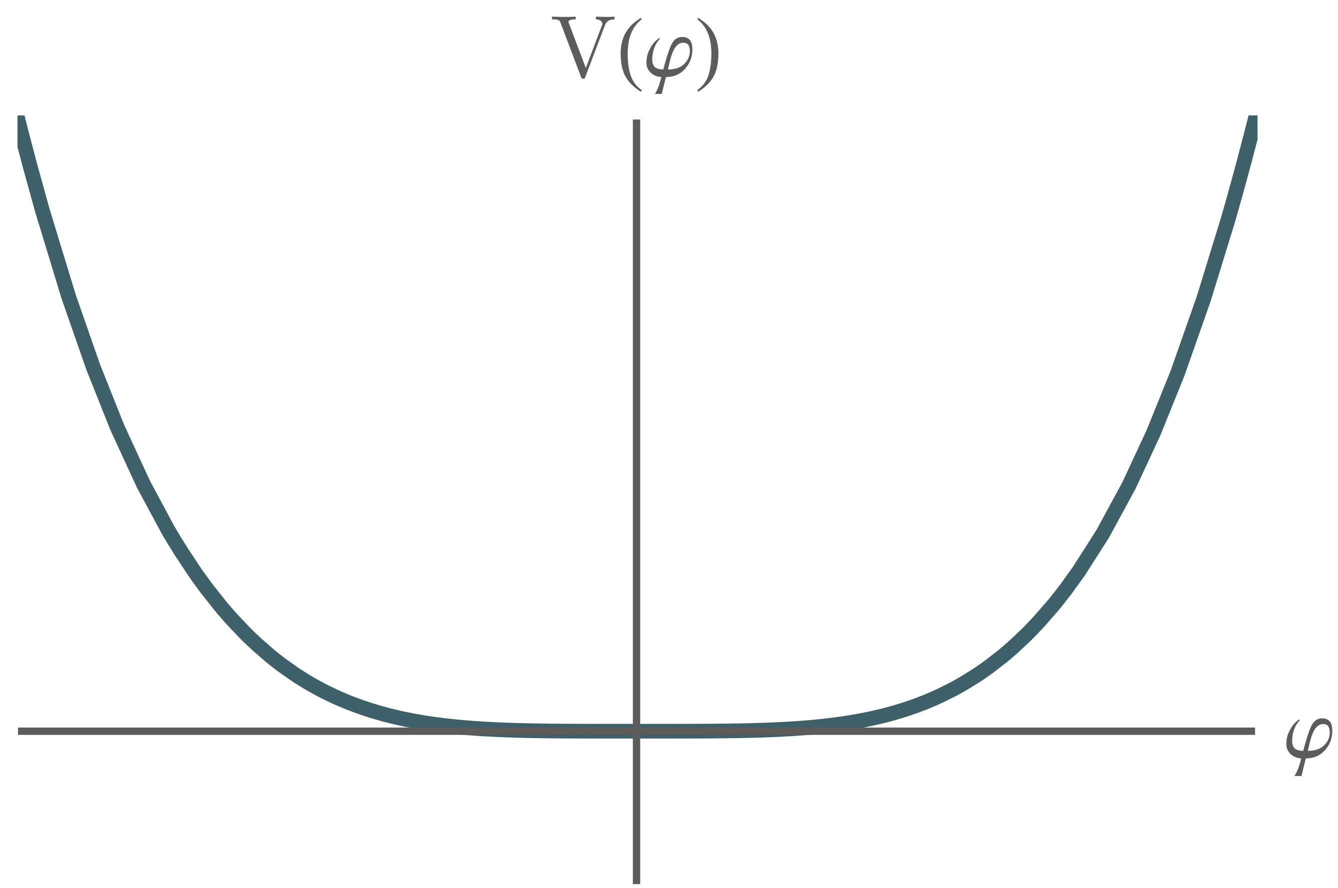
# INTRODUCTION

# WHY EFFECTIVE POTENTIAL?

- To determine the vacuum structure
- To asses stability of the vacuum state
- To determine loop-corrected couplings and masses




*[S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 18888]*

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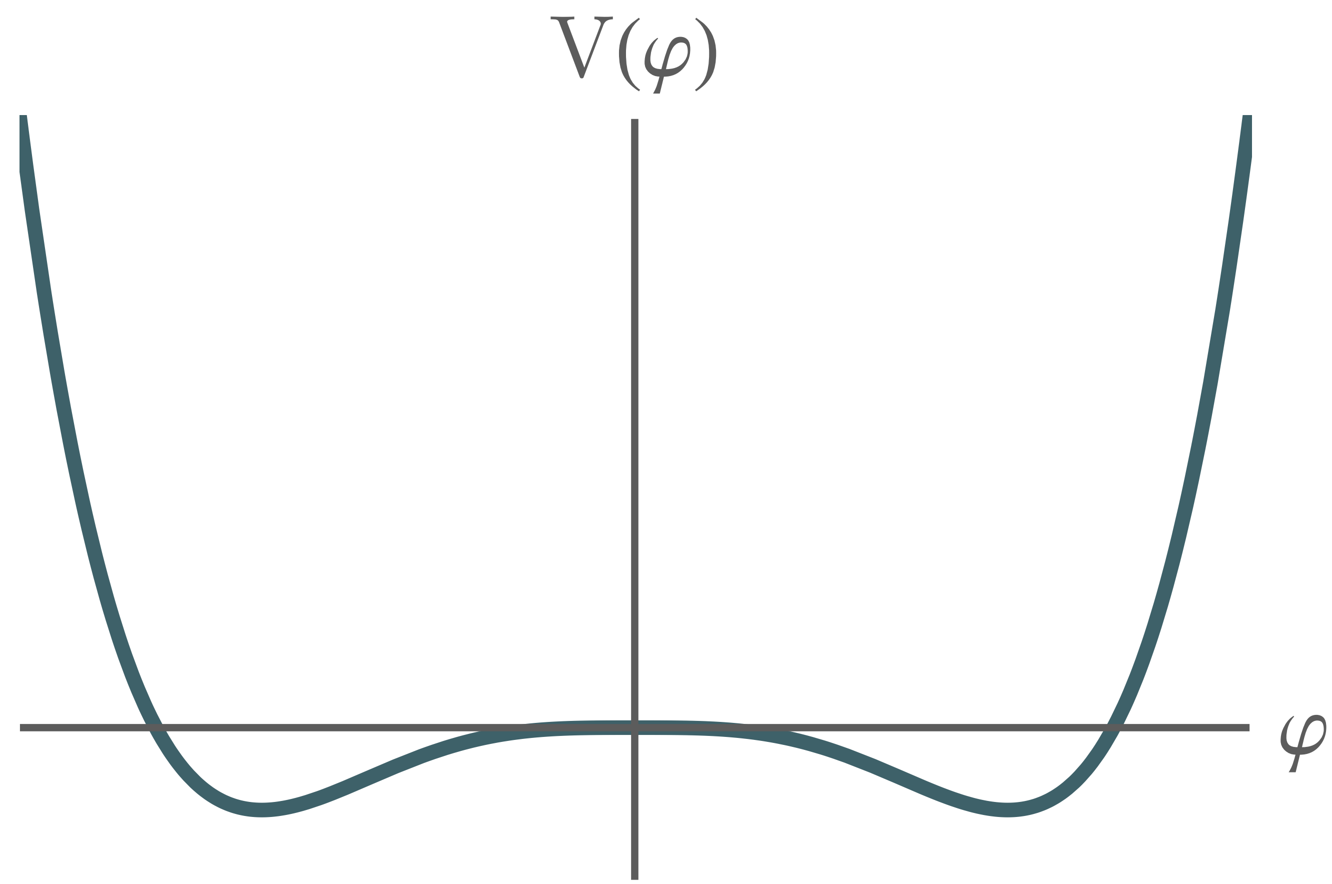



*[S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 18888]*

# WHY RGE?

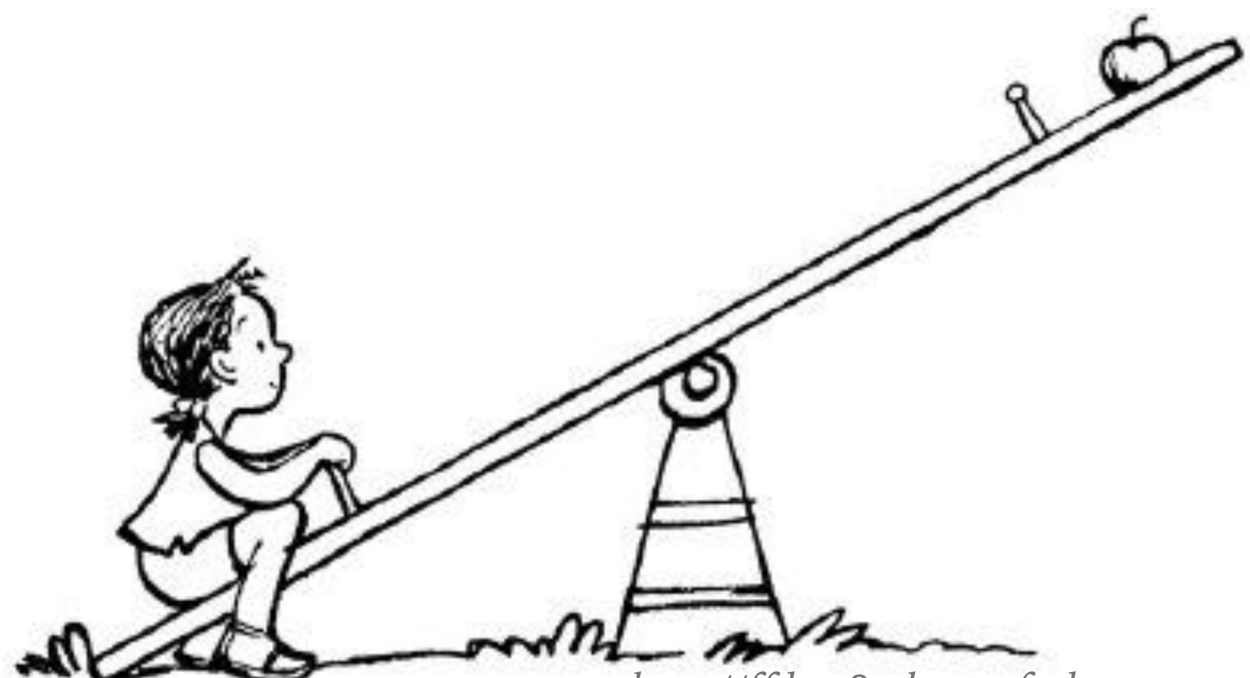
Loop expansion parameters: $\lambda, \ \lambda \log \frac{\phi^2}{\mu^2}$

RGE

Scale independence of the effective potential

RG improved potential expansion parameter: $\bar{\lambda}\left(\log \frac{\phi^2}{\mu^2}\right)$

# WHY IS THIS PROBLEMATIC?



http://ffden-2.phys.uaf.edu

*[See also: M.B. Einhorn, D.R.T. Jones, Nucl. Phys. B 230 (1984) 261, C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl. Phys. B 395 (1993) 17, C. Ford, Phys. Rev. D 50 (1994) 7531, C. Ford and C. Wiesendanger, Phys. Lett. B 398 (1997) 342, M. Bando, T. Kugo, N. Maekawa and H. Nakano, Prog Theor Phys (1993) 90, J.A. Casas, V. Di Clemente, M. Quirós, Nucl.Phys. B553 (1999) 511]*

# THE METHOD

# RENORMALISATION GROUP EQUATION

$$\mu \frac{\mathrm{d}V}{\mathrm{d}\mu}(\mu;\lambda,\phi) = \left(\mu\frac{\partial}{\partial\mu} + \sum_{i=1}^{N_\lambda}\beta_i\frac{\partial}{\partial\lambda_i} - \frac{1}{2}\sum_{a=1}^{N_\phi}\gamma_a\phi_a\frac{\partial}{\partial\phi_a}\right)V(\mu;\lambda,\phi) = 0$$

# METHOD OF CHARACTERISTICS

$$\mu \frac{\mathrm{d}V}{\mathrm{d}\mu}(\mu;\lambda,\phi) = \left(\mu\frac{\partial}{\partial\mu} + \sum_{i=1}^{N_\lambda}\beta_i\frac{\partial}{\partial\lambda_i} - \frac{1}{2}\sum_{a=1}^{N_\phi}\gamma_a\phi_a\frac{\partial}{\partial\phi_a}\right)V(\mu;\lambda,\phi) = 0$$

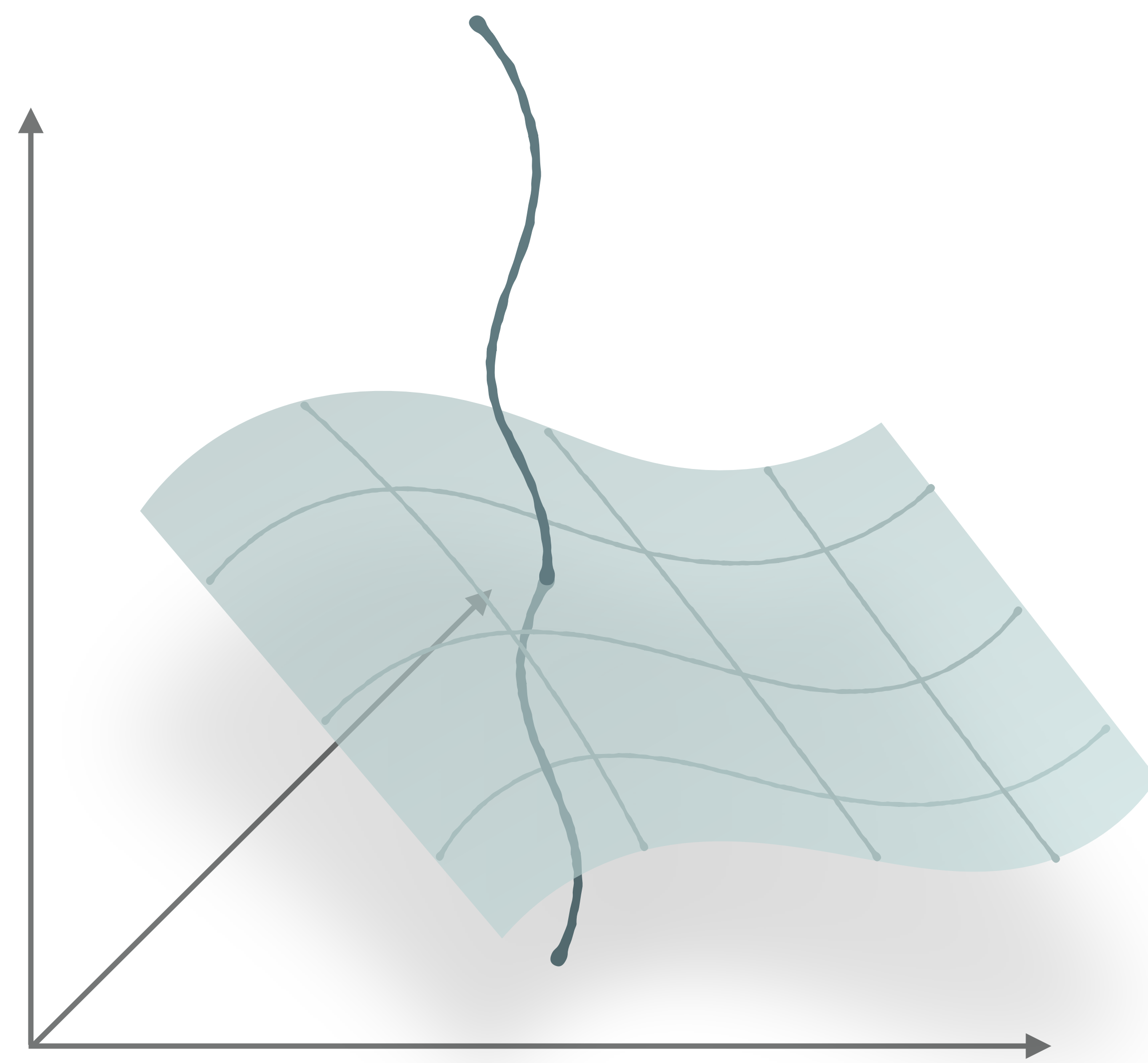
Characteristic curves

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\phi}_a(t) = -\frac{1}{2}\gamma_a(\bar{\lambda})\bar{\phi}_a(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) = 0$$



