Self-interacting dark matter from a Breit-Wigner resonance

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multicomponent scenarios and beyond

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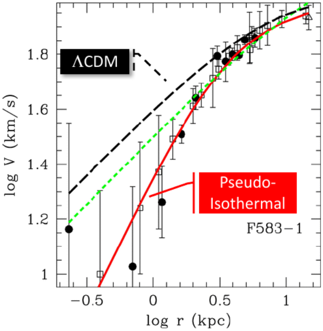
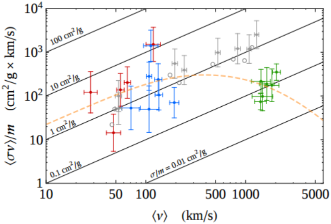
Self-interaction cross section

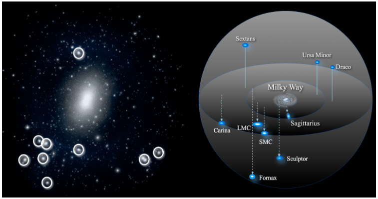
$$\frac{\sigma_{\text{self}}}{m} \approx 0.1 - 1\,\frac{\text{cm}^2}{\text{g}} \sim \frac{\text{barn}}{\text{GeV}}$$



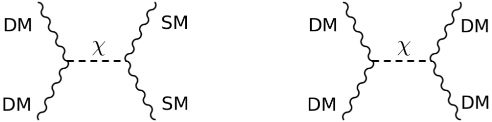
Kuzio de Narray+ 2008



Hai-Bo Yu+ 2015



Breit-Wigner resonance $2M_{DM} \approx M_\chi$



- special methods to treat resonantly enhanced annihilation

  Gondolo, Gelmini 1991, Griest, Seckel 1991

- enhancement of low velocity annihilation rates

  Ibe et al., 2009

- enhancement of the self-interaction cross-section

Boltzmann equation

$$\frac{dY}{dx} = -\alpha\frac{\langle\sigma v\rangle}{x^2}(Y^2 - Y_{\rm EQ}), \qquad \text{DM yield } Y = n/s, \qquad \alpha = \frac{s(m)}{H(m)}$$

- **entropy** s in coming volume is conserved, dimensionless parameter $x = m/T$

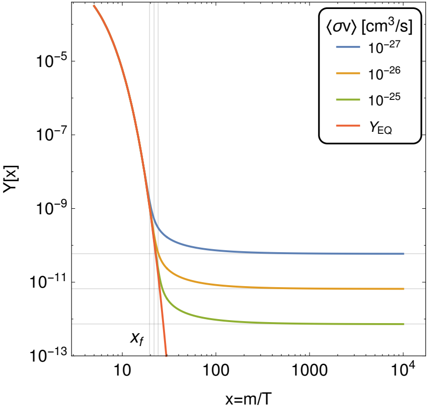
Approximate solutions

$$\frac{1}{Y_\infty} - \frac{1}{Y(x_d)} = \alpha\int_{x_d}^{\infty}\frac{\langle\sigma v\rangle}{x^2}$$

$\langle\sigma \mathbf{v}\rangle =$ **const**, $\qquad Y(x_d) \gg Y_\infty$

$$Y_\infty \approx \frac{x_d}{\alpha\langle\sigma v\rangle_0}$$

generalizations $\langle\sigma v\rangle \sim x^{-m}(1 + ax^{-n})$



WIMP miracle $m_{\rm DM} \sim 100$ GeV, $\langle\sigma v\rangle \approx 2\times10^{-26}\ {\rm cm^3 s^{-1}} \to \Omega_{DM}h^2 \approx 0.1$

Cross-section

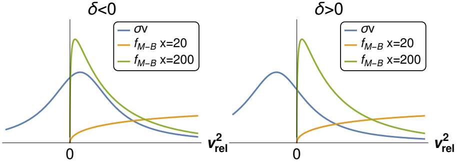
$$\sigma v_{\mathrm{rel}} \sim \frac{1}{(s - M_R^2)^2 + \Gamma^2 M_R^2} \approx \frac{1}{(\delta + v_{\mathrm{rel}}^2/4)^2 + \gamma^2}$$

