Non-Local Field Theory: From Gravity to Higgs

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SCALARS 2017

$$V \sim \frac{1}{r}$$

How to Resolve 1/r- Singularity?

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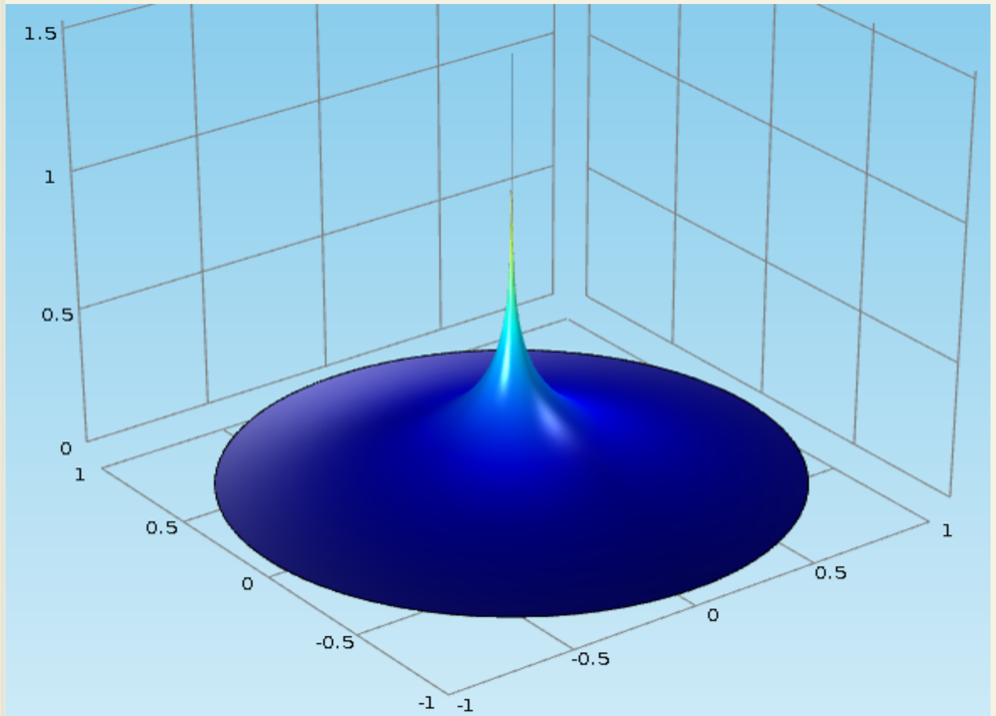
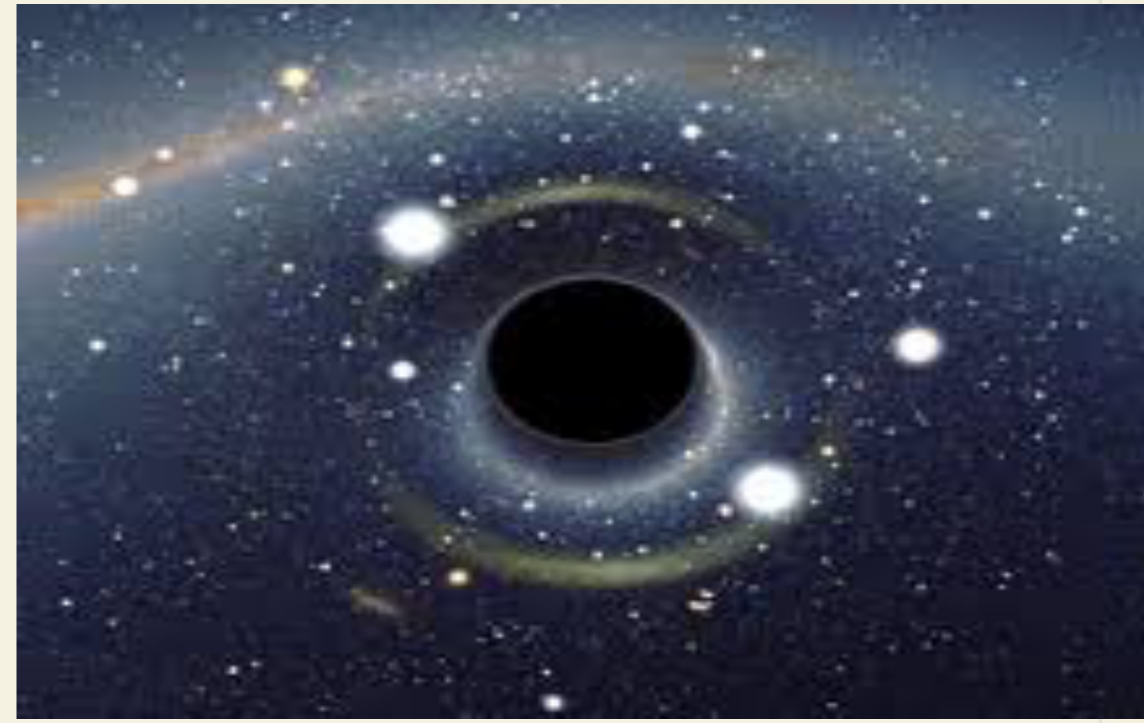
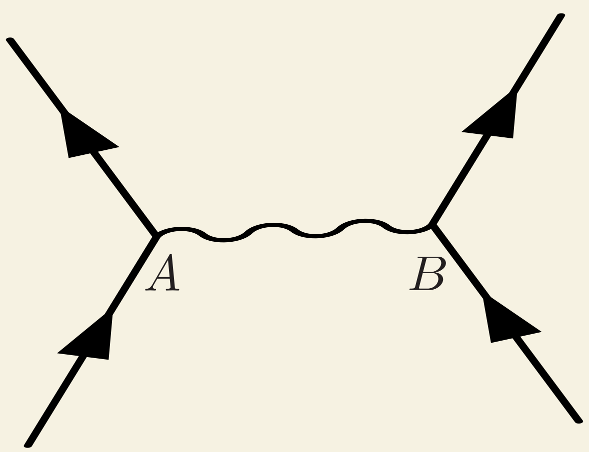
RESEARCH TRAINING GROUP Models of Gravity
Nikhef