# SINGLE-FIELD MODEL

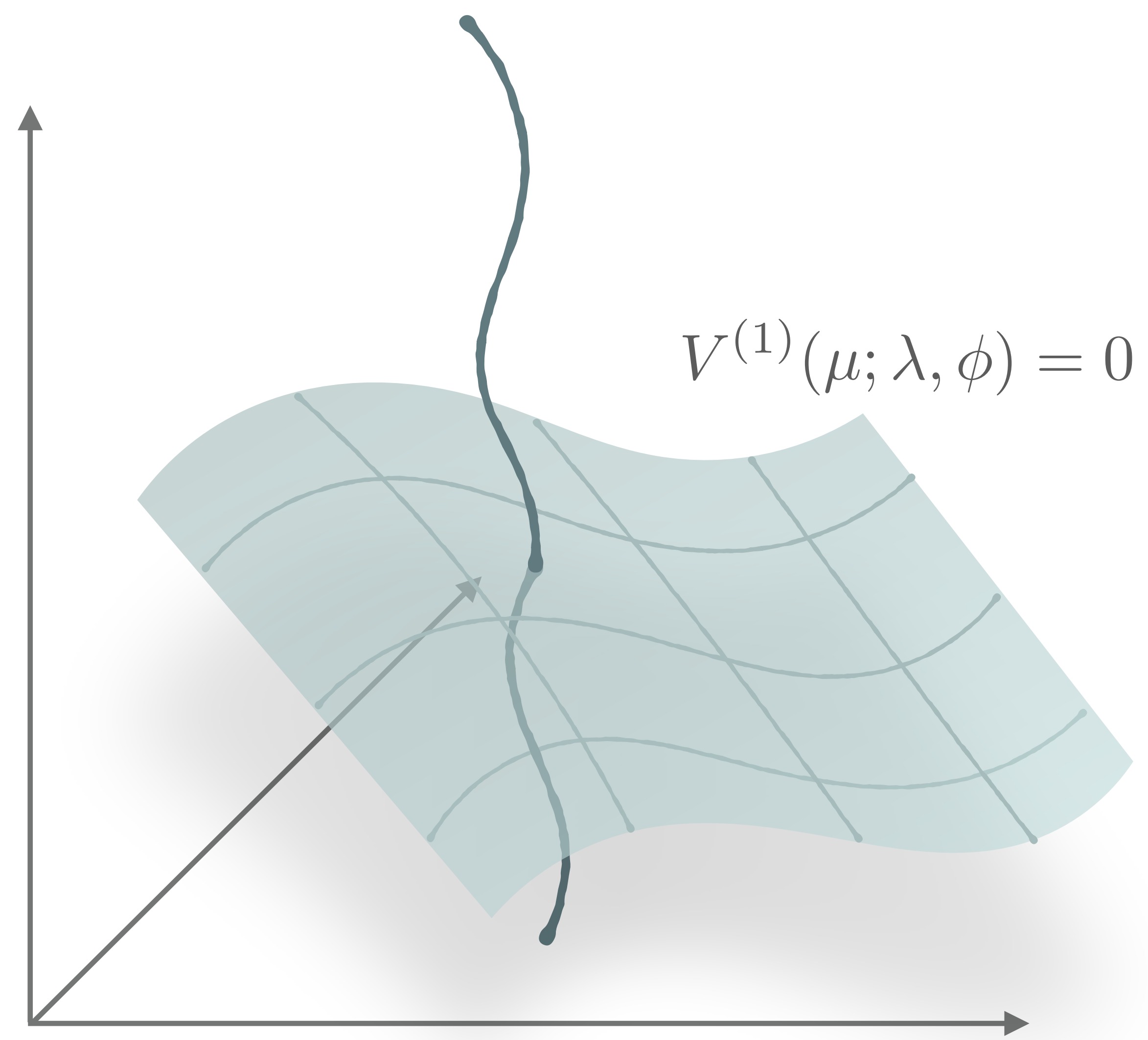
$$V_1(\mu;\lambda,\phi) = V^{(0)} + V^{(1)} = \frac{1}{4}\lambda\phi^4 + \frac{9\hbar\lambda^2\phi^4}{64\pi^2}\left[\log\frac{3\lambda\phi^2}{\mu^2} - \frac{3}{2}\right]$$

To compute $V(\mu,\lambda,\phi)$

$t_*$ Run to the tree-level surface

Evaluate

$$V(\mu,\lambda,\phi) = V^{(0)}(\bar{\lambda}(t_*,\lambda),\phi)$$



# MULTI-FIELD MODELS

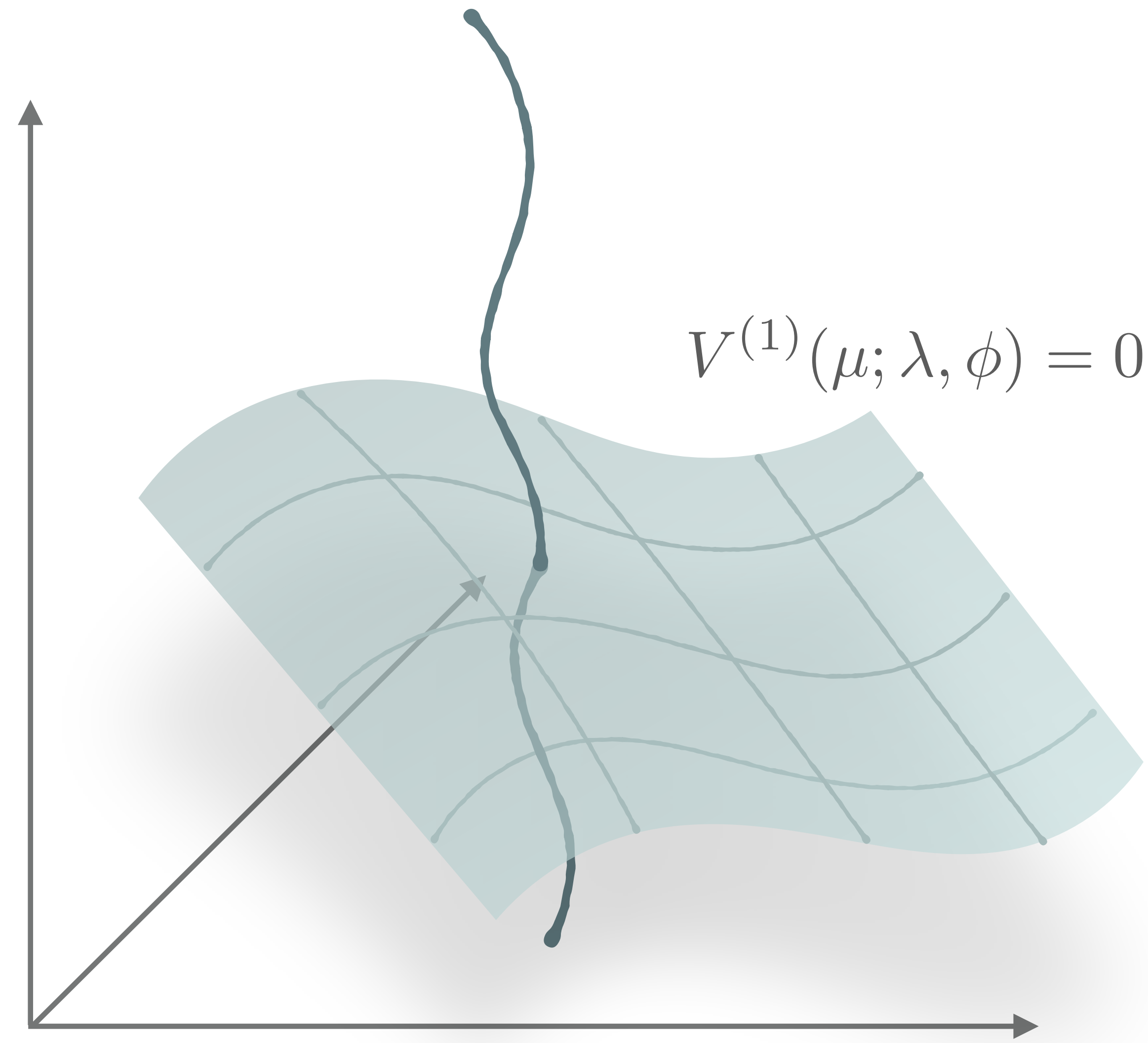
$$V^{(1)}(\mu,\lambda,\phi) = \frac{1}{64\pi^2}\sum_a n_a m_a^4(\lambda,\phi)\left[\log\frac{m_a^2(\lambda,\phi)}{\mu^2} - \chi_a\right]$$

To compute $V(\mu,\lambda,\phi)$

$t_*$ Run to the tree-level surface

Evaluate

$$V(\mu,\lambda,\phi) = V^{(0)}(\bar{\lambda}(t_*),\bar{\phi}(t_*))$$



# MULTI-FIELD MODELS

$$V^{(1)}(\mu,\lambda,\phi) = \frac{1}{64\pi^2} \ldots \left[\log\frac{m_a^2(\lambda,\phi)}{\mu^2} - \chi_a\right]$$

*M. Sher, Electroweak Higgs potentials and vacuum stability*

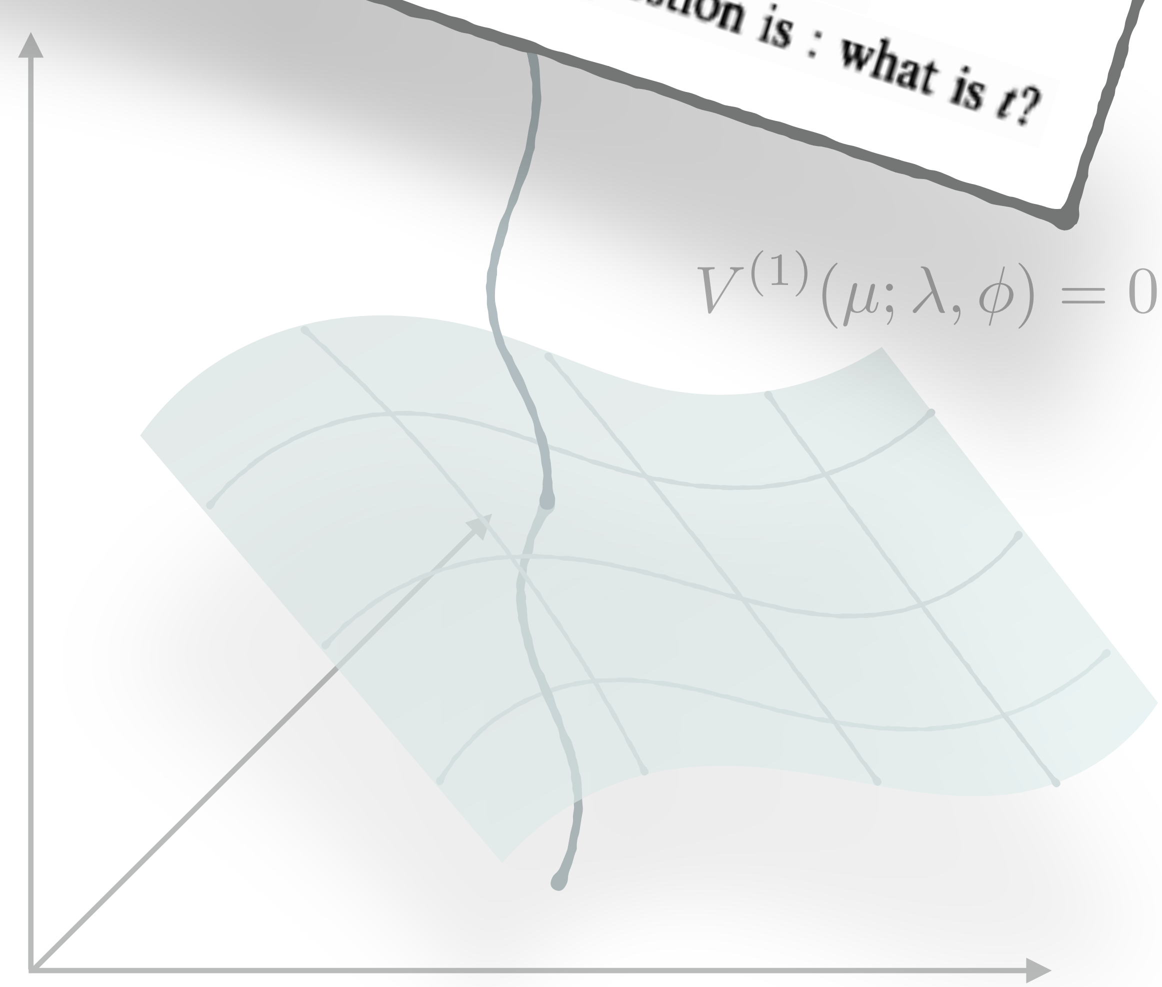
The form of this solution is not surprising; the only question is : what is *t*?