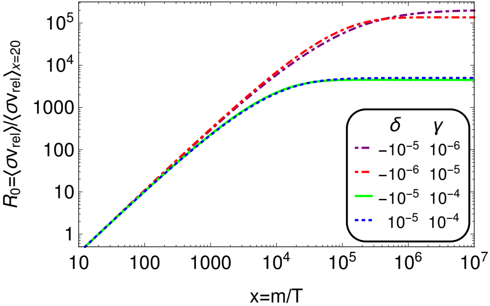
- M_R - mass of the resonance
- $\delta = 4m_{DM}^2/M_R^2 - 1 \ll 1$
- $\gamma = \Gamma/M_R \ll 1$

Thermal average with Maxwell-Boltzmann distribution

$$\langle \sigma v \rangle = \frac{x^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv_{\mathrm{rel}} v_{\mathrm{rel}}^2 e^{-x v_{\mathrm{rel}}^2/4} \sigma v_{\mathrm{rel}}$$

Physical/unphysical region





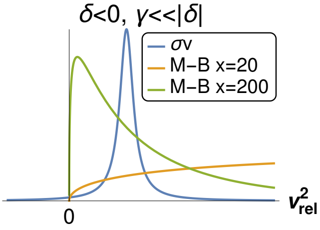
- $\langle \sigma v \rangle$ grows with decreasing T – annihilation lasts long after decoupling
- $\langle \sigma v \rangle$ reaches maximum for $x \approx (\max[|\delta|, \gamma])^{-1}$

Cross-section

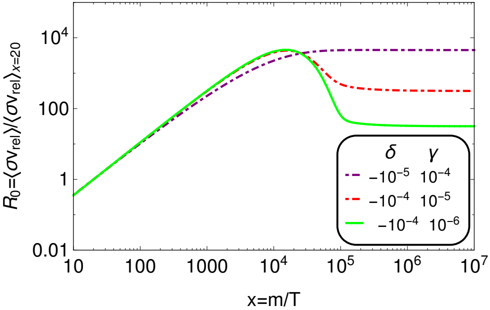
$$\sigma v_{\rm rel} \sim \frac{1}{(s-M_R^2)^2+\Gamma^2 M_R^2} \approx \frac{1}{(\delta + v_{\rm rel}^2/4)^2+\gamma^2}$$

- M_R - mass of the resonance
- $\delta = 4m_{DM}^2/M_R^2 - 1$
- $\gamma = \Gamma/M_R$

Narrow peak in the kinematically accessible region



- for a proper energy DM annihilates at the peak of the resonance
- **not suitable to enhance self-interaction**



- $\langle\sigma v\rangle$ has similar behaviour up to its maximum as in the previous case
- for $x \gtrsim (\max[|\delta|,\gamma])^{-1}$ $\langle\sigma v\rangle$ decreases by factor $\gamma/|\delta|$

Relic abundance - approximate formulas

$$\frac{1}{Y_\infty} - \cancel{\frac{1}{Y(x_d)}} = -\alpha \int_{x_d}^{\infty} \frac{\langle \sigma v \rangle}{x^2} = -\alpha \int_{x_d}^{\infty} dx \frac{1}{2\sqrt{\pi x}} \int_0^{\infty} dv_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \sigma v_{\rm rel}$$

Change the order - integral over x

$$\frac{\mathrm{erfc}(v_{\rm rel}\sqrt{x_d}/2)}{v_{\rm rel}} \approx \frac{1}{v_{\rm rel}} - \sqrt{\frac{x_d}{\pi}}$$

First approximation

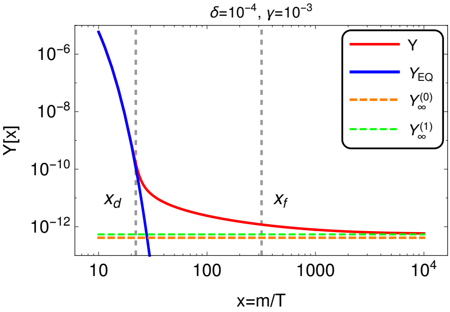
$$\frac{1}{Y_\infty^{(0)}} = \int_0^{\infty} \alpha \langle \sigma v \rangle_0 v_{\rm rel} \frac{\delta^2 + \gamma^2}{(\delta + v_{\rm rel}^2/4)^2 + \gamma^2} =$$

$$= \alpha \langle \sigma v \rangle_0 (\delta^2 + \gamma^2) \frac{\pi - 2\arctan(\delta/\gamma)}{\gamma}$$