Locality in space & time : From Blackhole to Cosmological Singularity

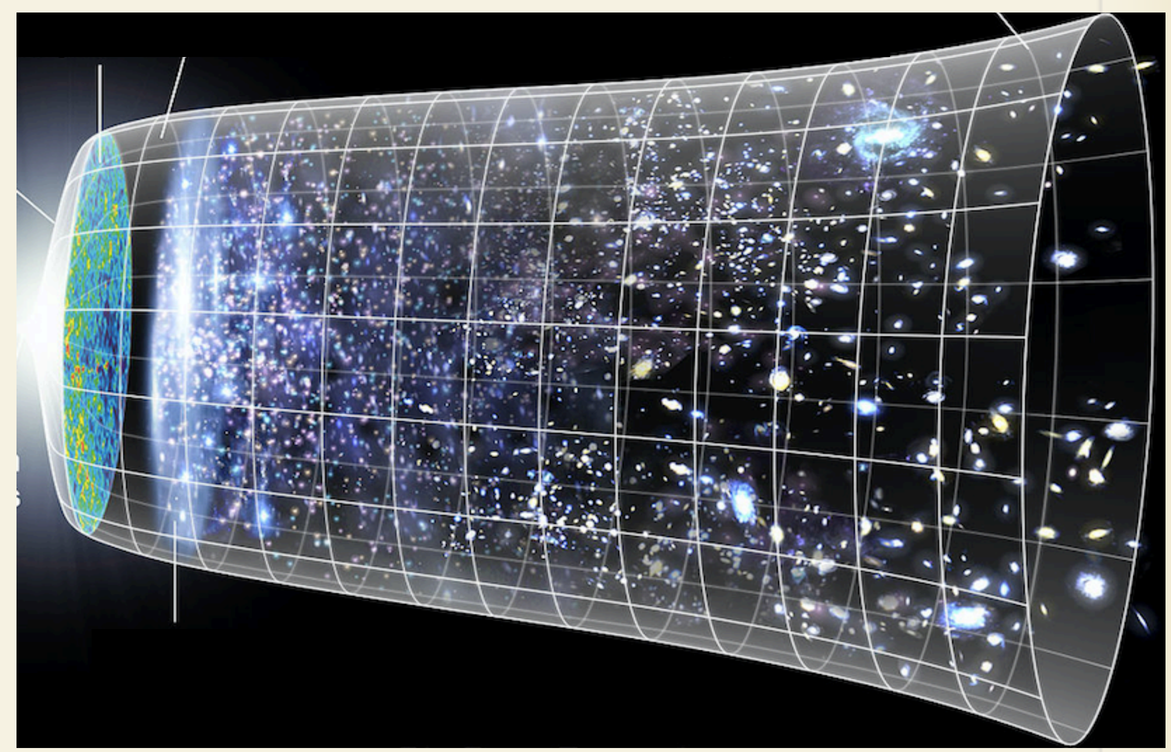






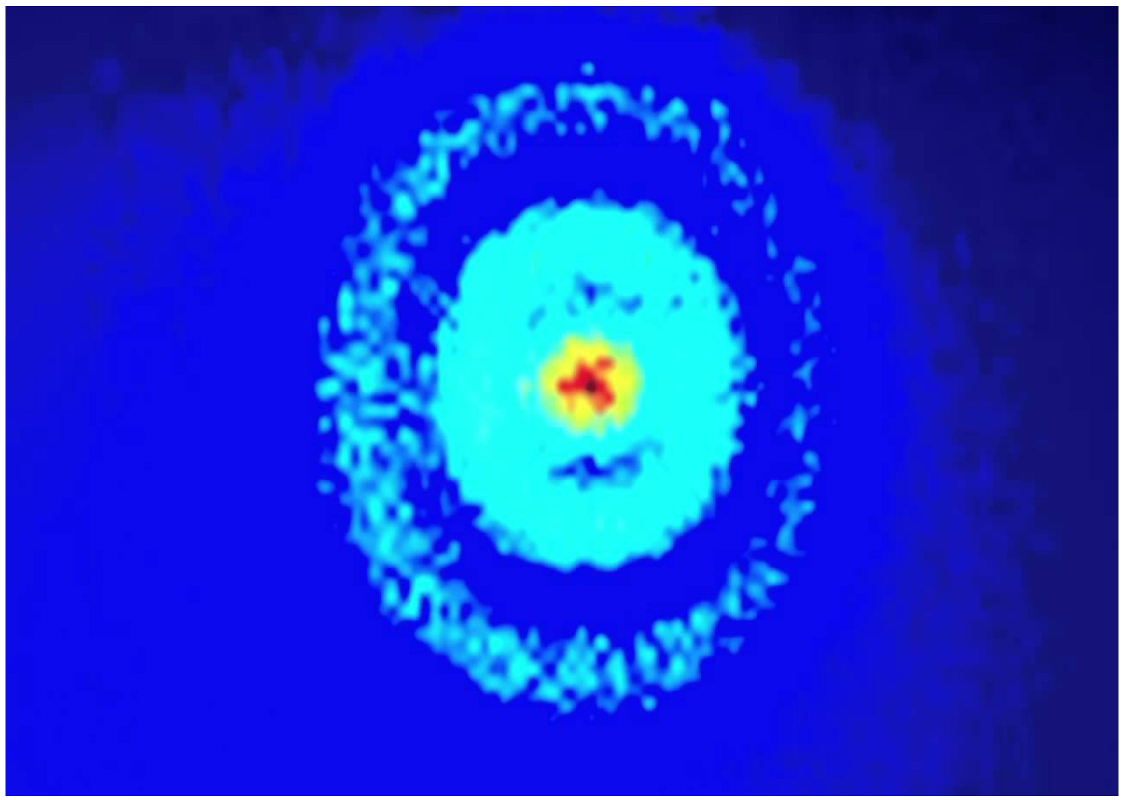
$$V \sim \frac{1}{r}$$

Graviton or Photon (mediator is massless)

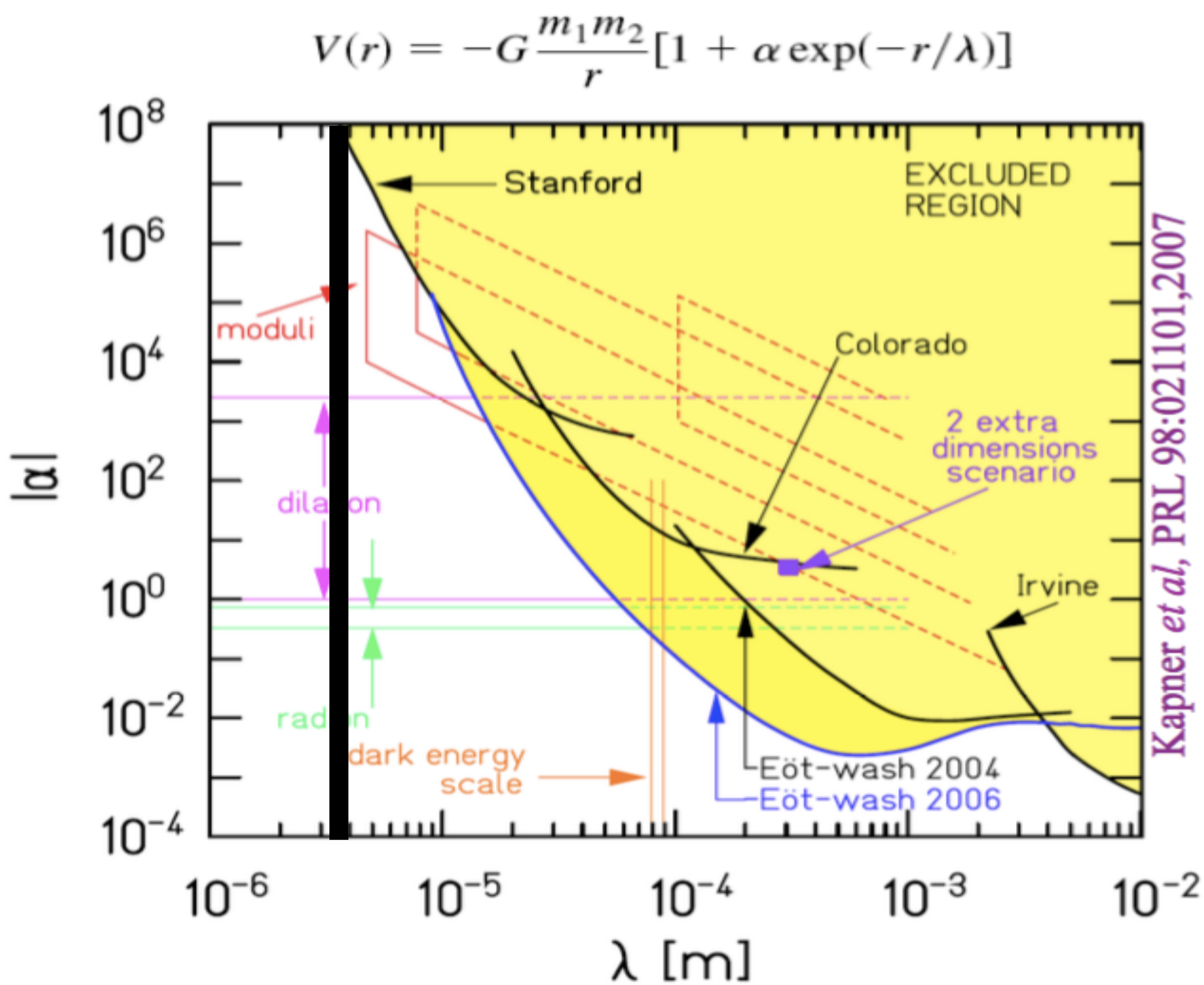


330 years & Gravity is Still Mysterious

Stodolna, et.al, (FOM Institute for Atomic and Molecular Physics), PRL 110:213001, 2013



Hydrogen atom $\sim 10^{-10}$ m



Beyond micro-meter Gravitational Interactions are not known!

or, $M \sim 10^{-2}$ eV

There ought to be a New Scale of Gravity which can ameliorate 1/r Singularity

Energy Ladder

New Scales in Gravity & Higgs Sectors

$(10^{27}\ \mathrm{eV})^4$

?