To compute $V(\mu,\lambda,\phi)$

$t_*$ Run to the tree-level surface

Evaluate

$$V(\mu,\lambda,\phi) = V^{(0)}(\bar{\lambda}(t_*),\bar{\phi}(t_*))$$



# MULTI-FIELD MODELS

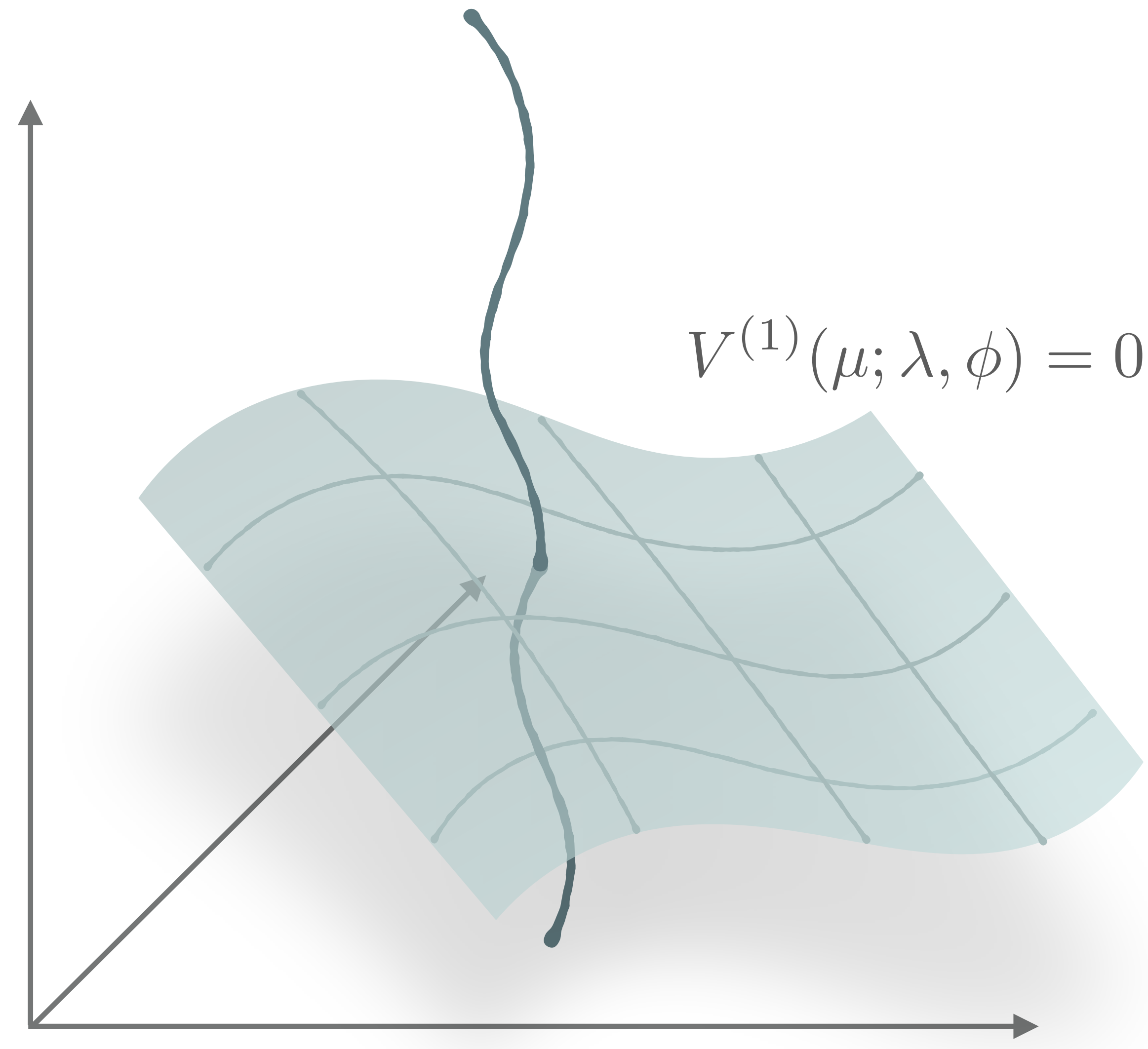
$$V(\mu,\lambda,\phi) = V^{(0)}(\bar\lambda(t_*),\bar\phi(t_*))$$

What is $t_*$?

$$V^{(1)}(\bar\mu(t_*);\bar\lambda(t_*),\bar\phi(t_*)) = 0$$

To leading order in $\hbar$

$$t_*^{(0)} = \frac{V^{(1)}(\mu,\lambda,\phi)}{2\mathbb{B}(\lambda,\phi)}$$



# VACUUM STABILITY

$$\boxed{\lim_{\phi\to\infty} V(\phi) = \,?}$$

One-loop potential unsuitable for this issue $\rightsquigarrow$ need of improvement

$$\boxed{V(\mu,\lambda,\phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))}$$

Enough to consider tree-level conditions evaluated at large scale.

# SUMMARY 1

- RG improved effective potential needed in multi-field models
- RG improvement by running to the hypersurface where (one-)loop corrections vanish
- The RG scale given implicitly (can be computed numerically) or approximately
- Applicable to study vacuum stability

# APPLICATION: SU(2)CSM

[L.Chataignier, T.Prokopec, M.G.Schmidt, B.Świeżewska, in preparation]

# SU(2)CSM

cSM:

$SU(2)_L \times U(1)_Y$

$\Phi$

Hidden sector:

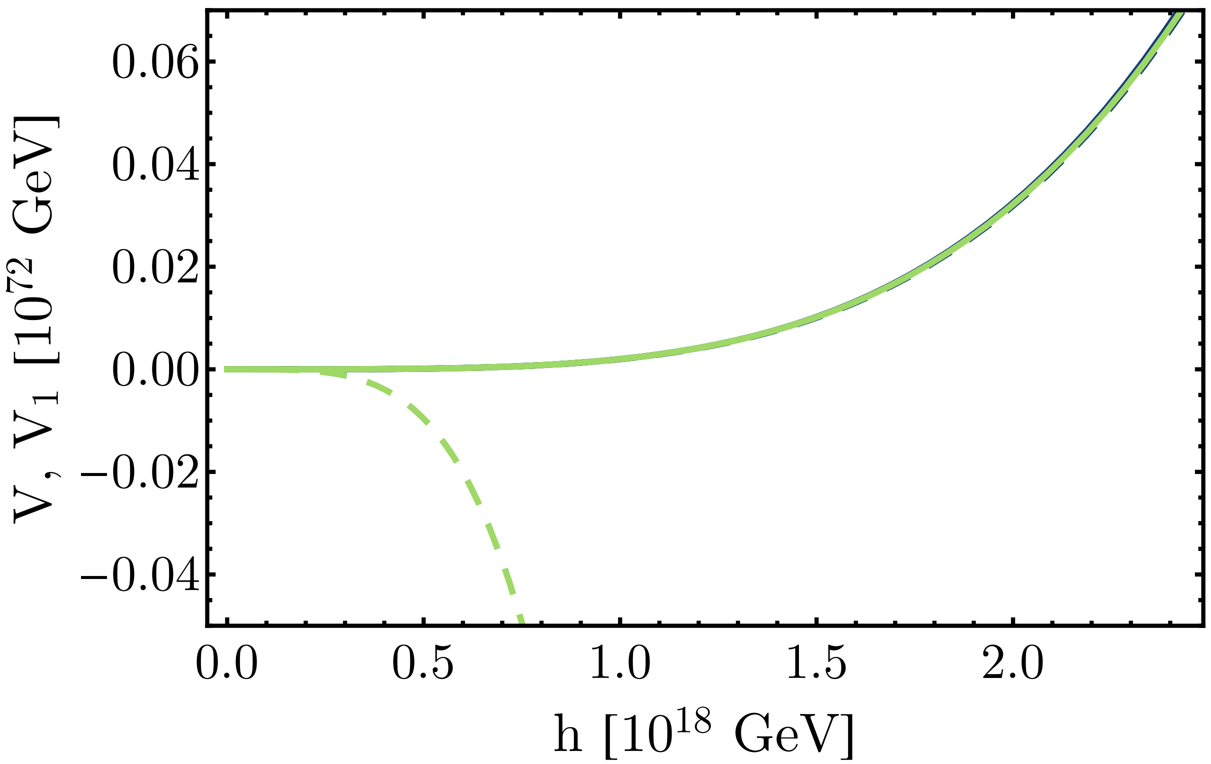
$SU(2)_X$

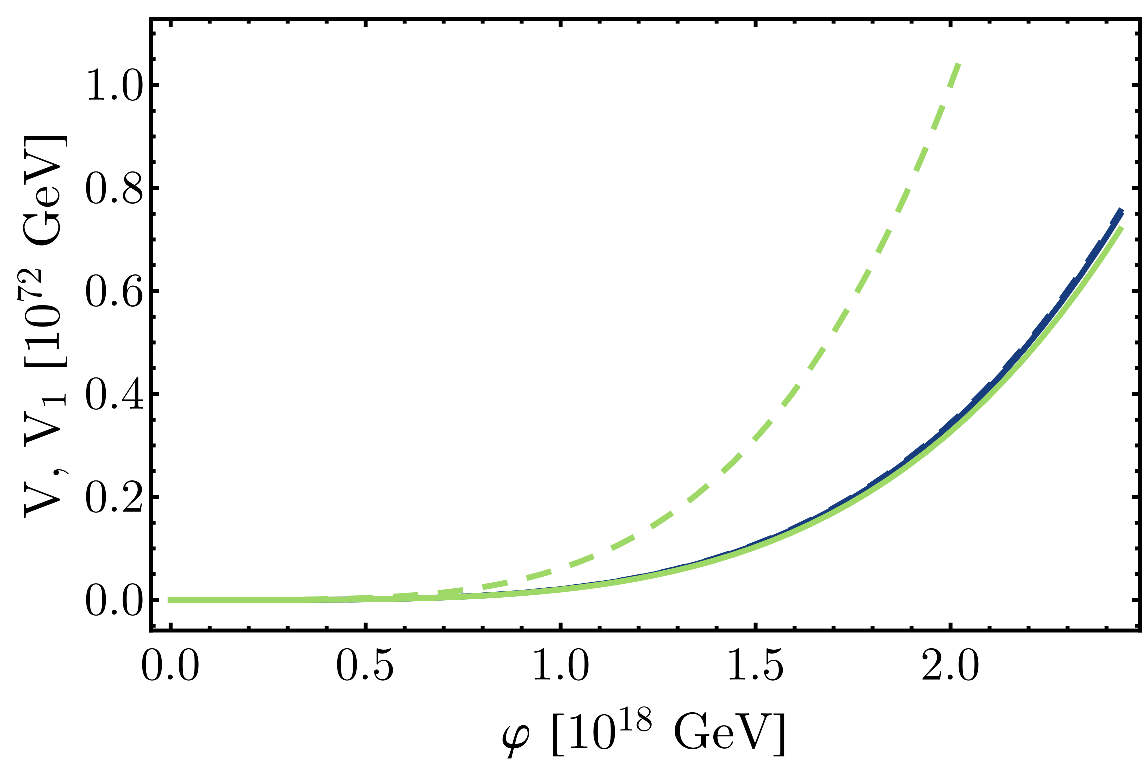
$\Psi$

$$V^{(0)}(\Phi, \Psi) = \lambda_1 \left(\Phi^\dagger \Phi\right)^2 + \lambda_2 \left(\Phi^\dagger \Phi\right) \left(\Psi^\dagger \Psi\right) + \lambda_3 \left(\Psi^\dagger \Psi\right)^2 ,$$

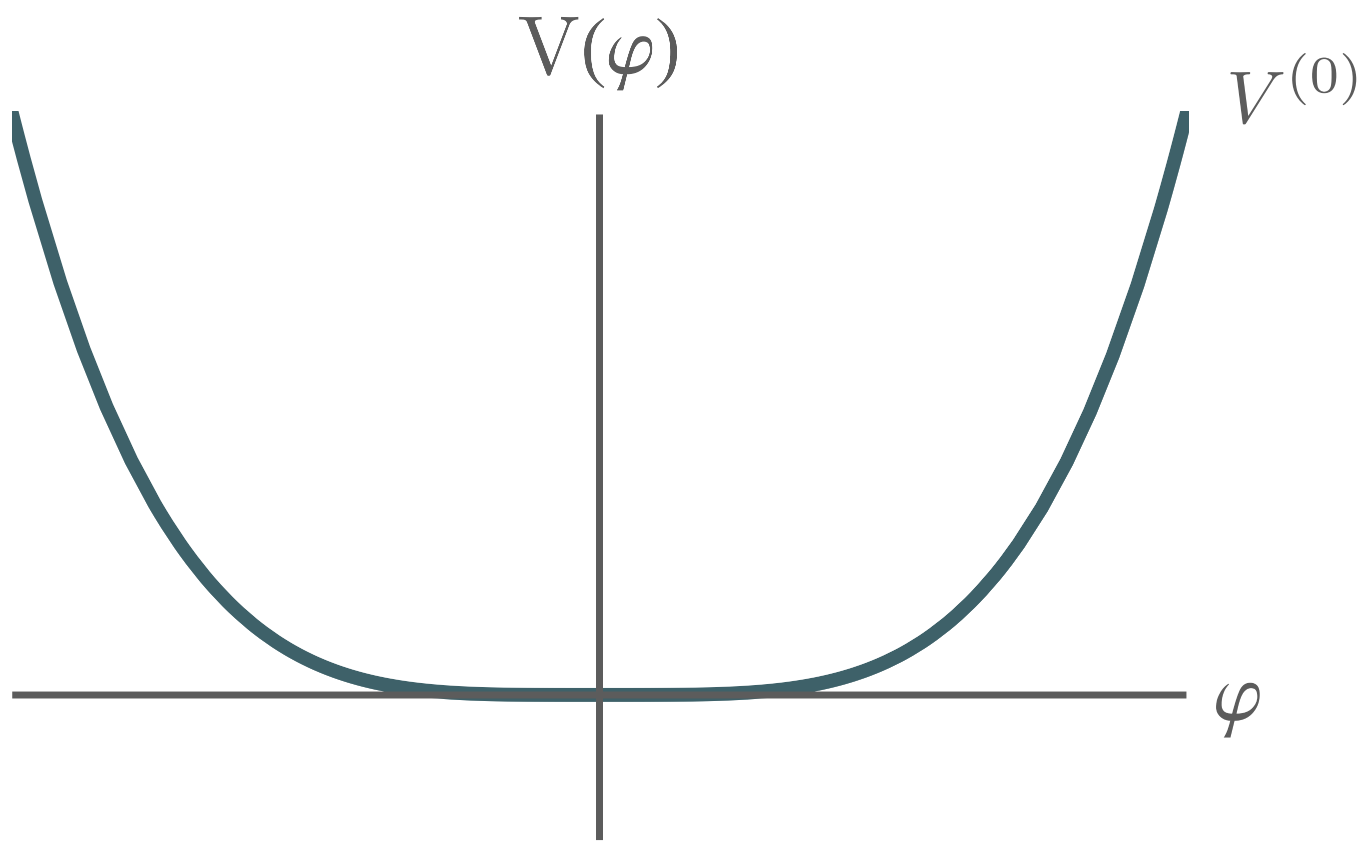
*[See also: T.Hambye, A.Strumia, PRD88 (2013) 055022, C.D.Carone, R.Ramos, PRD88 (2013) 055020, V.V.Khoze, C.McCabe, G.Ro, JHEP 08 (2014) 026]*