"Freeze-out" temperature

$$Y_\infty \approx x_f / (\alpha \langle \sigma v \rangle_{\rm max})$$

$$x_f = \begin{cases} (\pi\gamma)^{-1}, & \text{if } \gamma \gg |\delta|, \\ (2\delta)^{-1}, & \text{if } |\delta| \gg \gamma > 0, \end{cases}$$



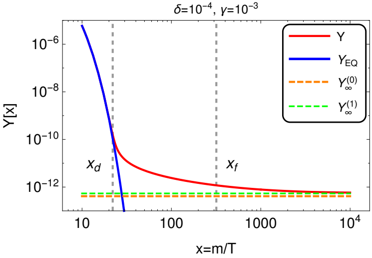
- effective annihilation after decoupling
- at "freeze-out" temperature $\langle \sigma v \rangle$ reaches its maximal value

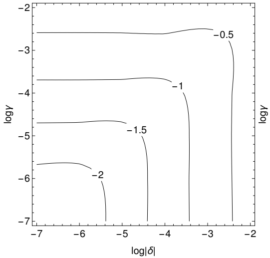
Relic abundance - approximate formulas

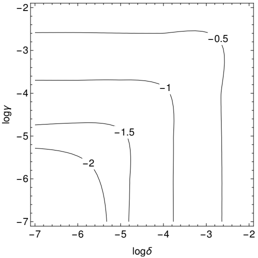
$$\int_{x_d}^{\infty} \frac{\exp(-x v_{\rm rel}^2/4)}{2\sqrt{\pi x}} dx = \frac{\mathrm{erfc}(v_{\rm rel}\sqrt{x_d}/2)}{v_{\rm rel}} \approx \frac{1}{v_{\rm rel}} - \sqrt{\frac{\mathbf{x_d}}{\pi}}$$

Second approximation - dependence on x_d

$$\frac{1}{Y_\infty^{(1)}} = \alpha\langle\sigma v\rangle_0 \times \begin{cases} \gamma(\pi - 2\sqrt{2\pi x_d \gamma}), & \text{if } \gamma \gg |\delta|, \\ \delta(2 - 2\sqrt{\pi x_d \delta}), & \text{if } \delta \gg \gamma > 0, , \\ \delta^2\gamma^{-1}(2\pi - 4\sqrt{\pi x_d |\delta|}), & \text{if } -\delta \gg \gamma > 0. \end{cases}$$



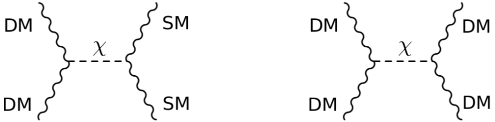




$Y_\infty \sim x_f/\langle\sigma v\rangle_0$ **$\langle\sigma v\rangle_0$ can be many times larger than 2×10^{-26} cm^{3}g^{-1}**

Self-interaction from the Higgs resonance

We need a mediator coupled to the SM and DM



Abelian vector dark matter

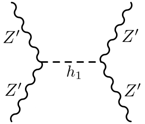
- extra complex scalar S charged under $U(1)_X$, VEV $\langle S \rangle = v_X$
- scalar mixing angle α, two mass eigenstates h_1, h_2
- **dark matter** candidate $U(1)_X$ **vector boson**, $M_{Z'} = g_x v_x \leftarrow$ Higgs mechanism

Higgs couplings

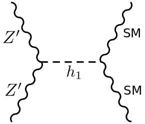
$$\mathcal{L} \supset \frac{h_1 \cos\alpha + h_2 \sin\alpha}{v} \left(2 M_W W_\mu^+ W^{\mu -} + M_Z^2 Z_\mu Z^\mu - \sum_f m_f \bar{f} f \right) + \frac{h_2 \cos\alpha - h_1 \sin\alpha}{v_x} M_Z^2 Z'_\mu Z'^\mu$$

Self-interaction from the Higgs resonance

Resonance with the SM-like Higgs $2M_{DM} \approx M_{h_1}$



- decay width $\Gamma_{h_1} \approx 4$ **MeV**, $\gamma = \Gamma_{h_1}/M_{h_1} \approx 3.2 \times 10^{-5}$
- no invisible Higgs decays $2M_{Z'} > M_{h_1}$,
- fine-tuning $\delta = 4M_{Z'}^2/M_{h_1} - 1 \ll \gamma$



$g_x < 4\pi$(petrubativity)