$(10^{-2}\ \mathrm{eV})^4$

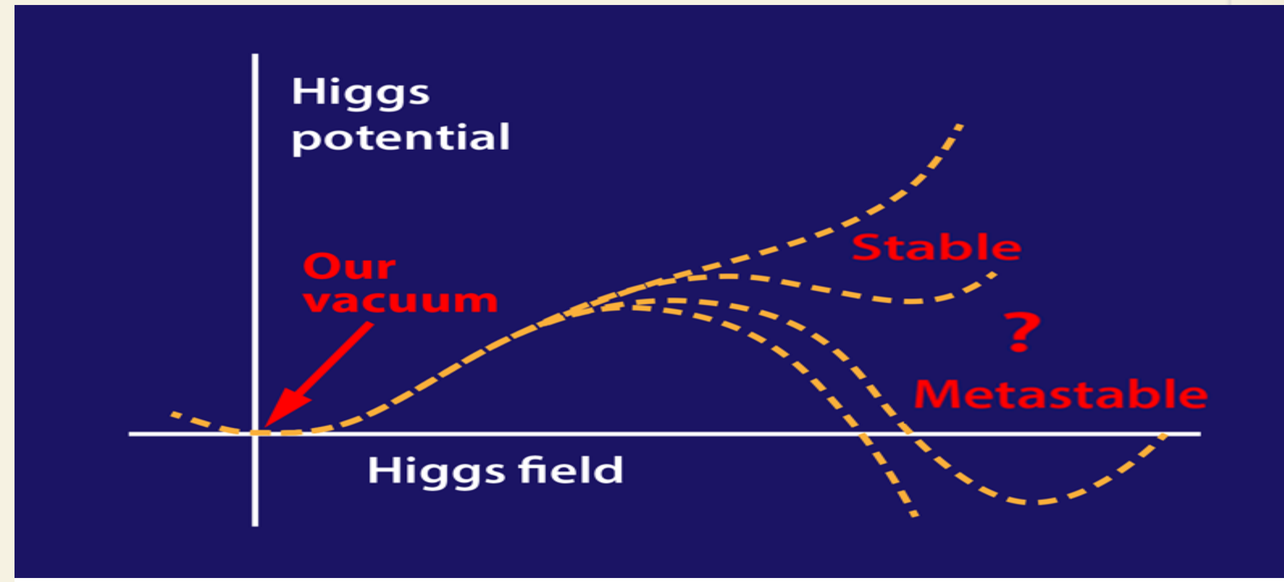
$(10^{-3}\ \mathrm{eV})^4$

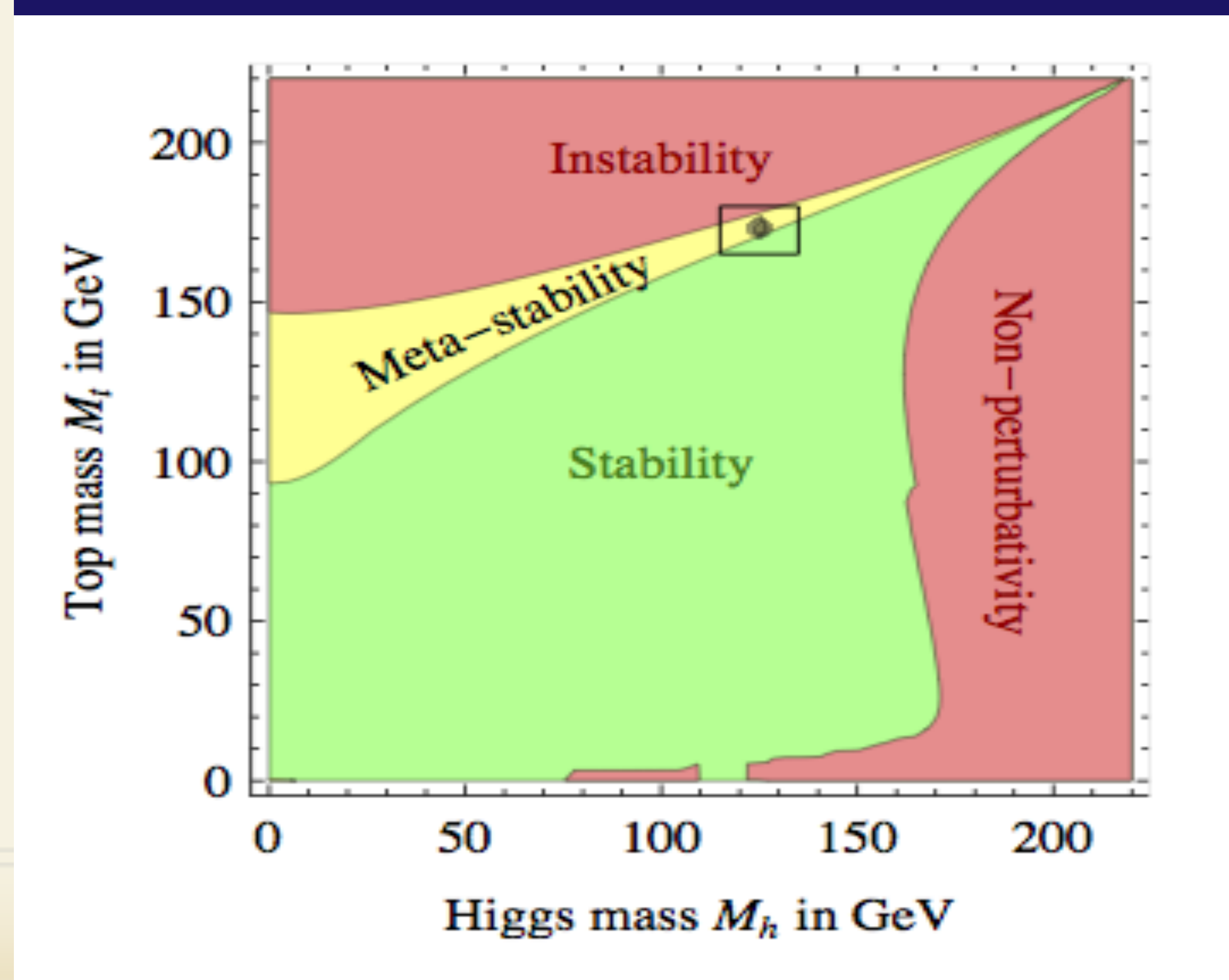
$(10^{28}\ \mathrm{eV})^4$

?

$(10^{12}\ \mathrm{eV})^4$

$(10^{-3}\ \mathrm{eV})^4$





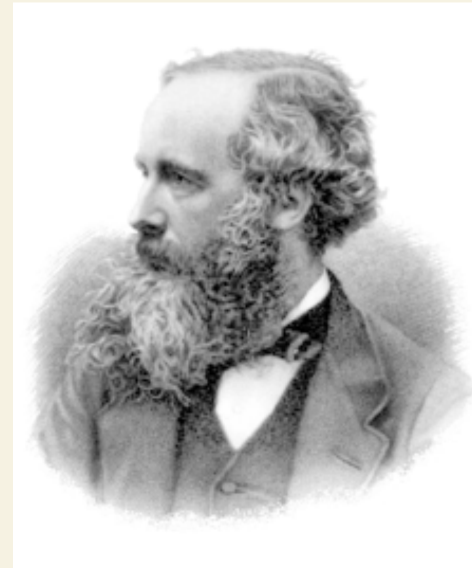
How to ameliorate the UV behaviour?

Maxwell's Theory

Gravity

Abelian-Higgs

Maxwell's Electromagnetism



Self energy of an electron is infinite in Maxwell's theory

1/r-fall of Coulomb's Potential

Quantum Electrodynamics (QED)

Classical approach: Born-Infeld

Born-Infeld resolves 1/r singularity in Coulomb Potential

$$\mathcal{L}_{\mathrm{Born-Infeld}} = b^2 \left[1 - \sqrt{1 - (\boldsymbol{E}^2 - \boldsymbol{B}^2)/b^2 - (\boldsymbol{E}\cdot\boldsymbol{B})^2/b^4} \right]$$

$$b \to \infty \qquad\qquad \mathcal{L}_{\mathrm{Born-Infeld}} \to \mathcal{L}_{\mathrm{Maxwell}}$$

Maxwell	**Born-Infeld**
$E_{\mathrm{tot}} = \frac{1}{2}\int (E.D + B.H) d^3 r$ $D = e\hat{r}/4\pi r^2, \quad E = e\hat{r}/4\pi\epsilon r^2, \quad B = H = 0$ $E_{\mathrm{tot}} = \frac{1}{32\pi^2}\int_0^\infty \frac{e^2}{r^4} 4\pi r^2 dr = \infty$	$\boldsymbol{\nabla}\cdot D = e\delta^{(3)}(\boldsymbol{r}) \qquad \boldsymbol{B} = 0$ $\boldsymbol{D} = \frac{e\boldsymbol{r}}{4\pi\vert\boldsymbol{r}\vert^3} \qquad \boldsymbol{D}^2/b^2 = \frac{q^2}{r^4}$ $E_{\mathrm{tot}} = 4\pi b^2 \int_0^\infty dr\, r^2 \left(\sqrt{1 + q^2/r^4} - 1\right)$ $= \frac{4\Gamma^2(5/4)\sqrt{e^3 b}}{3\pi} = 1.2361\sqrt{e^3 b}$

Fact-sheet for Einstein's Gravity

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable

Beyond two loops it is hard to compute, number of Feynman diagrams increases rapidly

Quadratic Curvature Gravity is renormalizable, but contains "Ghosts": Vacuum is Unstable

Utiyama (1961), De Witt (1961), Stelle (1977)
t'Hooft, Veltman (1974)

4th Derivative Gravity & Power Counting Renormalizability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k\,R + a\,R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}(b+a) R^2 \right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A} \left(\frac{1}{k^2} - \frac{1}{k^2 + A} \right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama (1960), De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: How to get rid of the extra dof ?