# VACUUM STABILITY IN SU(2)CSM



# RADIATIVE SYMMETRY BREAKING IN SU(2)CSM



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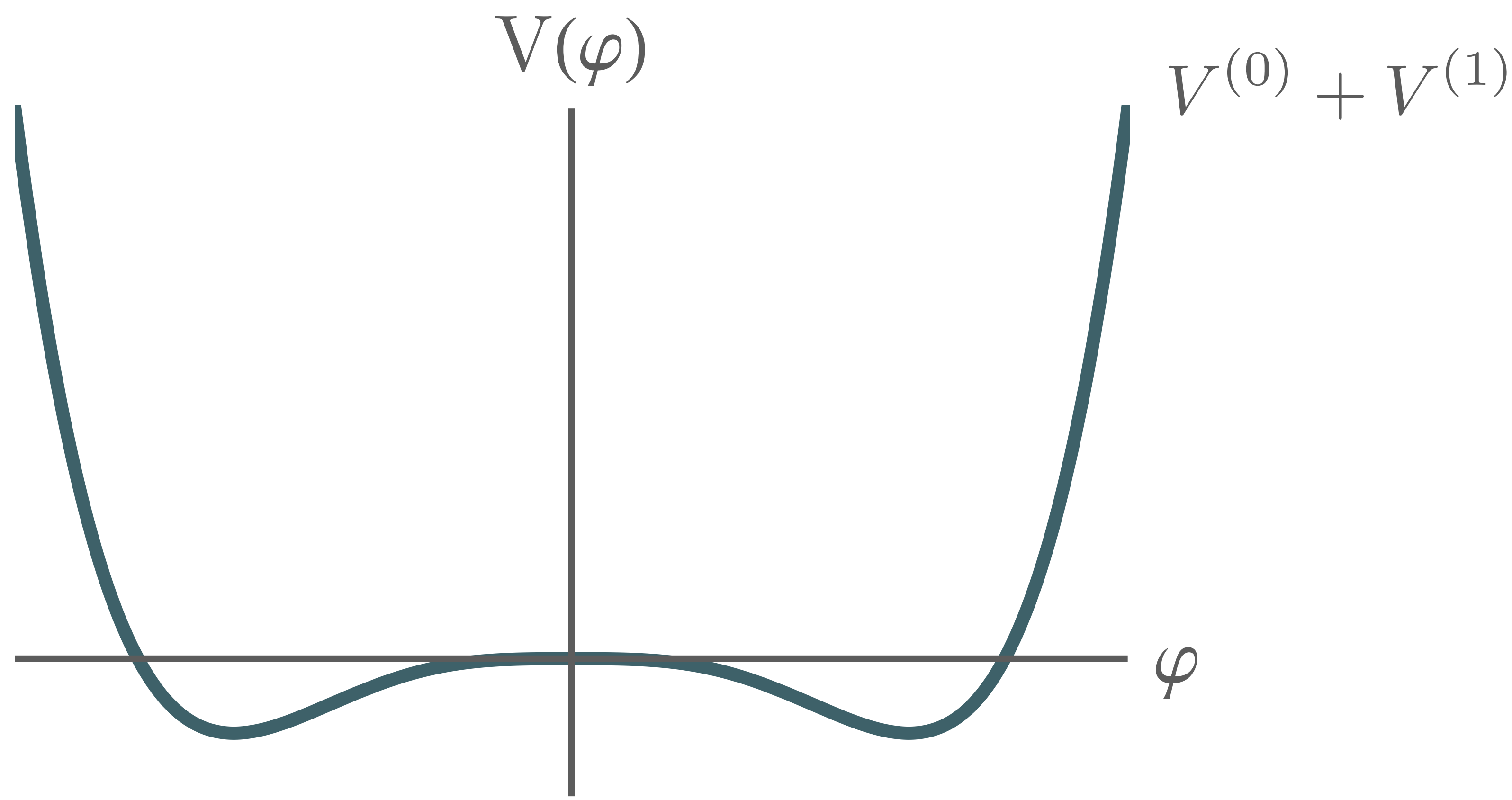

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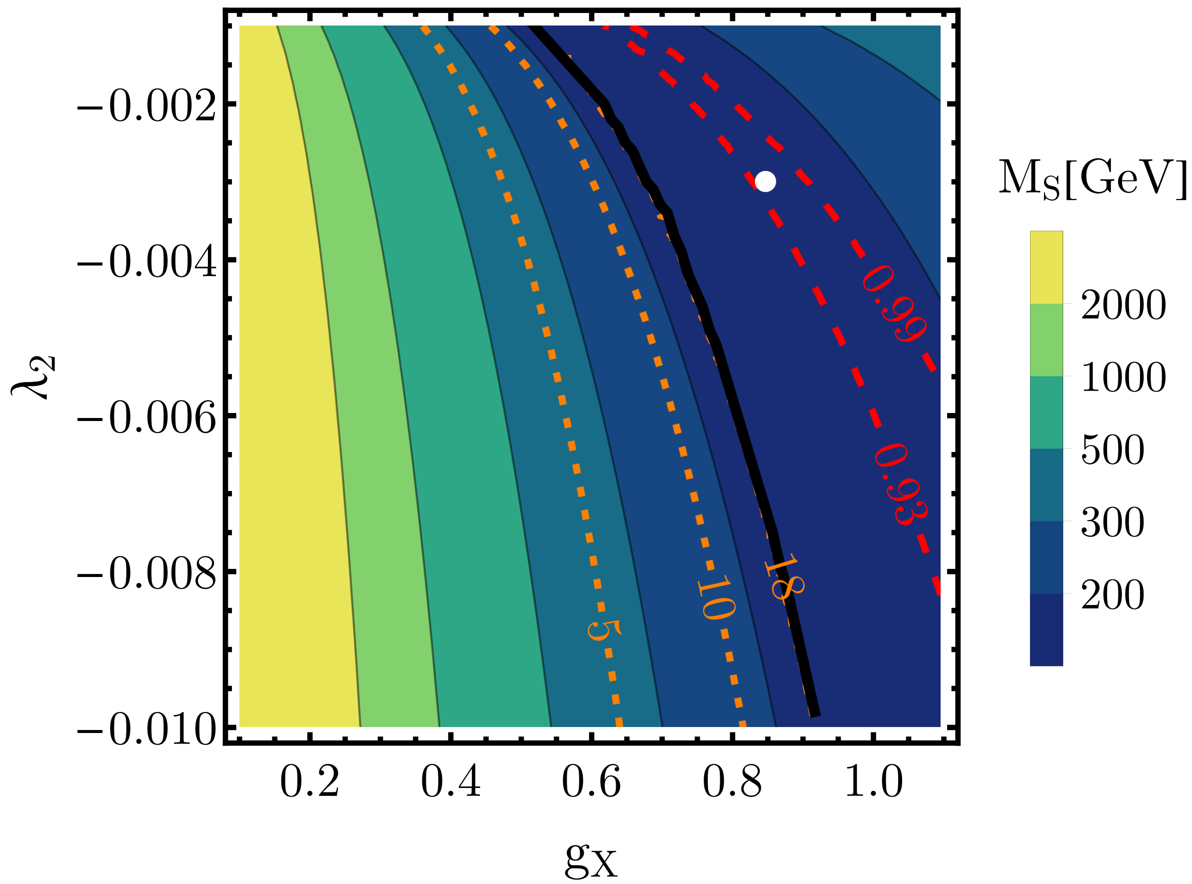
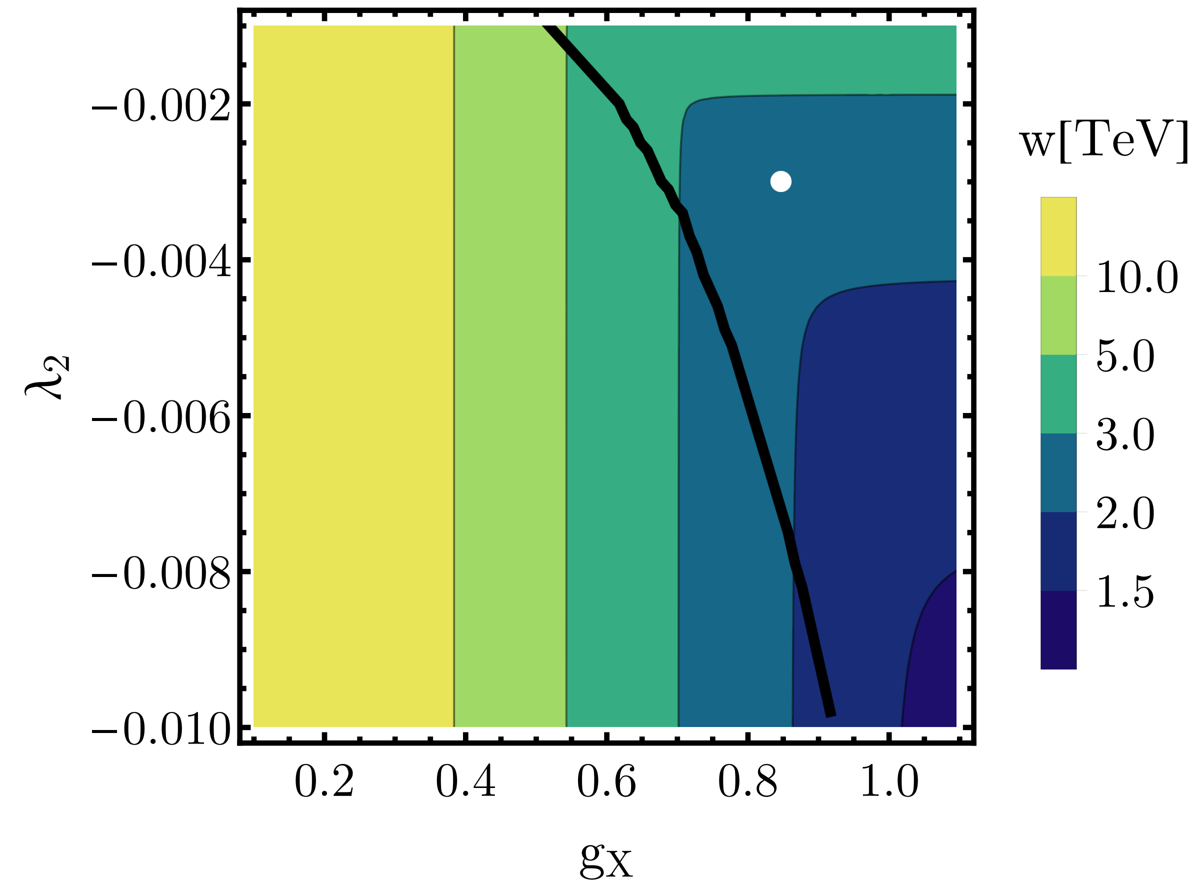
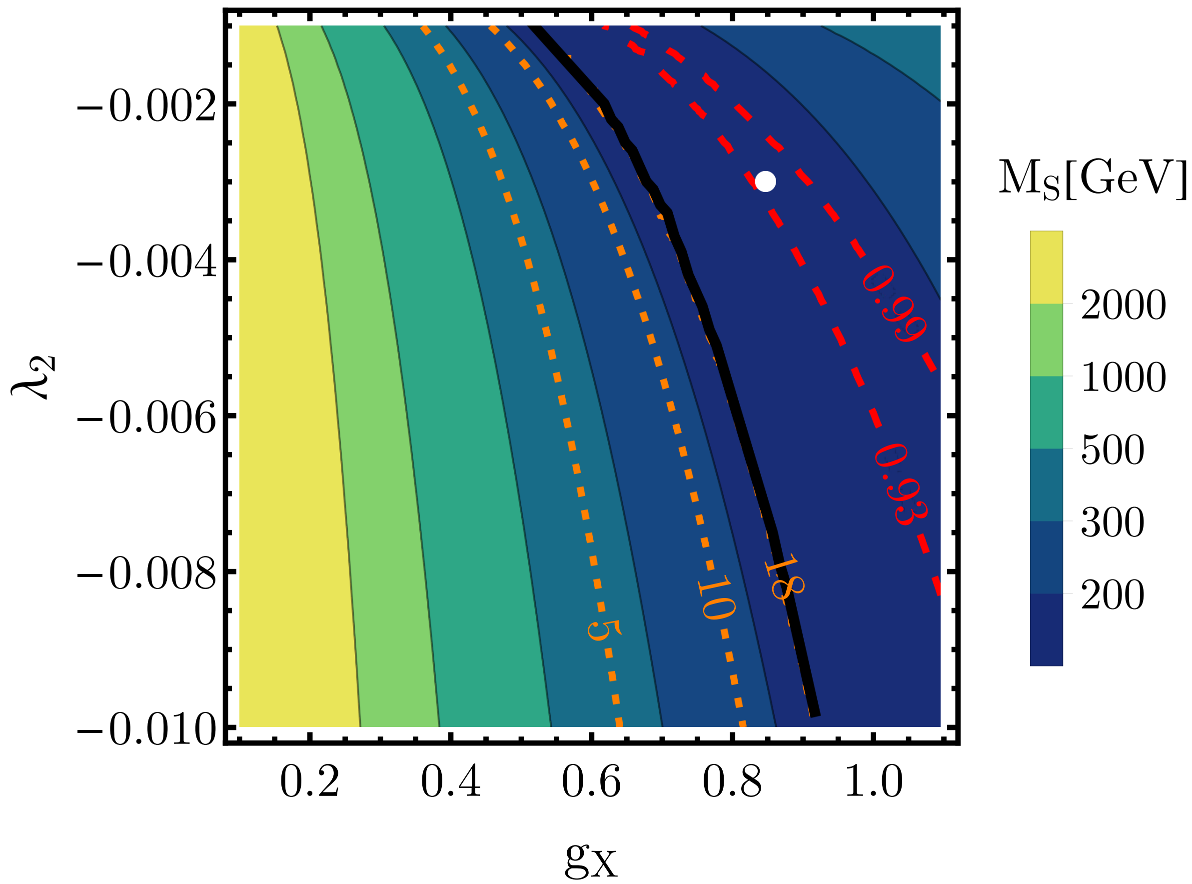
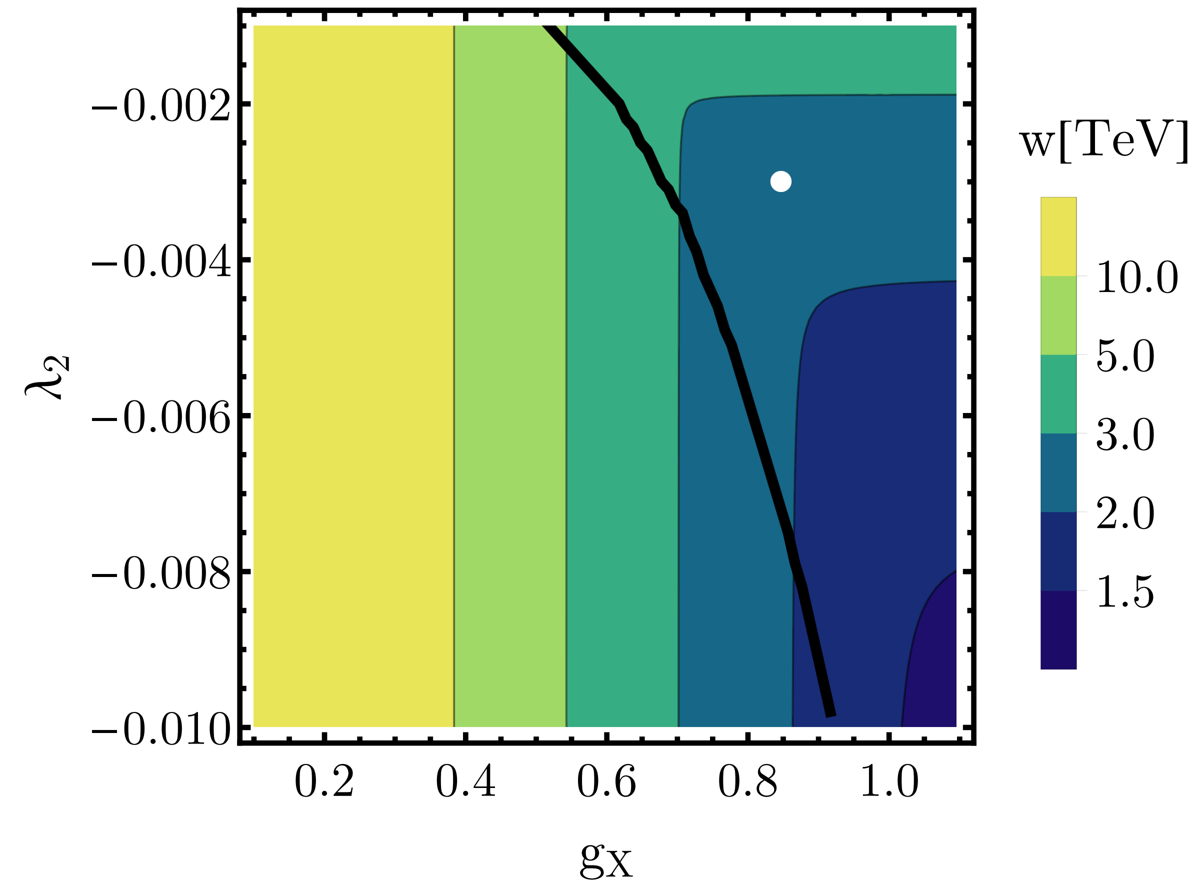



| | $\mu$ [GeV] | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $g_X$ | $w$ [GeV] | $V_{\mathrm{SM}}^{(1)}$[GeV$^4$] | $V_{\mathrm{X}}^{(1)}$[GeV$^4$] | $V^{(1)}/V^{(0)}$ |
|---|---|---|---|---|---|---|---|---|---|
| CW | 246 | 0.1236 | -0.0030 | -0.0047 | 0.8500 | 2411 | $2.38 \cdot 10^7$ | $3.18 \cdot 10^{10}$ | 0.802 |
| GW | 940 | 0.1055 | -0.0030 | $2 \cdot 10^{-5}$ | 0.8141 | 2722 | $6.28 \cdot 10^7$ | $-1.08 \cdot 10^{10}$ | 551 |
| RG | 738 | 0.1085 | -0.0030 | -0.0007 | 0.8202 | 2698 | $5.75 \cdot 10^7$ | $-4.27 \cdot 10^7$ | 0.002 |