$|\sin\alpha| < 0.36$ (ATLAS+CMS)

$$\frac{\sigma_{\rm self}}{M_{Z'}} \sim \frac{\sin^4\alpha}{\delta^2 + \gamma^2} < 1.1\ \mathbf{cm}^2\mathrm{g}^{-1}$$

DM abundance $\Omega_{DM}h^2 \sim x_f/\langle\sigma v\rangle_0$

non-resonant case	$\langle\sigma v\rangle_0 \approx 2 \times 10^{-26}\ \mathrm{cm}^3\mathrm{s}^{-1}$, $x_f = 20$
Higgs resonance	$\langle\sigma v\rangle_0 \approx 10^{-19}\ \mathrm{cm}^3\mathrm{s}^{-1}$, $x_f = 1/(\pi\gamma) = 10^4$

$$\sigma_{\rm self}/M_{Z'} \sim (g_x \sin\alpha)^4$$
$$\langle\sigma v\rangle_0 \sim (g_x \sin\alpha)^2$$

$$\sigma_{\rm self}/M_{Z'} \lesssim 10^{-8}\ \mathbf{cm}^2\mathrm{g}^{-1}$$

Resonance with the second scalar and bounds from indirect searches

Cross sections

$\sigma_{\text{self}}/M_{Z'} \propto (g_x \cos\alpha)^4$

$\langle\sigma v\rangle \propto (g_x \cos\alpha \sin\alpha)^2$

can be suppressed by

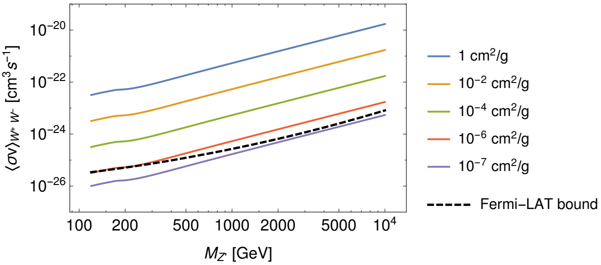
$\alpha \ll\sim 10^{-5} - 10^{-7}$.

$\delta > 0$: $\cancel{h_2 \to Z'Z'}$ $\delta \gg \gamma$

$\delta < 0$: $\gamma \gg |\delta|$

$\Gamma_{Z'Z'}$ phase space suppressed

Fermi-LAT constraints



Lower bound on annihilation rate $\langle\sigma v\rangle$

$$\langle\sigma v\rangle_0 \gtrsim \frac{2.2\cdot 10^2}{\epsilon\eta}\left(\frac{M_{DM}}{100\ \text{GeV}}\right)^{3/2}\left(\frac{\sigma_{\text{self}}/M_{DM}}{1\ \text{cm}^2/\text{g}}\right)^{1/2}\left(\frac{100}{g_*}\right)^{1/2}\left(\frac{0.12}{\Omega_{DM}h^2}\right) 2\times 10^{-26}\ \text{cm}^3\text{s}^{-1}$$

$\epsilon \in \{2, \pi\}$ – depends on the parameters of the resonance δ, γ

$\eta < \eta_{\max}$ – limited by perturbativity (VDM $\eta_{\max} = 3/16$)

$$\eta = \frac{\Gamma B_{DM}}{M\bar{\beta}_{DM}} = (2S+1)\frac{g^2\Lambda^2}{32\pi M^2}.$$

Summary

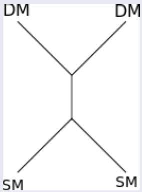
- Thermally averaged **cross-sections** for dark matter annihilation near the resonance **strongly depend on temperature**.
- There exist **approximate formulas for relic density** in terms of annihilation cross-section at low tempretures and parameters of the resonance
- **Self-interaction** rates are **limited** by the **indirect searches**
- **Future**:
  - influence of early kinetic decoupling on DM evolution and bounds indirect detection
  - bounds from late-time annihilations (CMB, BBN)

BACKUP SLIDES

Kinetic decoupling

For $T > T_d$ chemical and kinetic equilibrium is maintained by annihilation processes

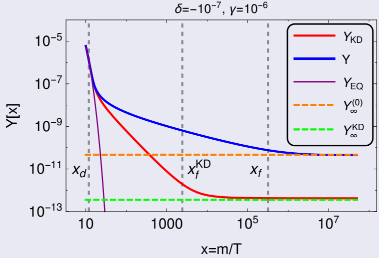
Kinetic decoupling $T = T_{kd}$



$$T > T_{kd}: \qquad T_{DM} = T_{SM} \sim 1/a$$
$$T < T_{kd}: \qquad T_{DM} \sim 1/a^2 \approx T_{SM}^2/T_{kd}$$