Resolution of Quantum Ghosts & Classical Instabilities

Higher derivative theories generically carry Ghosts (-ve Risidue)

$$S = \int d^4x\ \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) \sim \frac{1}{p^2} - \frac{1}{p^2 - m^2}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$S = \int d^4x\ \phi e^{-\Box/M^2}(\Box + m^2)\phi \Rightarrow e^{-\Box/M^2}(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{e^{-p^2/M^2}}{p^2 - m^2}$$

No extra states other than the original dof.

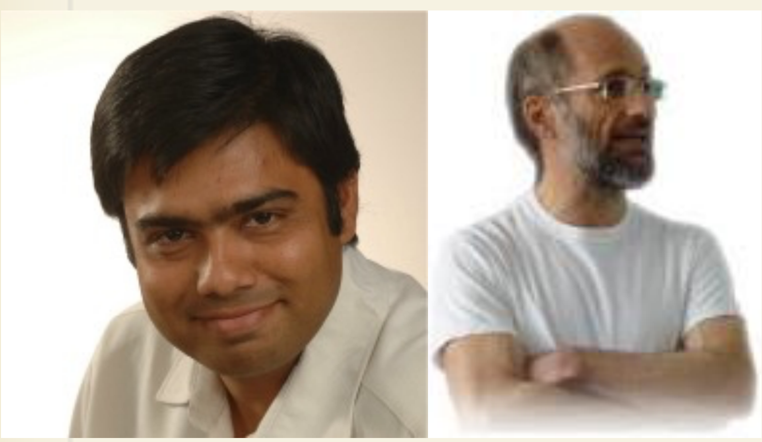
Woodard (1991), Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Born-Infeld Gravity

GR in IR **Corrections in UV**

M M_p

$M \to \infty$(Theory reduces to GR)



Biswas, AM, Siegel

Biswas, Gerwick, Koivisto, AM

Bouncing universes in string-inspired gravity, hep-th/0508194, JCAP (2006)

Towards singularity and ghost free theories of gravity, 1110.5249 [gr-qc], PRL (2012)

Higher Curvature Construction in Gravity

$$S_q = \int d^4x \sqrt{-g} R_{\mu_1\nu_1\lambda_1\sigma_1} \mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2} R^{\mu_2\nu_2\lambda_2\sigma_2}$$

All possible terms allowed by symmetry

Unknown Infinite Functions of Covariant Derivatives

$$\begin{aligned}
S_q &= \int d^4x \sqrt{-g}\big[R F_1(\Box) R + R F_2(\Box) \nabla_\mu \nabla_\nu R^{\mu\nu} + R_{\mu\nu} F_3(\Box) R^{\mu\nu} + R^\nu_\mu F_4(\Box) \nabla_\nu \nabla_\lambda R^{\mu\lambda} \\
&+ R^{\lambda\sigma} F_5(\Box) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\lambda R^{\mu\nu} + R F_6(\Box) \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda} F_7(\Box) \nabla_\nu \nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R^\rho_\lambda F_8(\Box) \nabla_\mu \nabla_\sigma \nabla_\nu \nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1} F_9(\Box) \nabla_{\mu_1} \nabla_{\nu_1} \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma R^{\mu\nu\lambda\sigma} \\
&+ R_{\mu\nu\lambda\sigma} F_{10}(\Box) R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda} F_{11}(\Box) \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1} F_{12}(\Box) \nabla^{\rho_1} \nabla^{\sigma_1} \nabla_\rho \nabla_\sigma R^{\mu\rho\nu\sigma} \\
&+ R_\mu^{\nu_1\rho_1\sigma_1} F_{13}(\Box) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1} F_{14}(\Box) \nabla_{\rho_1} \nabla_{\sigma_1} \nabla_{\nu_1} \nabla_{\mu_1} \nabla_\mu \nabla_\nu \nabla_\rho \nabla_\sigma R^{\mu\nu\lambda\sigma}\big]
\end{aligned}$$

Higher Curvature Action & Form Factors

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + R\mathcal{F}_1\left(\frac{\Box}{M^2}\right)R + R_{\mu\nu}\mathcal{F}_2\left(\frac{\Box}{M^2}\right)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3\left(\frac{\Box}{M^2}\right)R^{\mu\nu\lambda\sigma}\right]$$

Einstein-Hilbert Recovers IR

Ultra-violet modifications

$$\frac{\Box}{M^2} \qquad M \to \infty (\text{Theory reduces to GR})$$

Infinite Derivative Gravity (IDG)

Biswas, AM, Siegel, hep-th/0508194

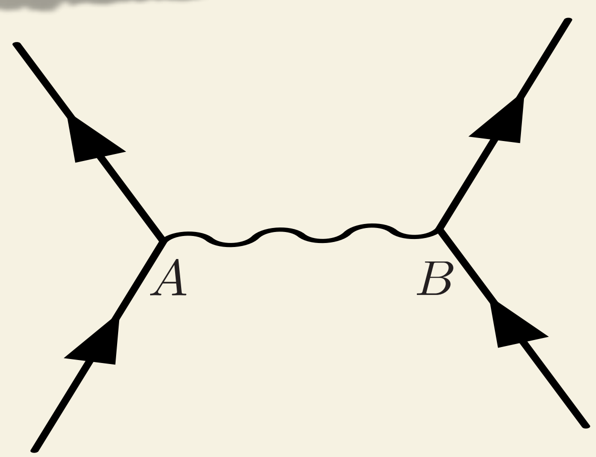
Biswas, Gerwick, Koivisto, AM, gr-qc/1110.5249

Biswas, Koshelev, AM, (extension for de Sitter & Anti-deSitter), arXiv:1602.08475 , arXiv:1606.01250

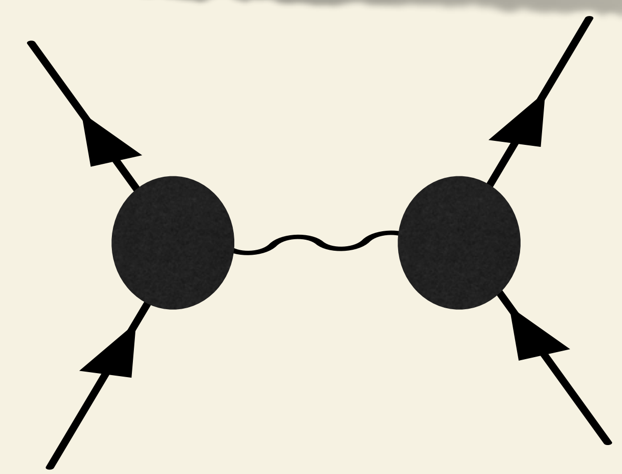
Infinite derivative Gravity

$$S = \int d^4x\sqrt{-g}\left[M_p^2\frac{R}{2} + R\left[\frac{e^{-\Box/M_s^2}-1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\Box/M_s^2}-1}{\Box}\right]R^{\mu\nu}\right]$$

$k < M_s$



$k \geq M_s$



$$\Pi(k^2) = \frac{1}{a(k^2)}\left[\frac{P^{(2)}}{k^2} - \frac{P^0}{2k^2}\right] \qquad a(k^2) = e^{k^2/M_s^2}$$