# RADIATIVE SYMMETRY BREAKING IN SU(2)CSM

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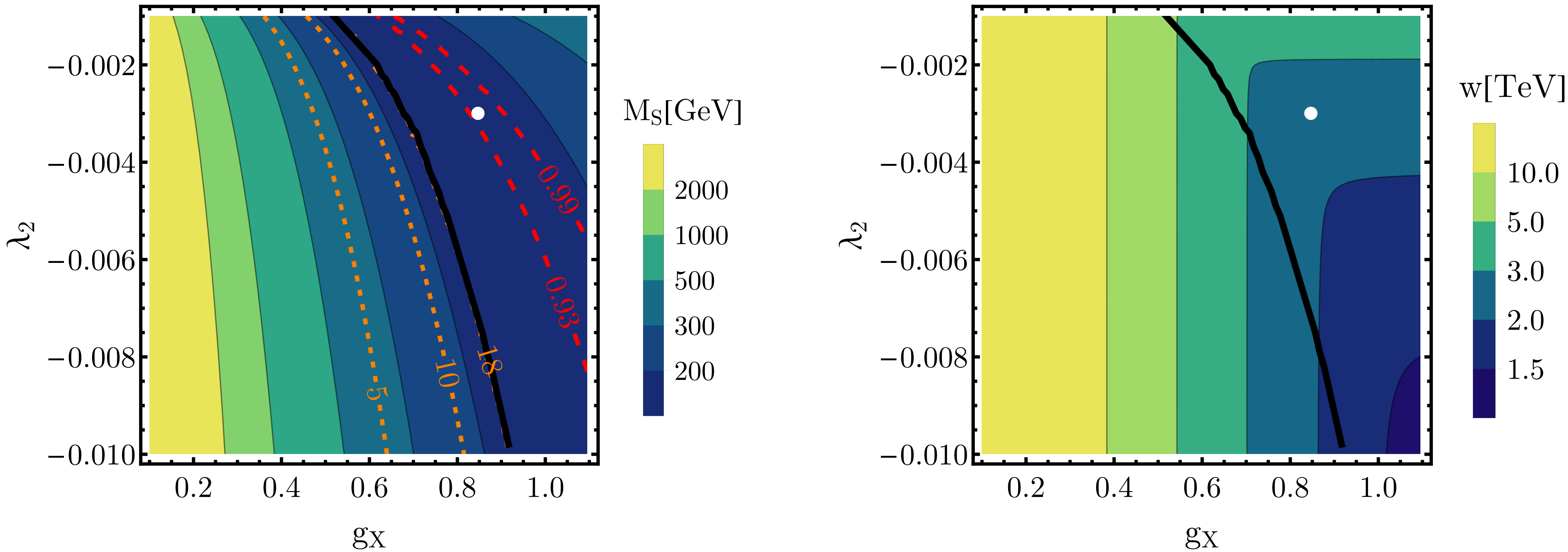


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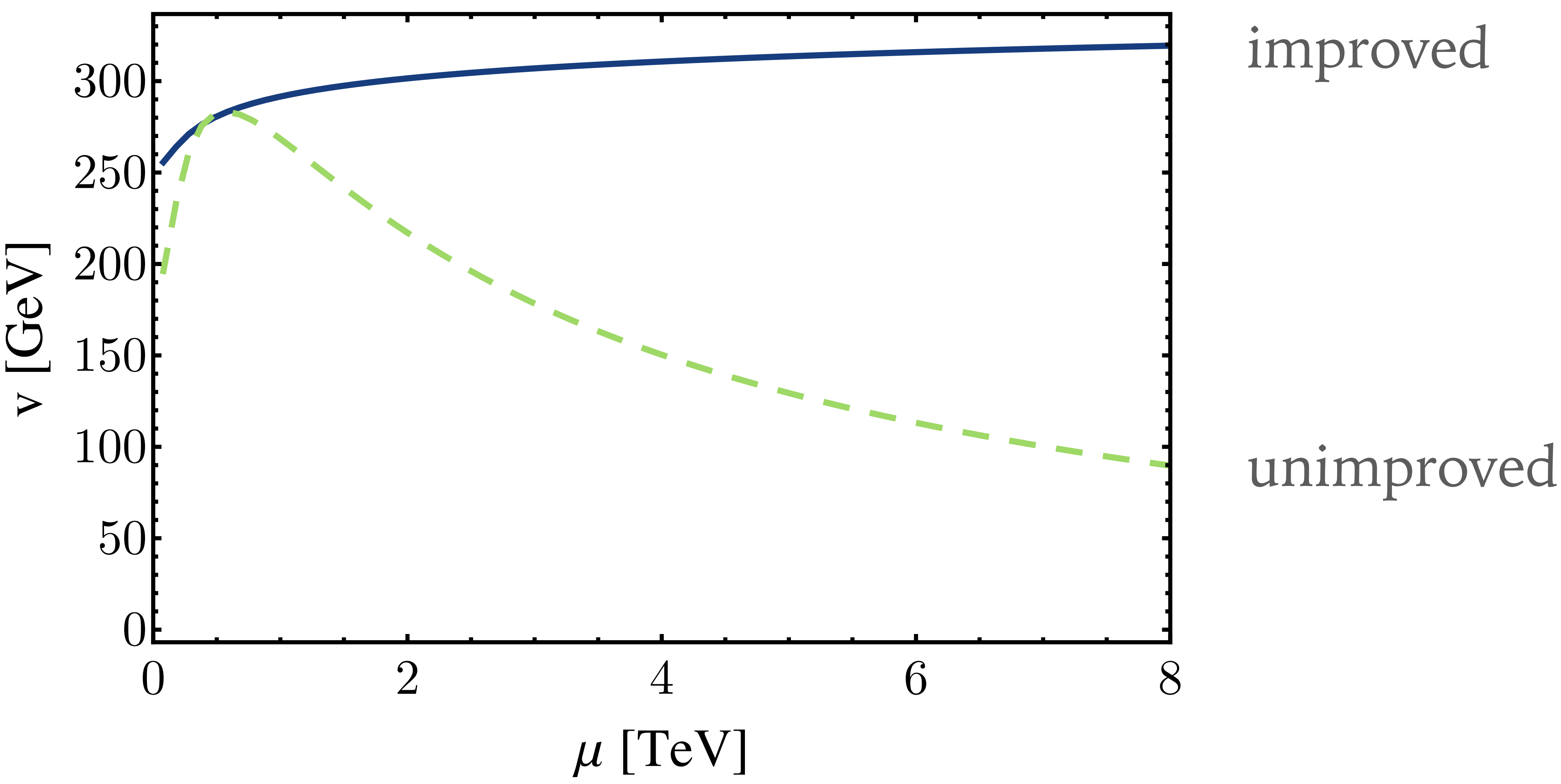
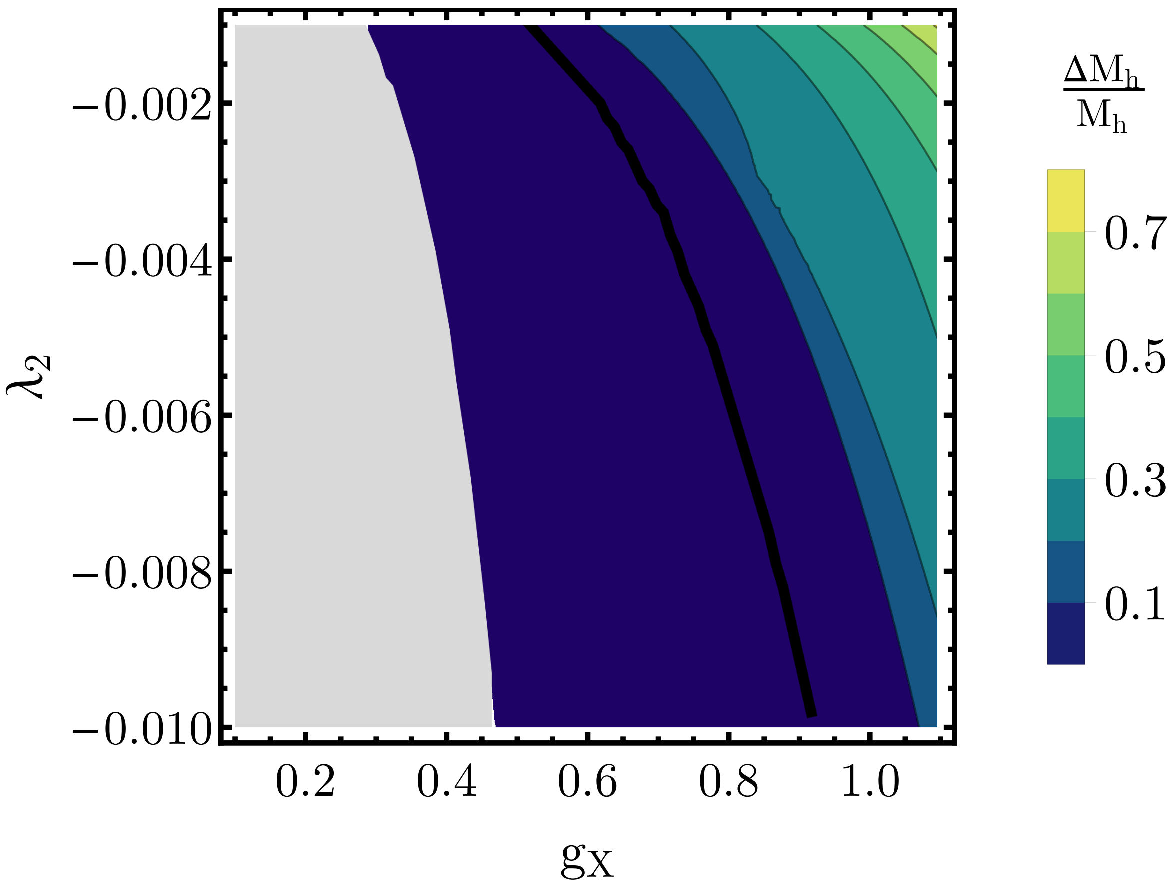
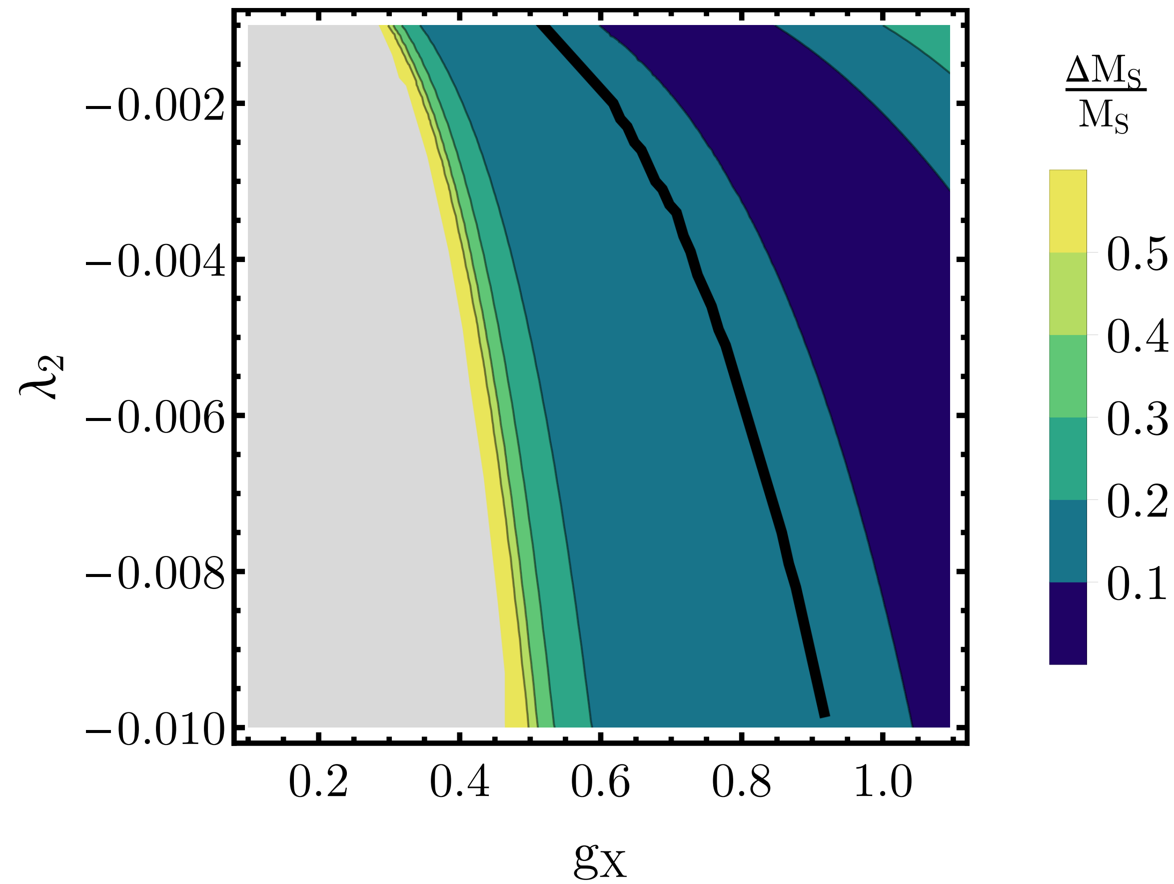
# RUNNING VEVS



