The fate of the oscillating false vaccum

Luc Darmé

NCBJ (Warsaw)

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Tunneling for a dynamical field

- Vacuum (un-)stability in QFT $\rightarrow$ key effect for many topics (e.g. Baryogenesis, GW, SM (meta)-stability, inflation, etc...)
- Since Coleman's seminal papers, much work devoted to thermal, gravitational and quantum corrections to the vacuum-to-vacuum vanilla case.
- Tunneling for a field classically evolving (e.g. inflation models, cosmological relaxation), where the initial state is time-dependent is less studied.
  $\rightarrow$ focus in this talk on a oscillating field around a local minimum.

Contents

Decay rate from an oscillating state

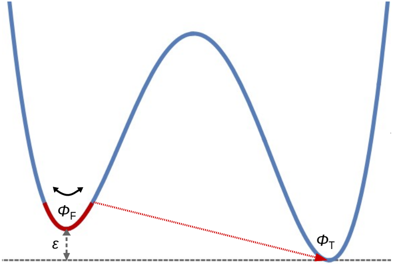
The fate of the false vacuum

- What is the probability in QFT for a quantum tunneling from one minimum to a deeper one?
- Find the “best” field configuration ϕ_0 interpolating between false and true vacuum
  - Be defined both in euclidean/imaginary time (during tunneling $\to$ Most Probable Escape Path (MPEP))
  - and real time (after the tunneling $\to$ classical “bubble” solution)

$$\frac{\Gamma}{V} = Ae^{-S}(1 + \mathcal{O}(\hbar)) \ .$$

- S is then related to the euclidean action for ϕ_0 and A to the quantum correction along this path

Let us be concrete



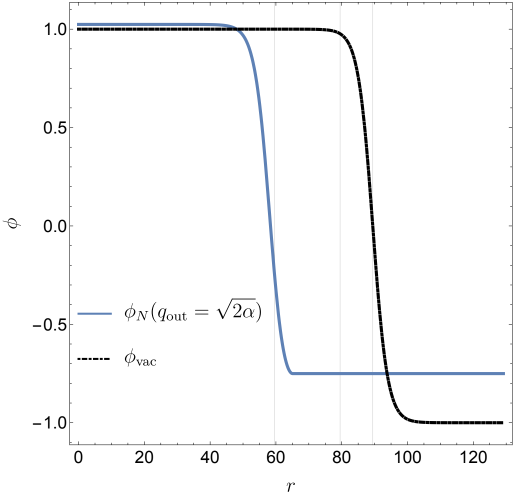
$$V = \frac{\mu^2}{8}(\phi^2 - 1)^2 - \frac{\varepsilon}{2}(\phi + 1)$$

- μ^{-1} is the "thickness" of the wall
- $R_{vac} = \frac{2\mu}{\varepsilon}$ is the radius of the bubble .
- "thin-wall" approximation $\alpha_{\rm tw} \equiv \frac{\mu^{-1}}{R_{vac}} \ll 1$

- Typically, a classical bubble of initial radius R_0 follows

$$\phi_0 = -\tanh\left[\frac{1}{2\alpha_{\rm tw}}\left(\frac{\sqrt{|x|^2 - t^2}}{R_0} - 1\right)\right]$$

Let us be concrete - 2



- Harmonic homogeneous oscillations

$$\varphi_{\text{out}} = (-1 + q_{\text{out}} \cos \mu t) \ ,$$

- Two scales involved: μ^{-1} for the oscillations and R_0 for the wall evolution

$\rightarrow$ thin-wall limit implies hierarchy of scales.

Membrane action

- Idea $\rightarrow$ parametrise the solution by the radius of the bubble $R(t)$, the tension σ of the wall and the differential pressure p

$$\sigma \equiv \int_{\varphi_{\text{in}}}^{\varphi_{\text{out}}} \sqrt{2V(\phi)} = \frac{2}{3}\mu \quad \text{and} \quad p \equiv \epsilon - \left(\frac{1}{2}\dot{\varphi}_{\text{out}}^2 - V(\varphi_{\text{out}})\right)$$

  - We are using the pressure here, not the energy density ρ!
  - For our oscillating state,

$$\rho = -\epsilon\,(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}}) \qquad \text{while} \qquad p = \epsilon\,(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}}\cos 2\mu t)$$

- This leads to the simple "bubble" Lagrangian

$$\mathcal{L}_m = -4\pi\sigma R^2\sqrt{1-\dot{R}^2} + \frac{4}{3}\pi p R^3$$

Estimating the decay rate

- The radius of the bubble is fixed by balancing the energy density gain inside the bubble and the wall tension:

$$R_0 = \frac{3\sigma}{|\rho|} = R_{vac}\left(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}}\right)^{-1} < R_{vac}$$

$\rightarrow$ Increasing the oscillations decreases the bubble radius

- At the extremum of the oscillations, $\dot{\phi} = 0 \rightarrow$, situation similar to the standard Coleman result:

$$\Gamma_{ext} \propto e^{-S_{ext}} \qquad \text{with} \qquad S_{ext} \equiv \frac{\pi^2\sigma}{2} R_0^3$$