Massless Graviton, massless spin-2 and spin-0 components propagate

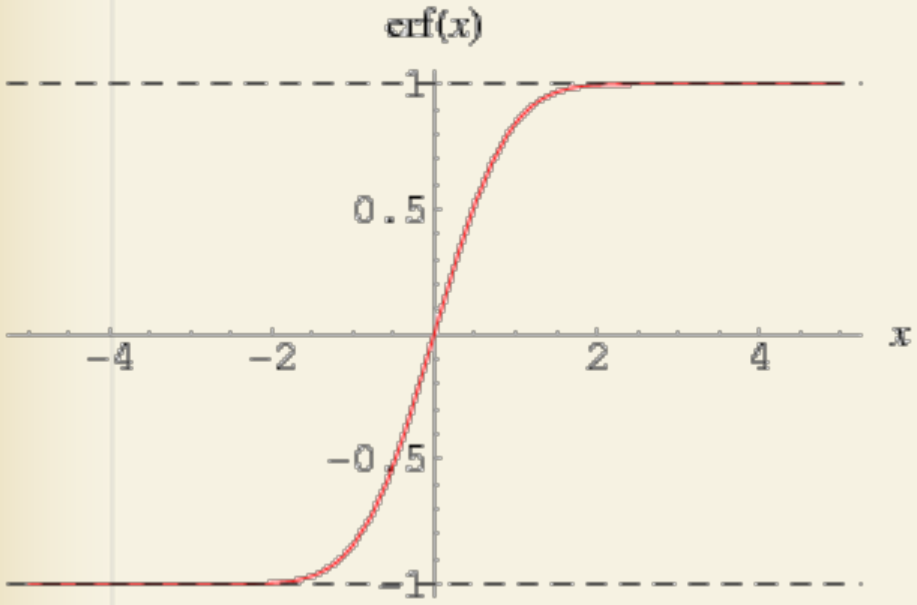
Biswas, AM, Siegel (2006) JCAP, Biswas, Gerwick, Koivisto, AM (2012) Phy. Rev. Lett.

Non-Local Gravitational Potential

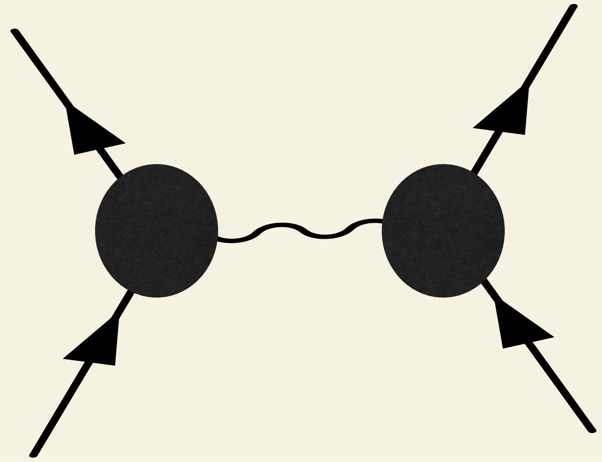
$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r}\mathbf{erf}\left(\frac{rM}{2}\right)$$



Interaction becomes Non-Local

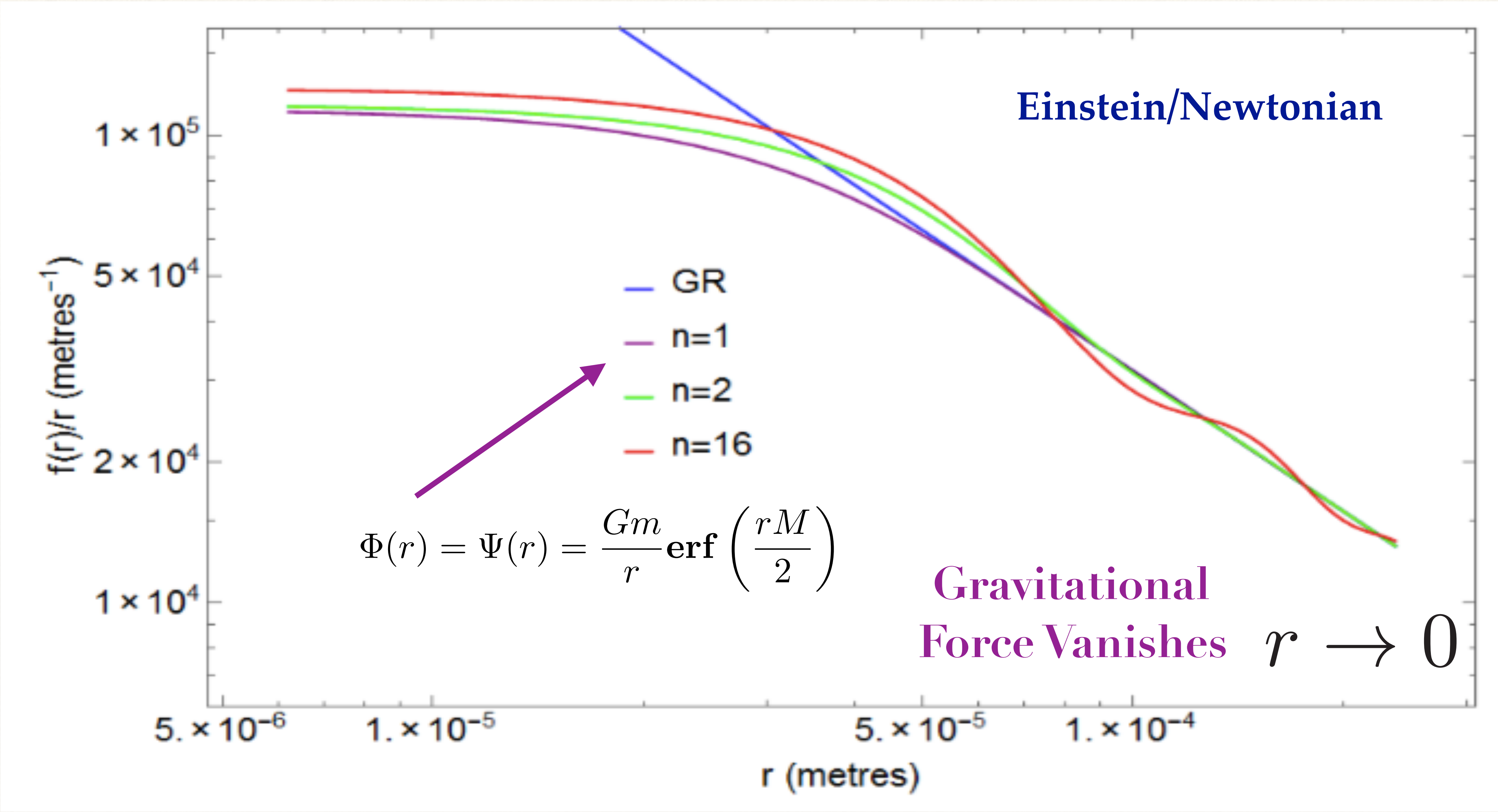


Biswas, Gerwick, Koivisto, AM, Phys. Rev. Lett. (2012) (gr-qc/1110.5249)

Resolution of Singularity at short distances

$$a(\Box) = e^{\gamma(\Box)}$$

Any Entire Function: $\gamma(\Box) = -\frac{\Box}{M^2} - \sum_N a_N \left(\frac{\Box}{M^2}\right)^N$



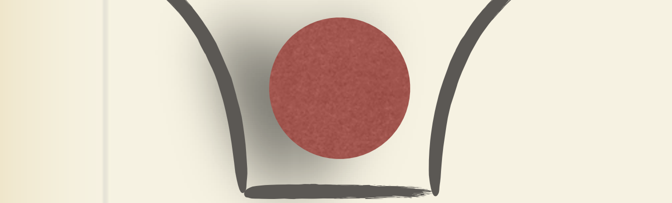
$$mM \ll M_p^2 \implies m \ll M_p$$

Current Bound : M > 0.01 eV $\quad m \leq 10^{25}$ grams

Biswas, Gerwick, Koivisto, AM (2012), Edholm, Koshelev, AM (2016), Frolov & Zelnikov (2015, 2016)