# RUNNING MASSES



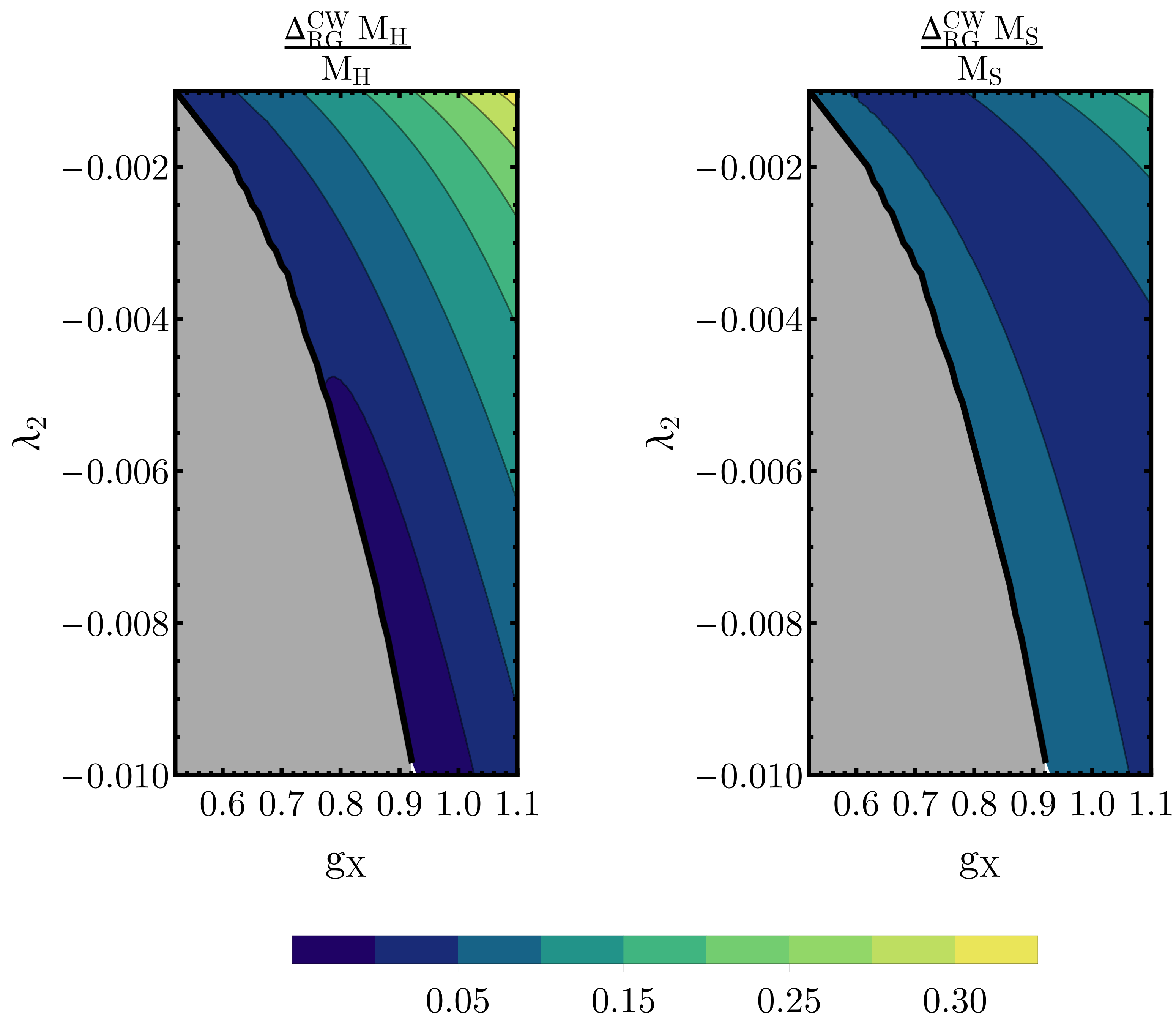




Change induced (mainly) by

$$\mu = 246\,\mathrm{GeV} \to \mu = 940\,\mathrm{GeV}$$

# RUNNING MASSES

Preliminary



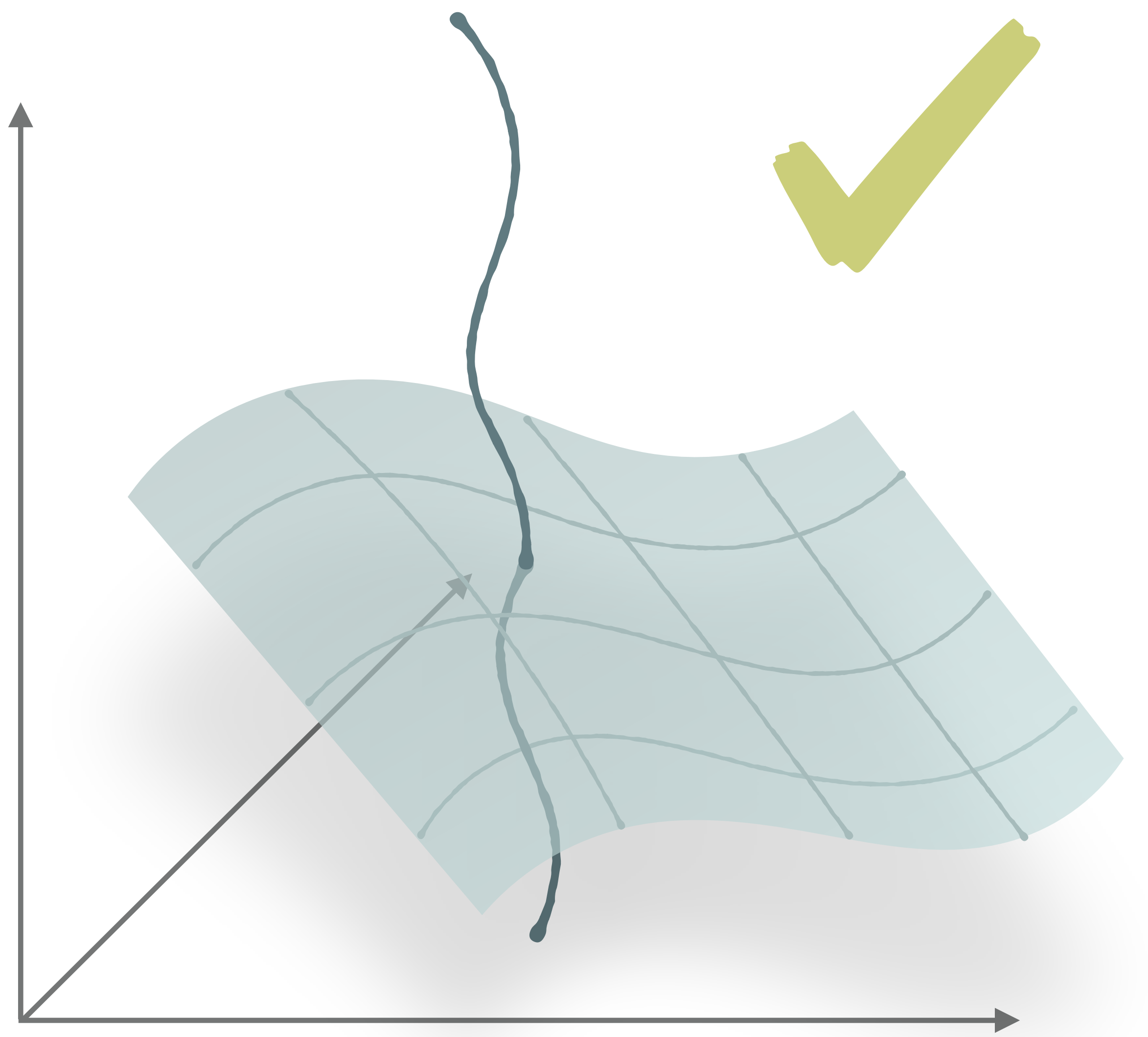
# SUMMARY 2

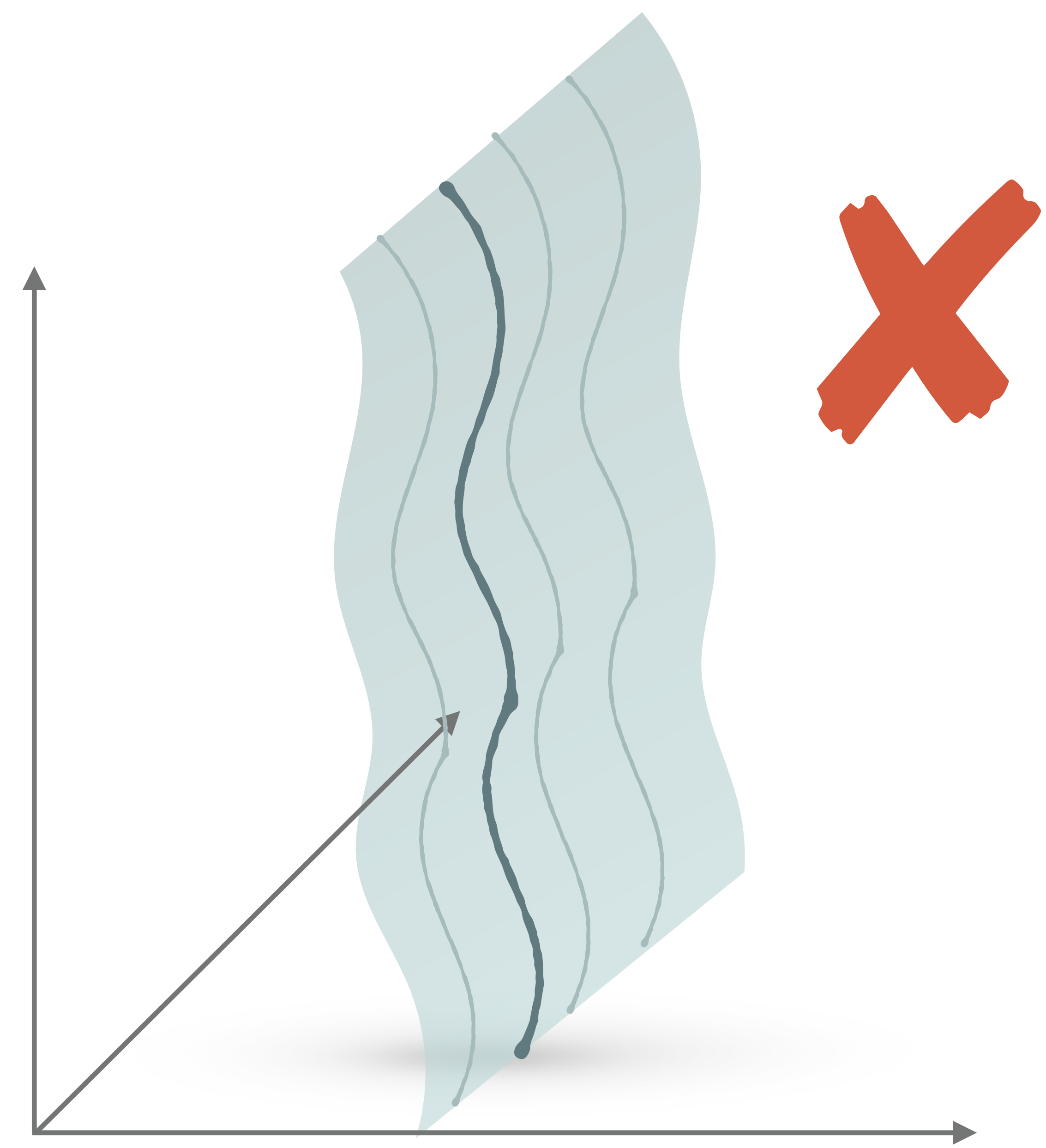
- RG improved effective potential gives VEVs that are less scale dependent
- RG improves perturbative behaviour of the expansion
- Less scale dependent effective potential gives less scale dependent masses

# MORE TECHNICAL

# VALIDITY OF THE METHOD

Boundary surface must be non-characteristic





# VALIDITY OF THE METHOD

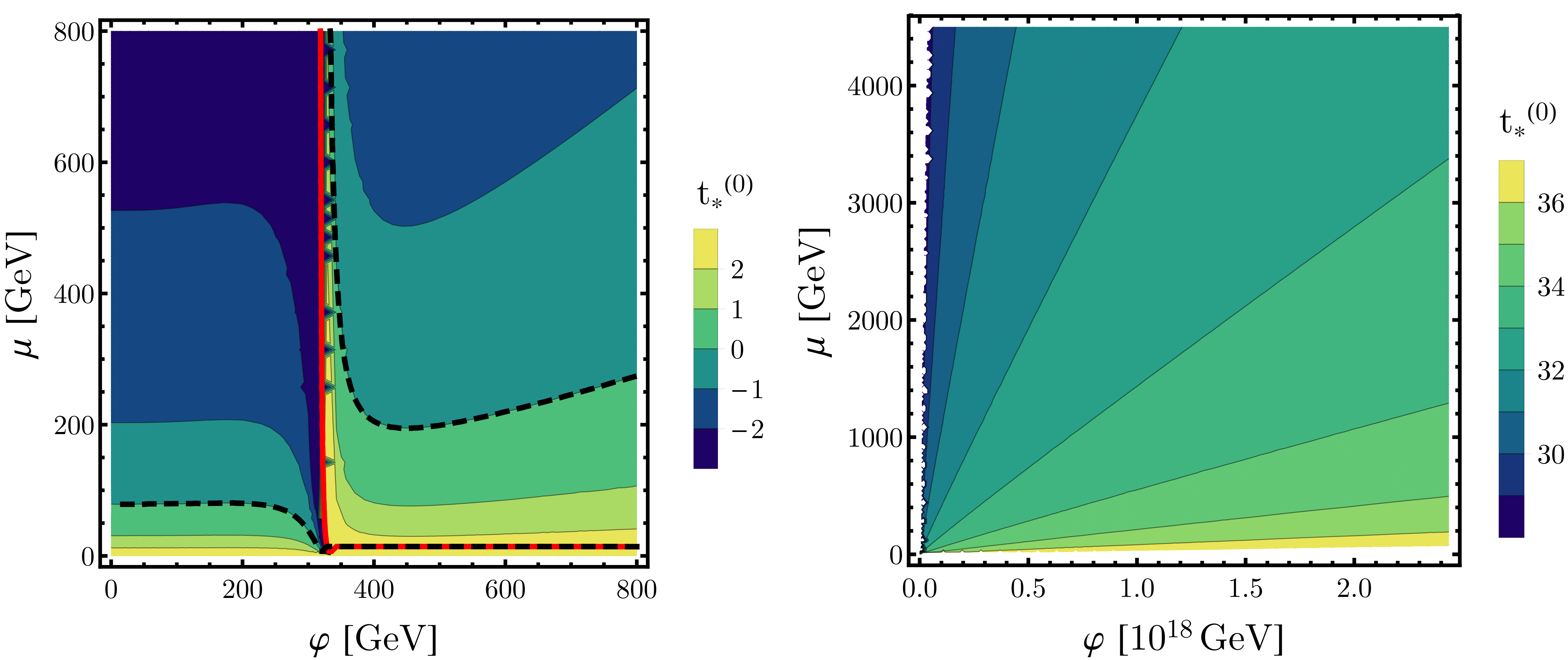
Is there a unique solution for $t_*$?

$$t_* = \frac{V^{(1)}\left(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*)\right)}{2\mathbb{B}\left(\bar{\lambda}(t_*), \bar{\phi}(t_*)\right)} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

# VALIDITY OF THE METHOD

To first approximation $V^{(1)}(\mu; \lambda, \phi) = 0$ is characteristic when $\mathbb{B} = 0$

$\mathbb{B} = 0$ is characteristic.