- Since the radius is smaller, the exponential term is reduced by

$$\frac{S_{ext}}{S_{vac}} = \left(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}}\right)^{-3}$$

Numerical approach

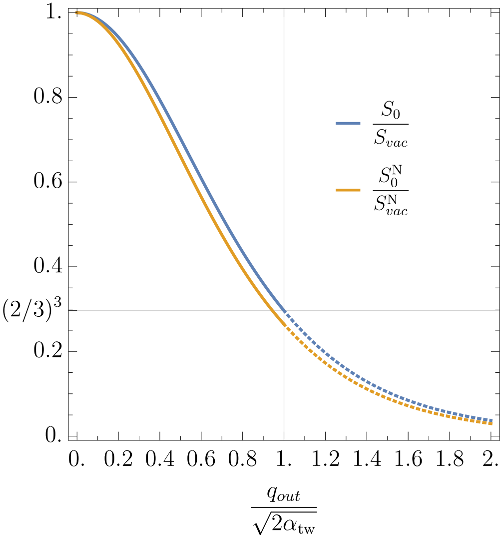


Figure: Ratio of the decay rate exponent and of the bubble radius over the no-oscilation case for $g = 1/10$, $b = 1/300$ and $c = 1$. The dotted line indicates a contracting bubble.

Time-dependent decay rate

- During the oscillations, the decay rate varies as

$$\Gamma(t) \propto \exp\left[-S_{ext}\left(1+\mathcal{O}(1)\frac{q_{\text{out}}^2}{\alpha_{\text{tw}}}\sin^2\mu t\right)\right] \ .$$

- If the field oscillations are fast compared to the other relevant time scale (e.g. the Hubble rate) $\rightarrow$ integrate over a period:

$$\left\langle\ \Gamma(t)\ \right\rangle \ \propto\ \frac{1}{\sqrt{S_{ext}q_{\text{out}}^2/\alpha_{\text{tw}}}}\exp\left[-S_{ext}\right]\ , \qquad (1)$$

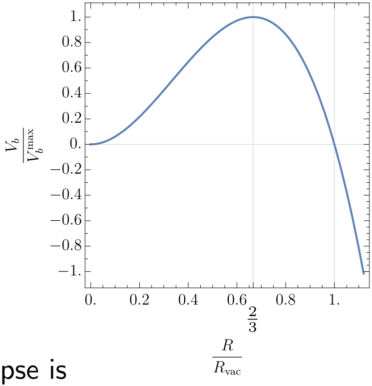
- As expected, the probability is dominated by the extremum contribution.

Growing or collapsing bubble?

Initial evolution of the bubble

- Given a bubble with a fixed tension, differential pressure and radius, will it grow or collapse?
- For small wall velocity, expand the action in potential + kinetic term.

$$V_{bubble} \;=\; 4\pi\sigma R^2 - \frac{4}{3}\pi p R^3 \;.$$



- The critical radius for growth/collapse is

$$R_c = \frac{2}{3\mu\alpha_{\rm tw}}(= \frac{2}{3}R_{vac})$$

Criterium for initial growth/collapse

- What is V_{bubble} in the oscillating field case?
  - $\mu^{-1} \ll R_0$ implies the field oscillated fast compared to the bubble evolution $\rightarrow$ one can average.
  - Since

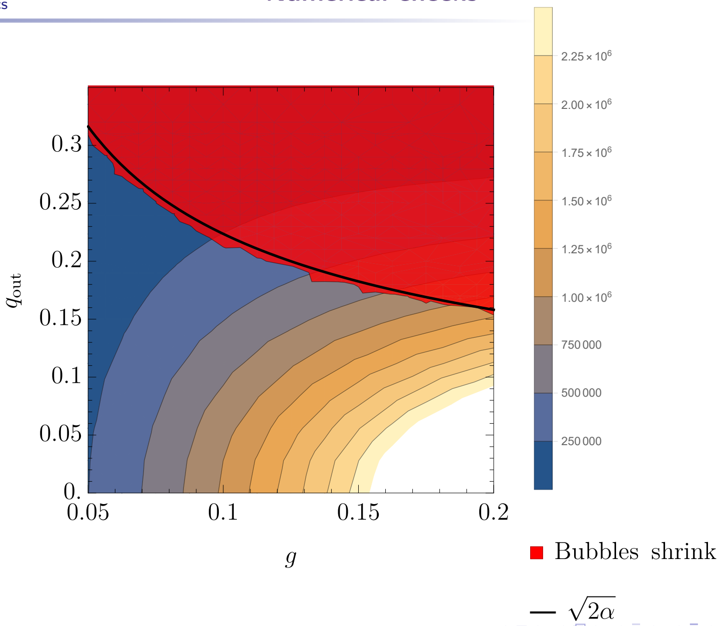
$$\langle p \rangle = \left\langle \, \epsilon \, (1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}} \cos 2\mu t) \right\rangle \sim \epsilon$$

$\rightarrow$ the potential has the same form as the vaccum one.