Astrophysical Blackhole

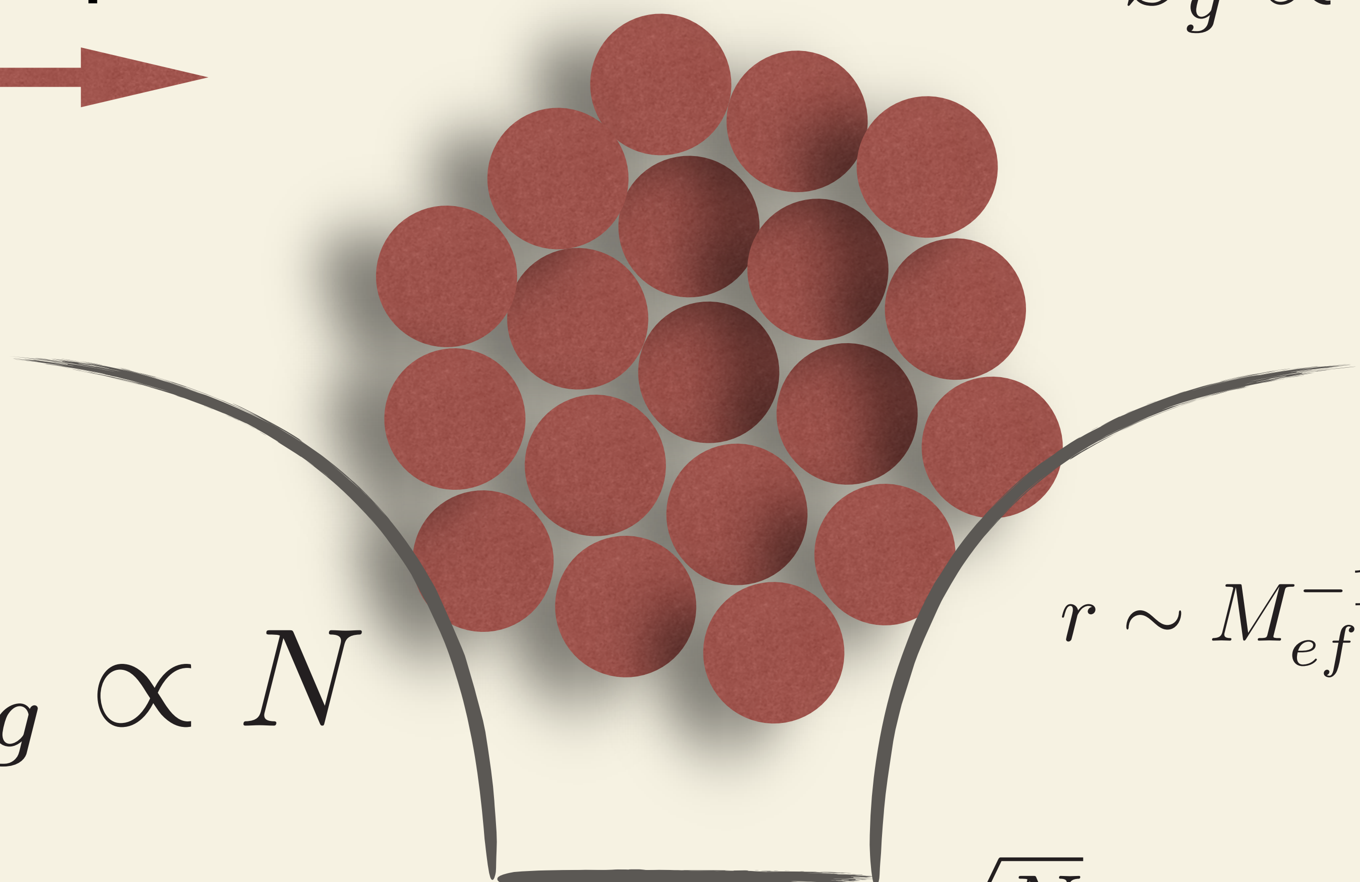
m



N-copies

$$M_{ns} = mN$$

$$S_g \propto \frac{Area}{4G}$$



$$r \sim M_s^{-1}$$

$$\Phi \sim \frac{mM_s}{M_p^2} < 1$$

$$S_g \propto \frac{Area}{4G}$$

$$S_g \propto N$$

$$r \sim M_{eff}^{-1} \sim \frac{\sqrt{N}}{M_s}$$

$$r \sim \frac{\sqrt{N}}{M_s} \geq r_{sch} \sim \frac{2M_{ns}}{M_p^2}$$

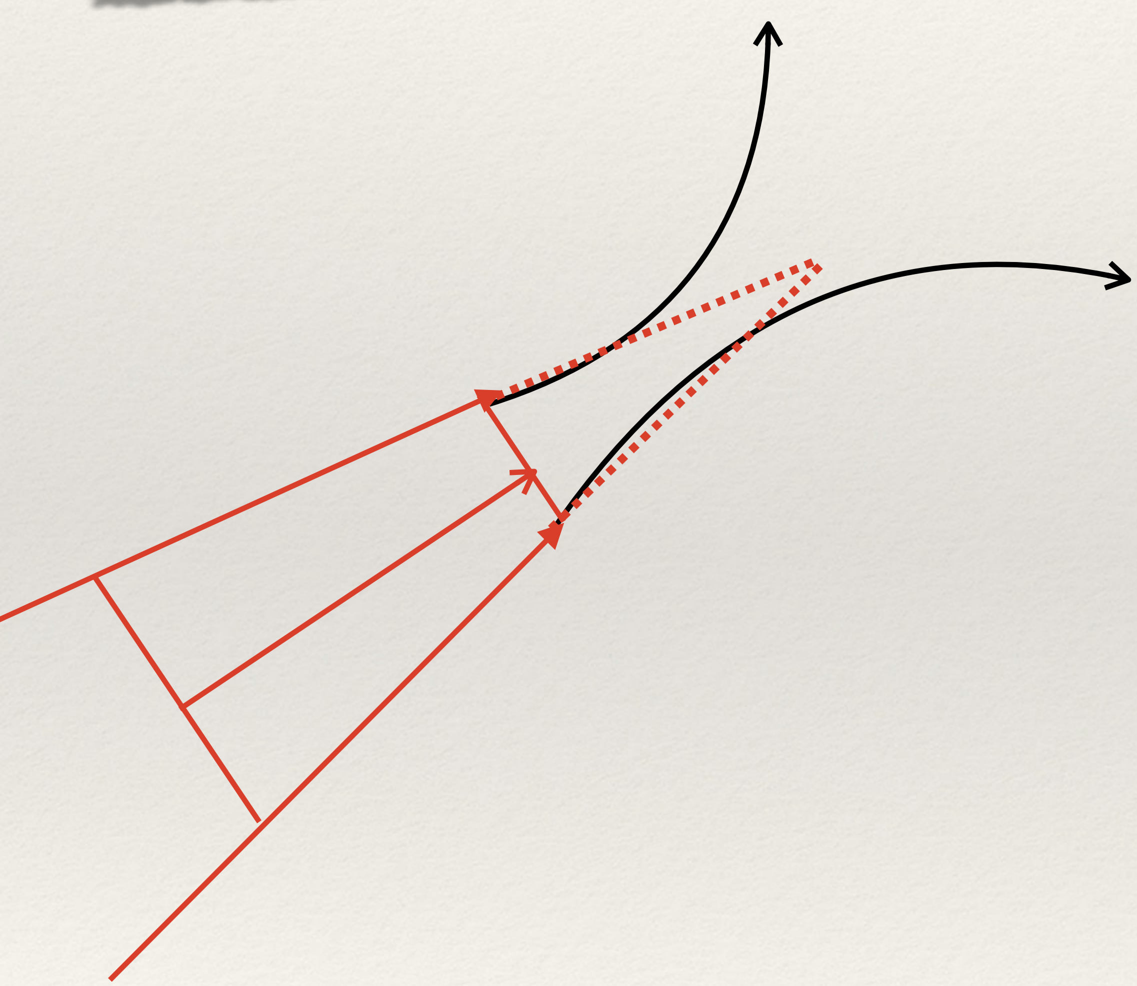
Length scale of Non-locality shifts from UV to IR