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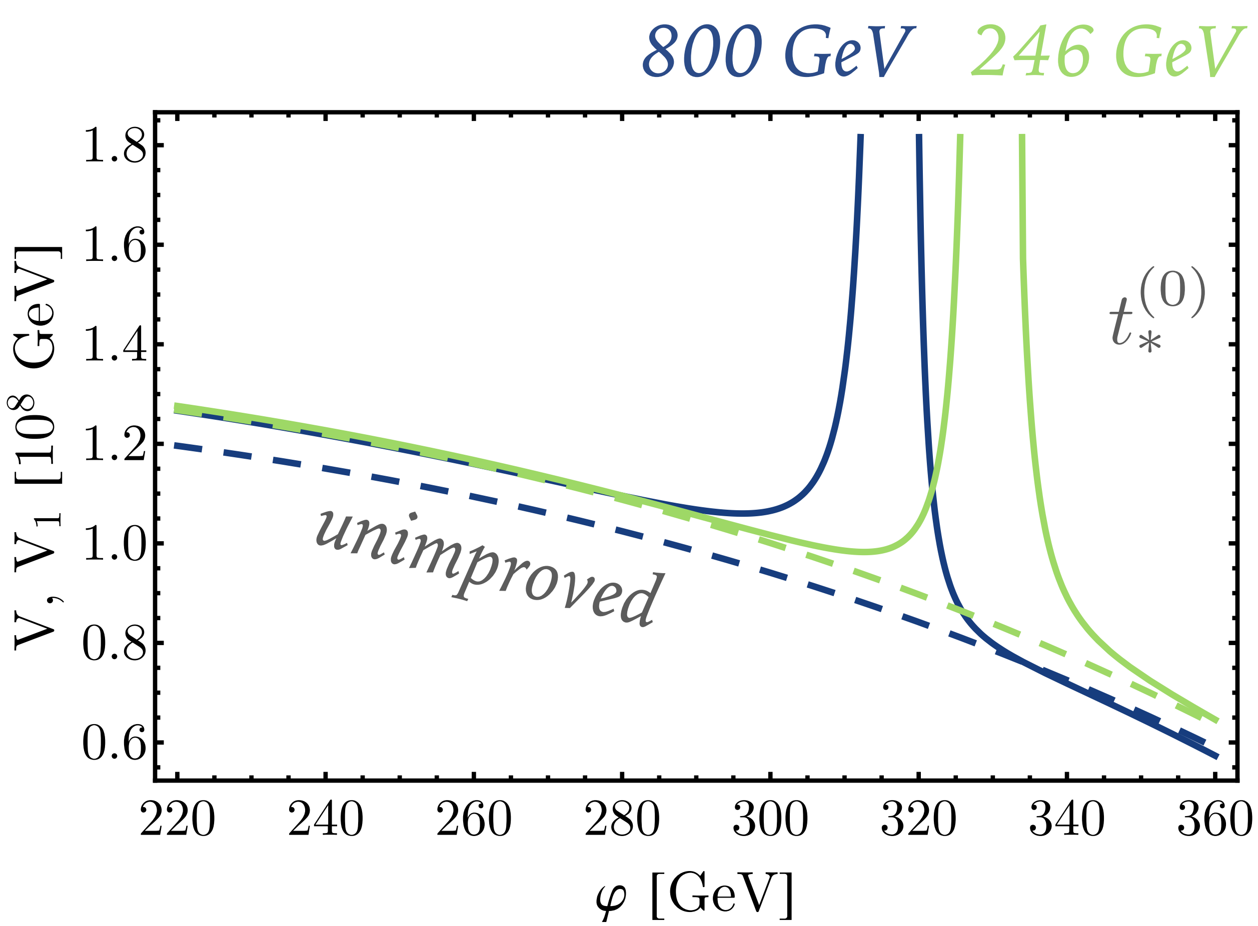
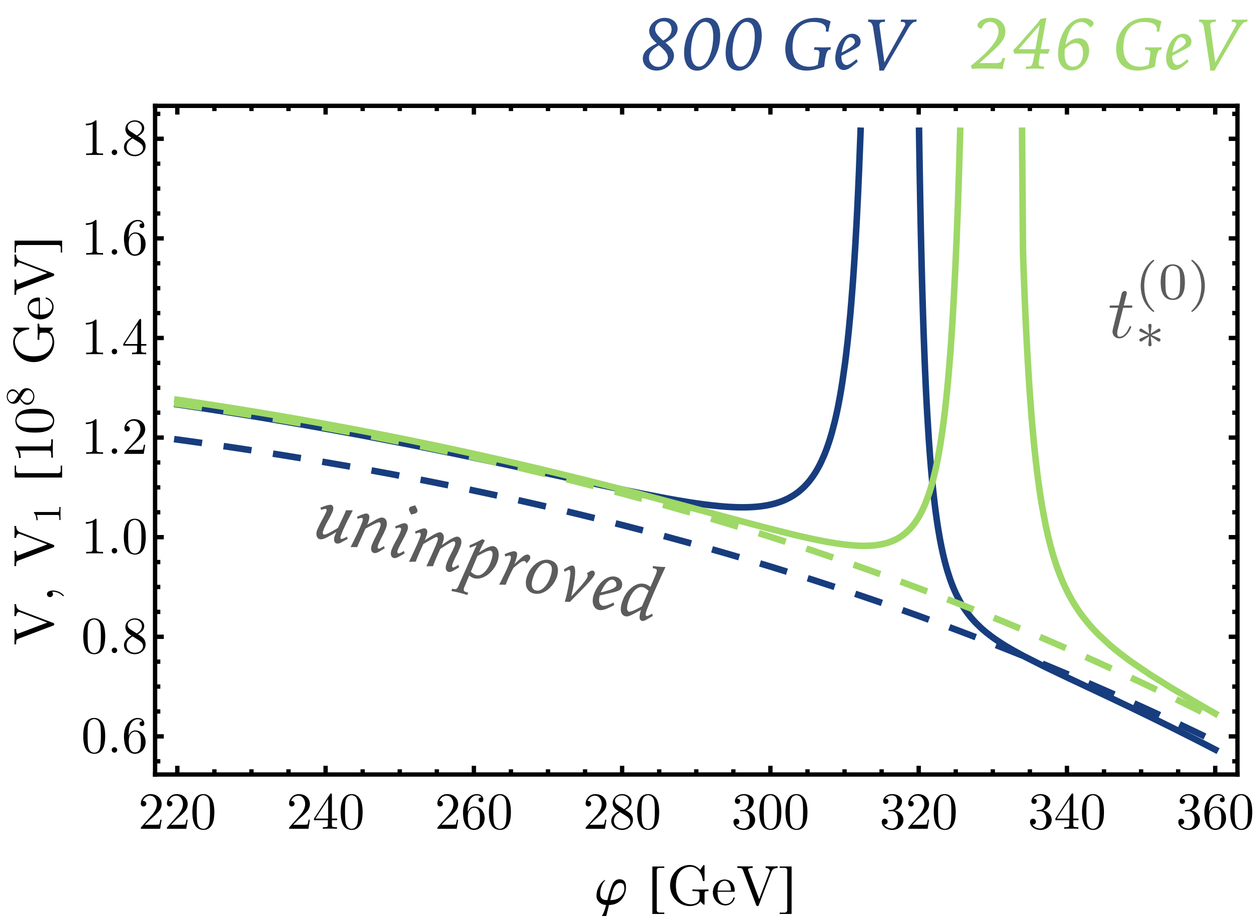
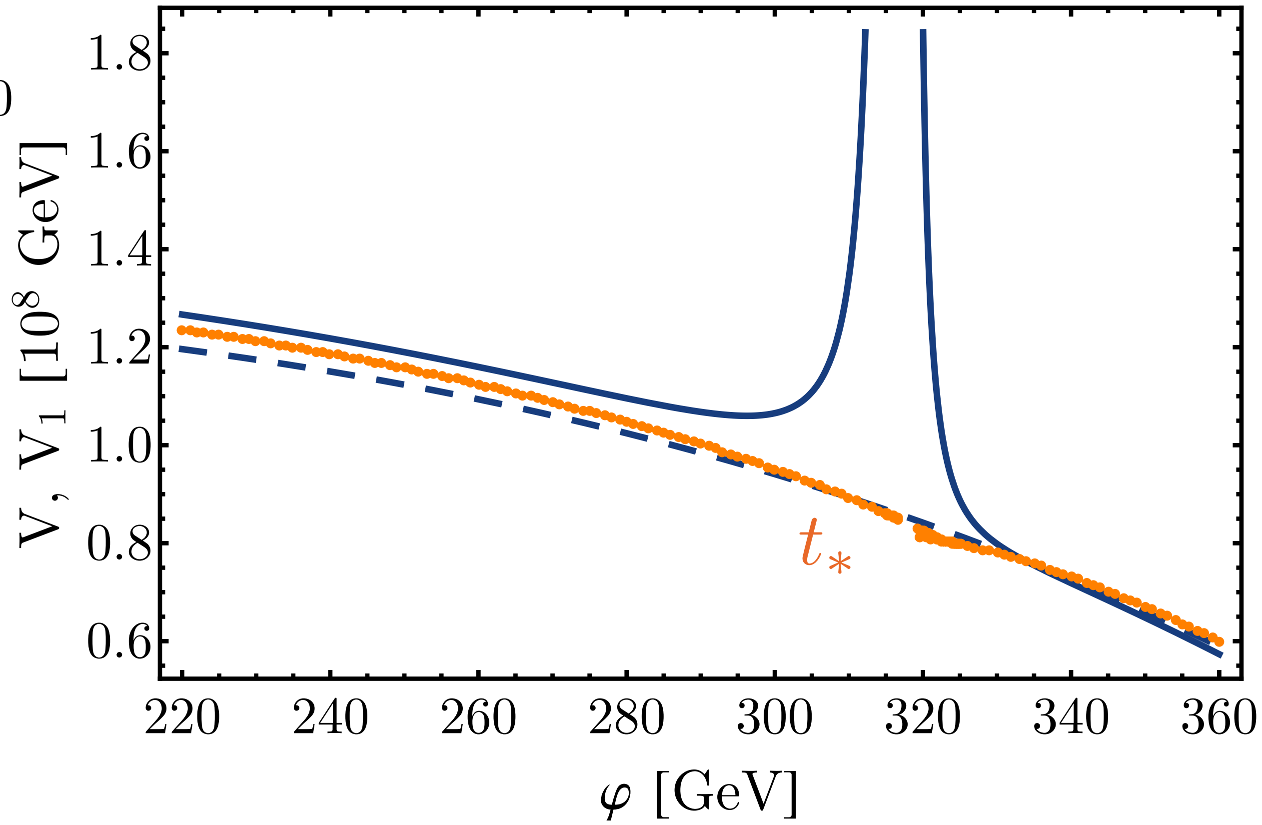



$t_*$

# VALIDITY OF THE METHOD



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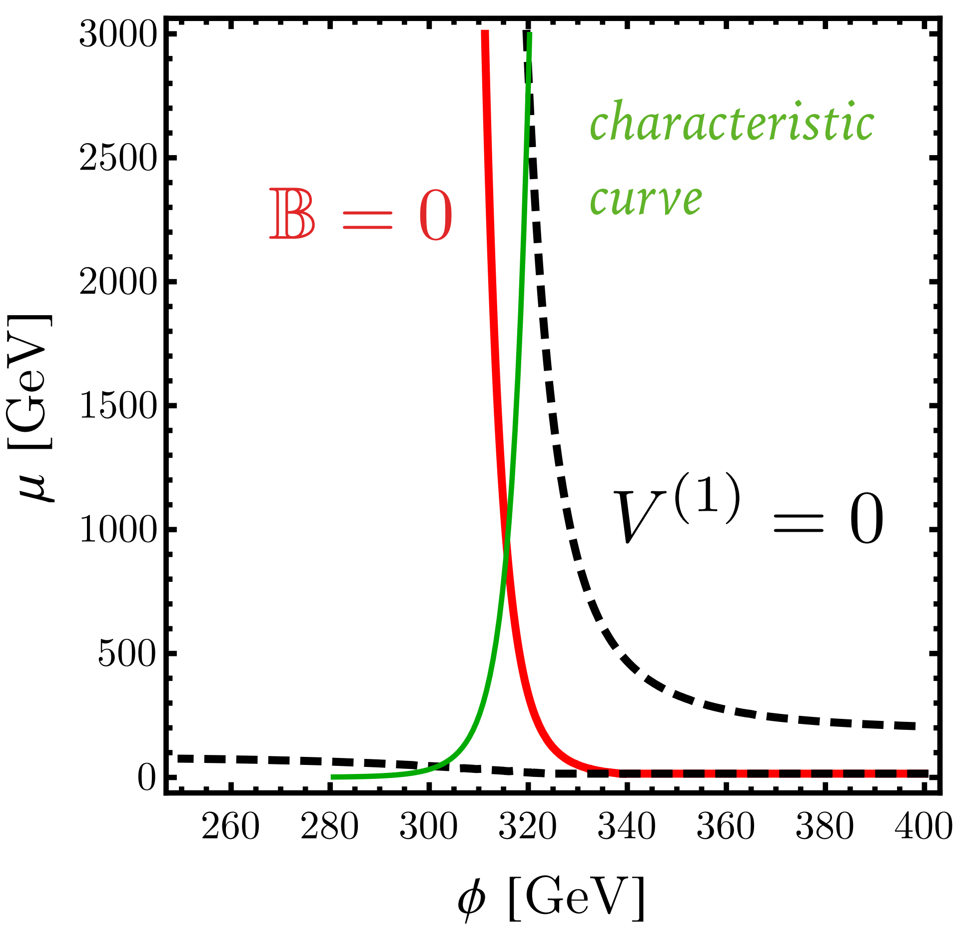
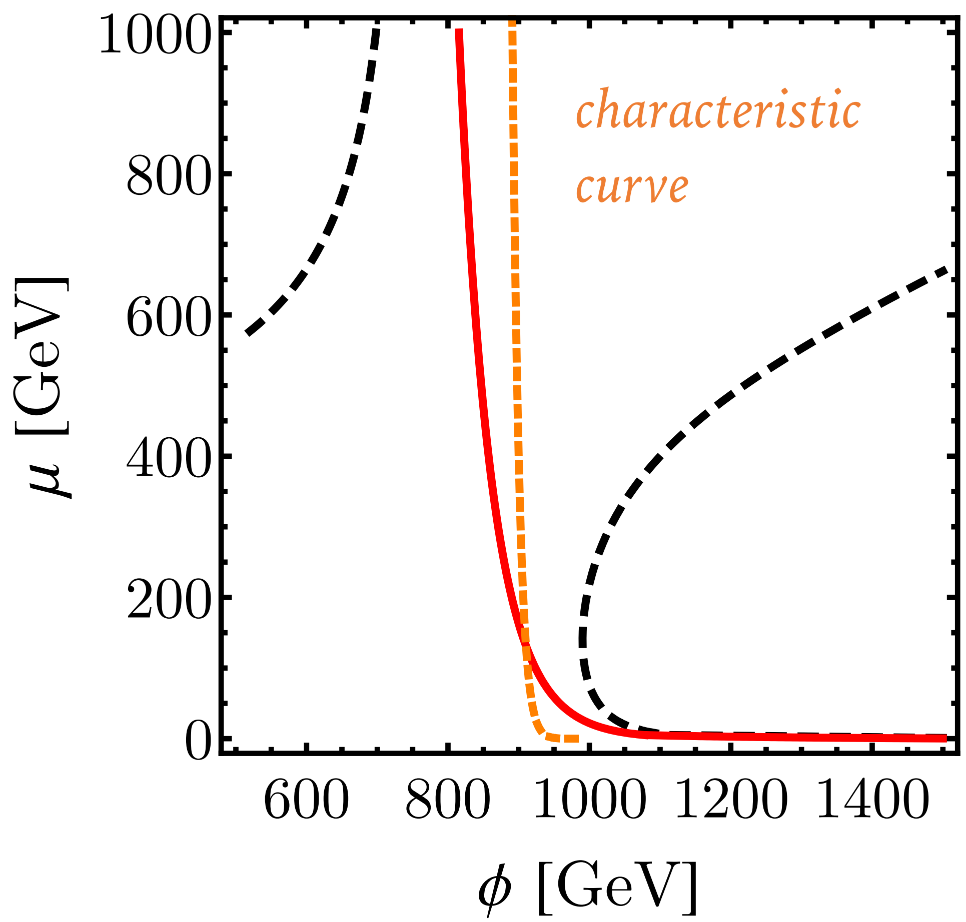



Higgs-Yukawa model

# RESUMMATION

The leading logarithms are not resummed. And what is?

$$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*)) \text{ resums powers of } t_*$$

one-field case

$$V(\mu; \lambda, \phi) = \frac{1}{4}\bar{\lambda}(t_*, \lambda)\phi^4 = \frac{1}{4}\phi^4 \sum_{n=0}^{\infty} \lambda^{n+1} \left[\frac{9\hbar}{16\pi^2} \log \frac{3\lambda\phi^2}{\mu^2}\right]^n + \cdots$$

multi-field case

$$t_* = \frac{V^{(1)}\left(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*)\right)}{2\mathbb{B}\left(\bar{\lambda}(t_*), \bar{\phi}(t_*)\right)} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

# RESUMMATION

$$t_* = \frac{V^{(1)}\left(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*)\right)}{2\mathbb{B}\left(\bar{\lambda}(t_*), \bar{\phi}(t_*)\right)} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$

pivot log

$\mu$-independent

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of the pivot log.