- The initial radius $R_0 = R_{vac} \left(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}}\right)^{-1} < R_{vac}$ is time-independent
  $\rightarrow$ the bubble becomes unstable when $R_0 = \frac{2}{3} R_{vac}$
- Overall we find

$$q_{\text{out}}^2 > 2\alpha_{\text{tw}} \; . \qquad \Leftrightarrow \qquad q_{\text{out}}^2 > \epsilon \mu^{-2}$$

Numerical checks



Outlooks and conclusion

Much more to do

- The time-dependence of the tunneling rate should be better understood/controlled
- For $q^2_{\text{out}} > \epsilon\mu^{-2}$, what is the "true" phase transition rate?
- Quantum effects must be included, e.g.:
  - the usual Coleman corrections
  - fluctuations growth (e.g. parametric resonance ...)
- Explore phenomenological consequences
  - Check for vacuum stability in dynamical scenarios
  - Gravitational Wave production in this setup
  - Possibility of resonant tunneling.

Take-home status

- First steps toward comprehensive understanding of tunneling from an oscillating state.
- Expected: tunneling rate dominated by its value at the extremum of the oscillations
  - Exponential increase of the tunneling rate
- Unexpected: for $q_{\text{out}}^2 > \epsilon\mu^{-2}$, the most probable bubbles collapse $\to$ suppressed rate.
  - Checked numerically $\to$ robust criterium

Backup slides

Time evolution - Theory

- Initial radius obtained from energy conservation $\rightarrow$ constant, same as extremum case
- First estimation: the field is "frozen" during the tunneling
  - The pressure is

$$p_0 = \epsilon \left(1 + \frac{q_{\text{out}}^2}{4\alpha_{\text{tw}}} \cos 2\mu t_0\right) .$$

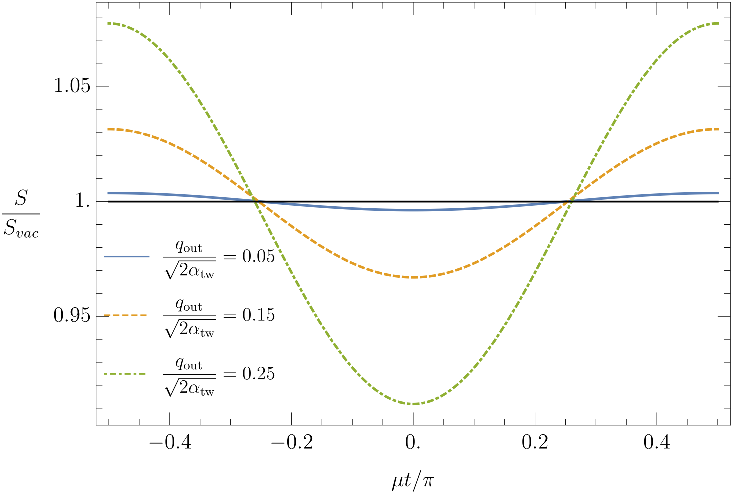
  - The action in Euclidean time is

$$U = i \int_{-R_0}^{0} d\tau \left[-4\pi\sigma R^2 \sqrt{1 + R'^2} + \frac{4}{3}\pi p_0 R^3\right]$$

  - Suppose bubble radius grows as the vacuum-to-vacuum case

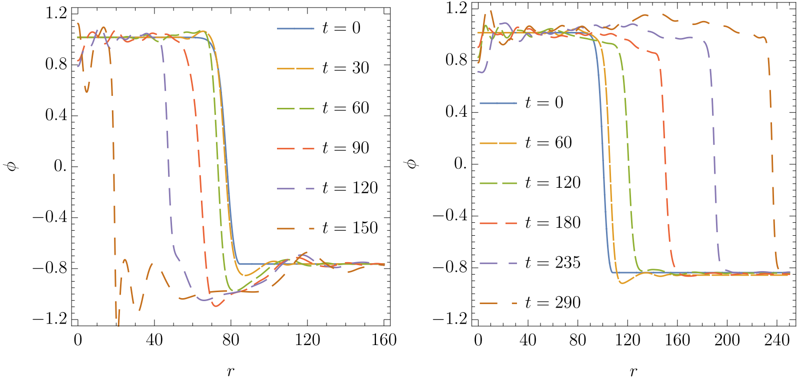
$$R = \sqrt{R_0^2 - \tau^2} \text{ with } \tau \subset [-R_0, 0\ ] .$$

Time evolution



Ratio of the decay rate exponential exponent S over the vacuum-to-vacuum one S_{vac} as function of $\mu t/\pi$ for various values of $q_{\rm out}/\alpha_{\rm tw}$. The extremum of the oscillation occurs at $t = 0$.

Evolution examples



Field profiles showing lattice bubble evolution for oscillation reaching $1.2\sqrt{2\alpha_{\rm tw}}$ (left panel) and $0.8\sqrt{2\alpha_{\rm tw}}$ (right panel). The values defining the potential were set to $g = 1/10$, $b = 1/300$ and $c = 1$.