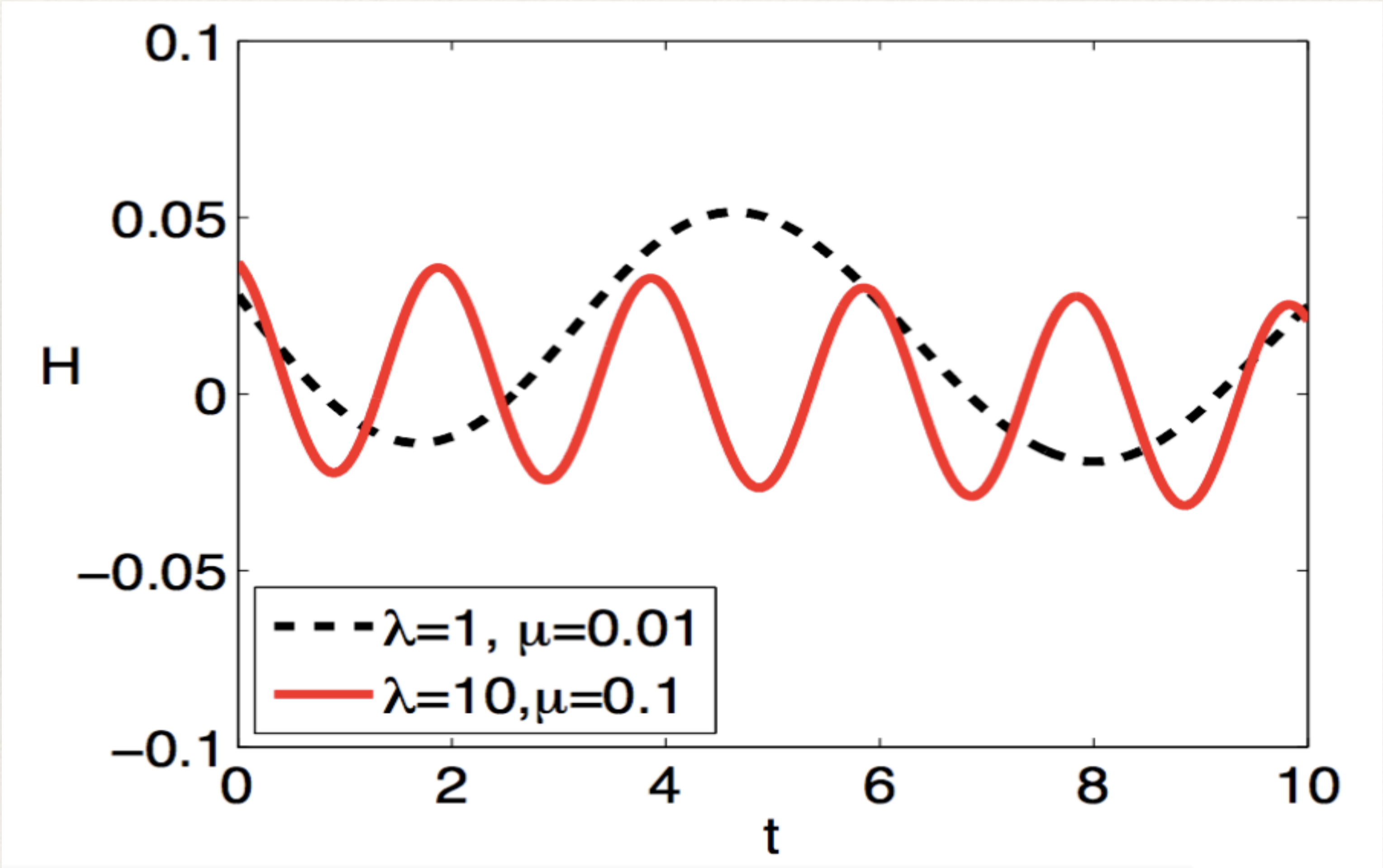
$$M_{eff} = \frac{M_s}{\sqrt{N}}$$

Koshelev, AM, PRD (2017)

Non-singular Cosmology

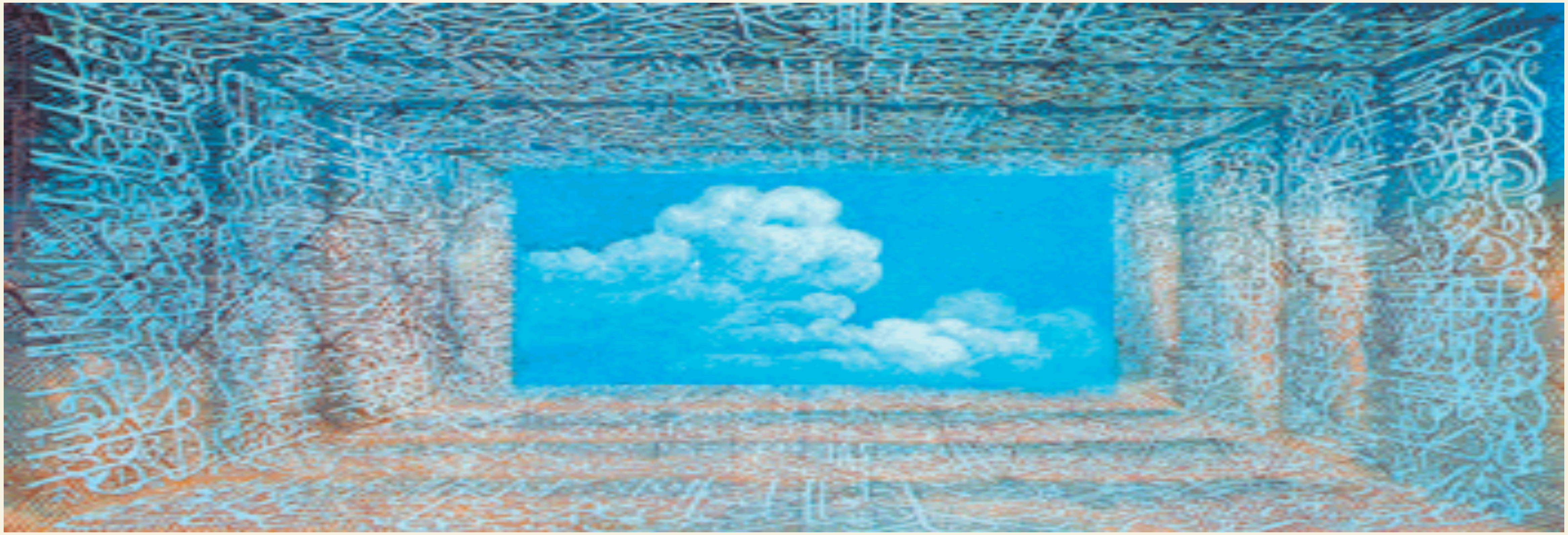
$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}}-1}{\Box}\right]R^{\mu\nu}\right]$$





Biswas, AM, Siegel (2007), Biswas, Koivisto, AM (2011), Biswas, Koshelev, AM , Vernov (2014)

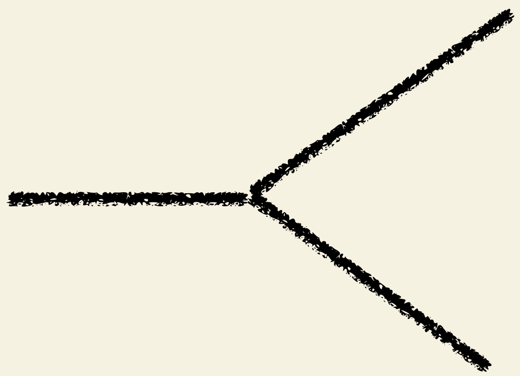
Applications of Non-locality in Higgs



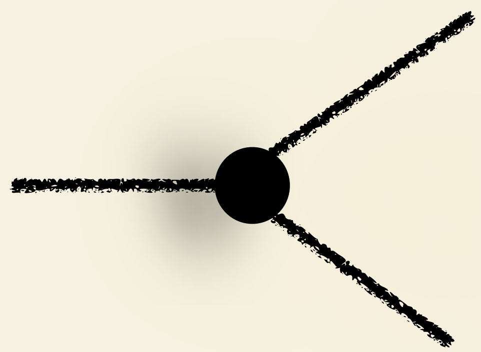
At high energies higher derivatives in the Higgs sector !

Local vs Non-Local Field Theory

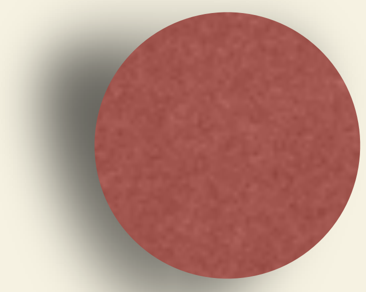
$$S = \int d^4x \left[-\frac{1}{2}\phi e^{\frac{\Box + m^2}{M^2}}(\Box + m^2)\phi - \frac{\lambda}{4!}\phi^4 \right] \qquad \Pi(p^2) = -\frac{ie^{-\frac{p^2+m^2}{M^2}}}{p^2+m^2}$$



$P^2 < M^2$



$P^2 \geq M^2$



$r \sim M^{-1}$

Scale of Non-Locality

$$\delta m^2 \sim \lambda M^2 \qquad \Gamma_4 \sim -\lambda^2 e^{-2m^2/M^2}\left[1 + \mathcal{O}(m^2/M^2)\right]$$