If $\left|\log \frac{\mathcal{M}^2}{\mu^2}\right| \gg \max_a \left\{\left|\log \frac{m_a^2(\lambda, \phi)}{\mathcal{M}^2}\right|\right\}$ these are the dominant terms.

# GENERALISATION TO HIGHER ORDERS

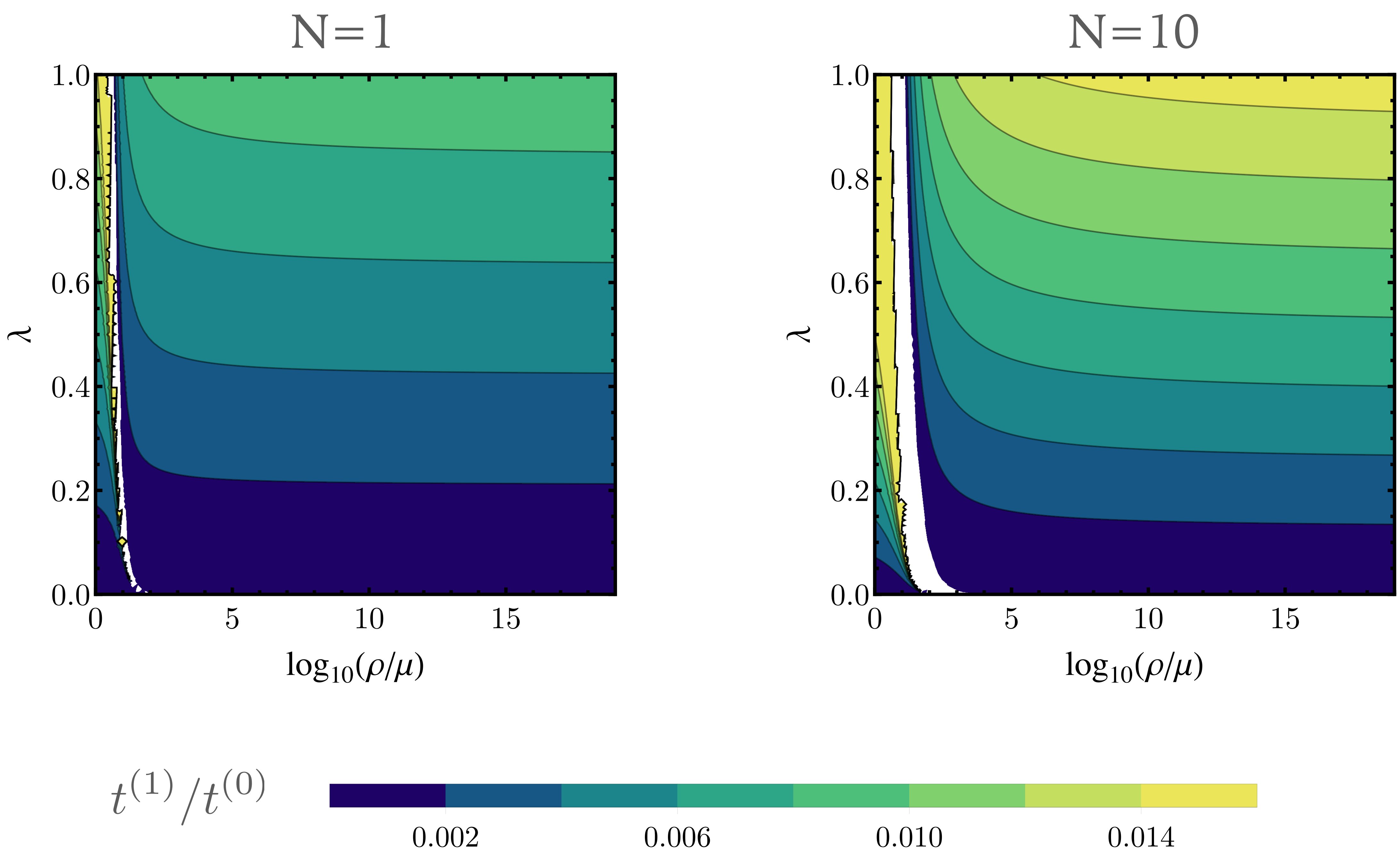
$$V(\mu;\lambda,\phi)=V^{(0)}(\lambda,\phi)+q(\mu,\lambda,\phi)=V^{(0)}(\lambda,\phi)+\sum_{l=1}^{\infty}\hbar^l V^{(l)}(\mu,\lambda,\phi)$$

$$q(\bar{\mu}(t_*),\bar{\lambda}(t_*),\bar{\phi}(t_*))=0$$

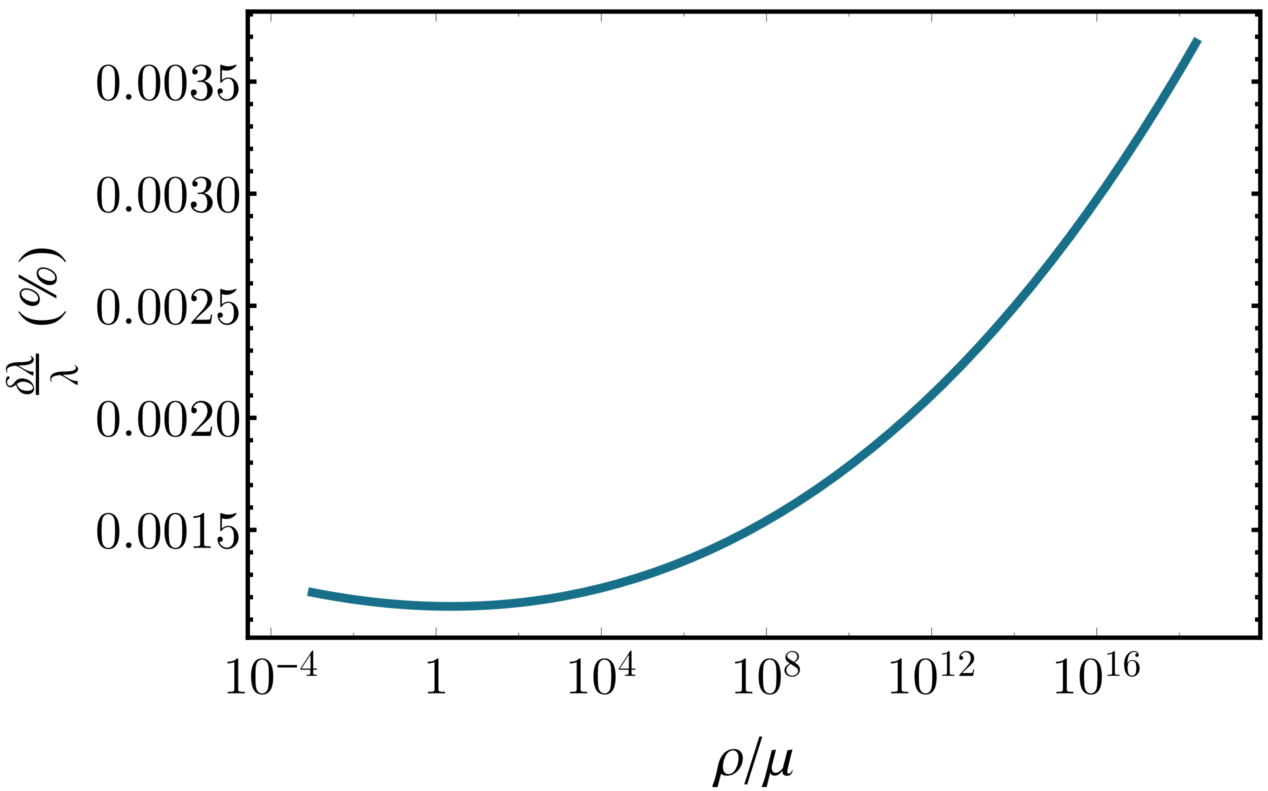
Solve for $t_*$ perturbatively in $\hbar$

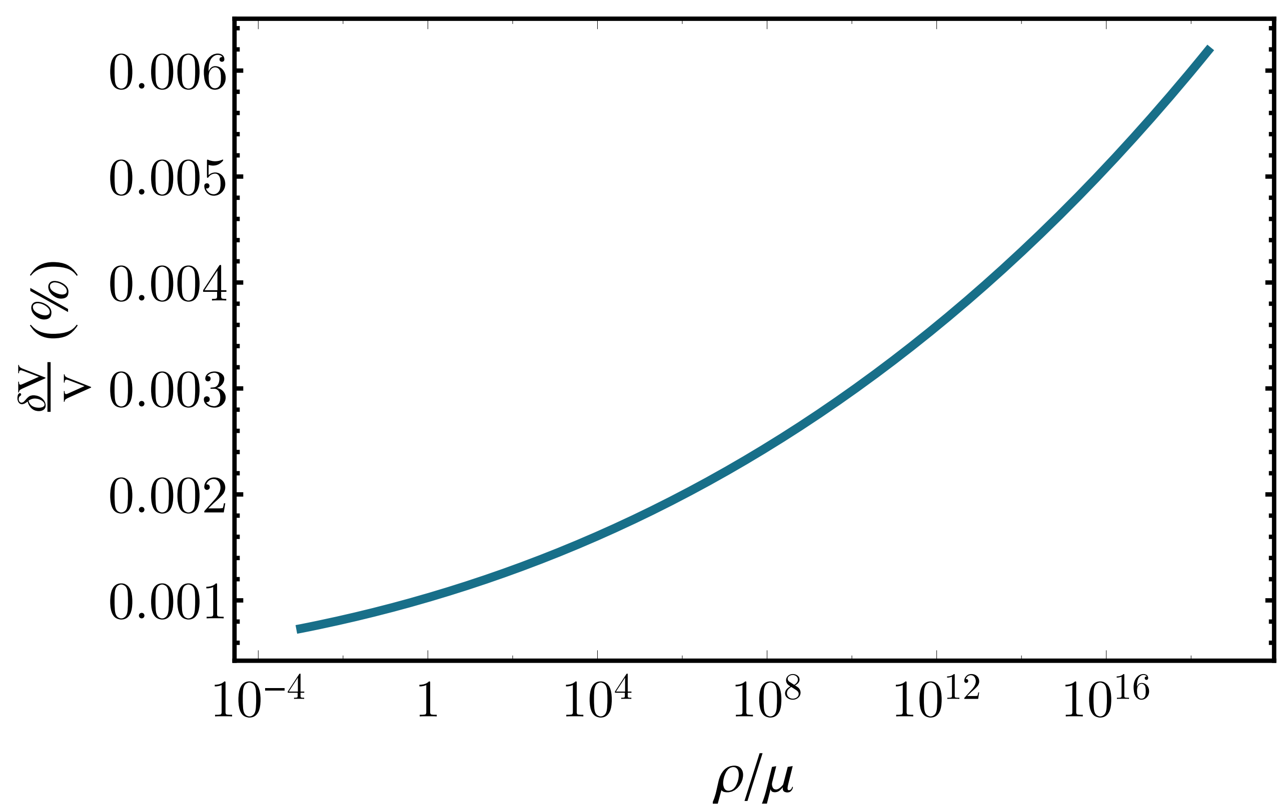
$$t_*^{(1)}=\frac{V^{(2)}(\mu,\lambda,\phi)-\left[\mathrm{d}^{(2)}V^{(0)}\right]_{t=0}t_*^{(0)}-\frac{1}{2}\left[\left(\mathrm{d}^{(1)}\right)^2V^{(0)}\right]_{t=0}\left(t_*^{(0)}\right)^2}{\left[\mathrm{d}^{(1)}V^{(0)}\right]_{t=0}}$$

# O(N)-SYMMETRIC MODEL AT TWO-LOOP LEVEL



# O(N)-SYMMETRIC MODEL AT TWO-LOOP LEVEL



# SUMMARY 3

- Can be implemented to any loop order
- Resums the pivot logarithm
- Some difficulties when the tree-level hypersurface non-characteristic

# RECAP

To compute $V(\mu, \lambda, \phi)$

$t_*$

Evaluate

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Solve numerically

$$V^{(1)}(\bar{\mu}(t_*); \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

Use approximation

$$t_*^{(0)} = \frac{V^{(1)}(\mu, \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)}$$

No explicit logs!

Form preserved at higher orders difference can only come from running

# RECAP

Applicable to analysis of vacuum stability

To compute $V(\mu, \lambda, \phi)$

$t_*$

Evaluate

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Solve numerically

$$V^{(1)}(\bar{\mu}(t_*); \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

Use approximation

$$t_*^{(0)} = \frac{V^{(1)}(\mu, \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)}$$

No explicit logs!

Form preserved at higher orders difference can only come from running

# RESUMMATION

$$t_* = t_*^{(0)} + \mathcal{O}(\hbar) = \frac{1}{2}\log\frac{\mathcal{M}^2}{\mu^2} + \frac{1}{2}\frac{\mathbb{A}\left(\lambda,\phi,\mathcal{M}\right)}{\mathbb{B}\left(\lambda,\phi\right)} + \mathcal{O}(\hbar)$$

$$V^{(0)}(\bar{\lambda}(t_*),\bar{\phi}(t_*)) = \sum_{n=0}^{\infty}\hbar^n w_n^{(n)}(\lambda,\phi)(2t_*)^n = \sum_{n=0}^{\infty}\hbar^n w_n^{(n)}(\lambda,\phi)\left(\log\frac{\mathcal{M}^2}{\mu^2}\right)^n + \cdots$$