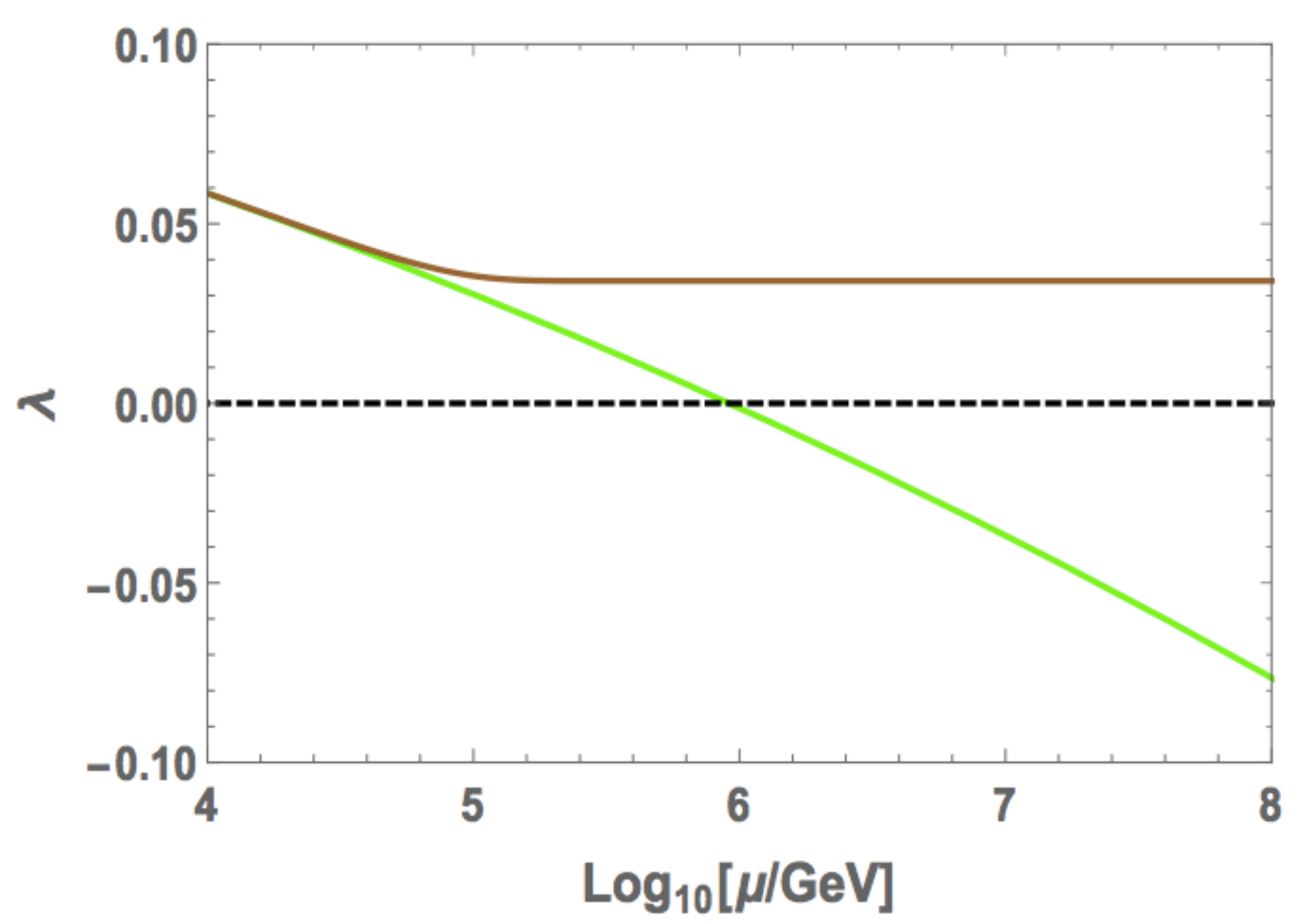
$$\sigma_{NL}(f\bar{f} \to f'\bar{f}') = e^{-s/M^2}\sigma_L(f\bar{f} \to f'\bar{f}')$$

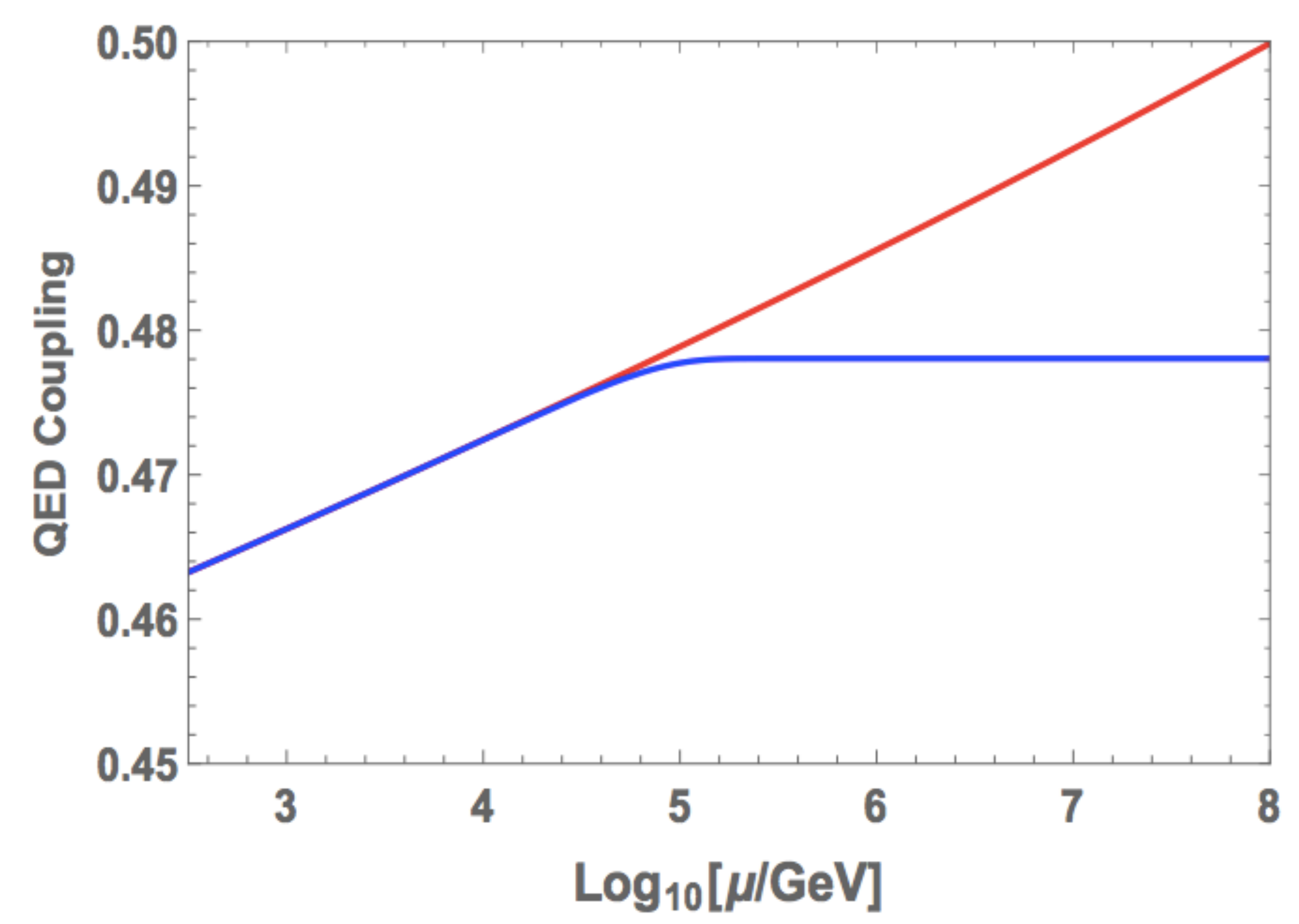
Moffat Phys.Rev.D (1990), Biswas + Okada, Nucl. Phys. B (2016)

Freezing Higgs Interactions

$$\mathcal{L} = -\frac{1}{2}\phi e^{\frac{\Box+m_\phi^2}{M^2}}\left(\Box + m_\phi^2\right)\phi + i\bar{\psi}e^{\frac{\Box+m_\psi^2}{M^2}}\left(\gamma^\mu\partial_\mu - m_\psi\right)\psi$$
$$-\lambda\phi^4 - y\phi\bar{\psi}\psi + h.c. \qquad \textbf{Abelian Higgs} \qquad (1)$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}e^{\frac{\Box}{M^2}}F_{\mu\nu} + i\bar{\psi}e^{\frac{\nabla^2}{M^2}}\gamma^\mu D_\mu\psi + h.c.$$





Ghoshal, AM, Okada, Villalba (2017)

Implications for Higgs Cosmology beyond the scale of Non-locality

Towards Asymptotic Freedom

Weakens Interactions in the UV

Weakening of Gravitational Interaction Smears out Singularities & Event Horizon

Provide Stability to the Abelian Higgs Potential

Higgs Interactions in the UV become frozen, the Abelian-Higgs becomes non-dynamical in the UV

Blackhole vs Non-Singular Compact Object (NSCO)