Resonant annihilation cross-section grows faster in the expanding universe – more effective annihilation

Xiao-Jun et al. 2011



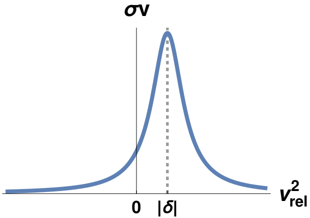
$$T_{kd} \approx T_d$$

$$x_f = \begin{cases} (\pi 5^{1/4}\sqrt{2}\Gamma(\frac{1}{4})\gamma^{3/4})^{-1}, & \text{if } \gamma \gg |\delta|, \\ (\pi 5^{1/4}\Gamma(\frac{1}{4})/2\delta^{3/4})^{-1}, & \text{if } |\delta| \gg \gamma > 0. \end{cases}$$

Annihilation near the resonance

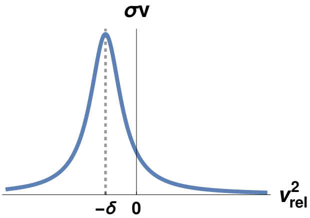
$$\sigma v_{\rm rel} = \frac{\omega}{s}\beta_f \frac{4M^2\bar\Gamma^2 B_i B_f}{\bar\beta_f\bar\beta_i}\frac{1}{(s-M^2)^2+\Gamma^2M^2} \approx \frac{4\omega}{M^2\bar\beta_i}\frac{\bar\gamma^2 B_i B_f}{(\delta+v_{\rm rel}^2/4)^2+\gamma^2}$$

- initial states m_i, final states m_f, resonance M
- statistical factor $\omega = (2S_R+1)/(2S_i+1)^2$
- resonance decay branching ratios B_i, B_f
- phase space $\beta = \frac{1}{8\pi}\sqrt{1-4m^2/s}$, $\bar\beta = \beta|_{s=m}$
- small parameters $\delta = 4m_i^2/M^2 - 1$, $\gamma = \Gamma/M$



Resonance peak in physical region

$2m_i < M$, $\delta < 0$,
$\bar\Gamma = \Gamma$ - physical width
peak is kinematically accesible



Resonance peak in unphysical region

$2m_i > M$, $\delta > 0$,
$\bar\Gamma B_i/\bar\beta \sim g_i$ - coupling constant

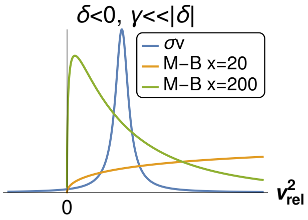
Thermally-averaged cross-sections - another case

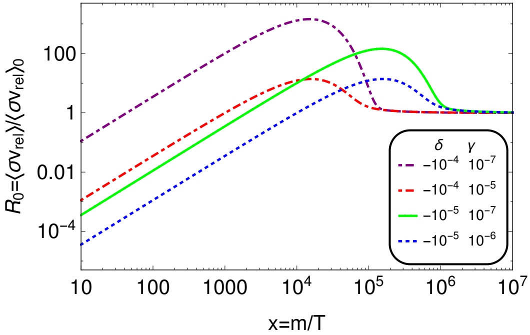
$$\frac{4\omega}{M^2\bar{\beta}_i}\frac{\bar{\gamma}^2 B_i B_f}{(\delta + v_{\rm rel}^2/4)^2+\gamma^2}$$

$\langle v_{\rm rel}^2\rangle = 6/x$

Narrow resonance in physical region $\delta < 0$, $\gamma \ll |\delta|$

- maximum of $\langle\sigma v\rangle$ at $x \approx |\delta|^{-1}$,
- $\langle\sigma v\rangle_{\rm max} = \delta/\gamma\langle\sigma v\rangle_0$





Abelian vector dark matter

Additional complex scalar field S

- singlet of $U(1)_Y \times SU(2)_L \times SU(3)_c$
- charged under $U(1)_X$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + (D_\mu S)^* D^\mu S + \tilde{V}(H, S) \tag{1}$$

$$V(H, S) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 - \mu_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |S|^2 |H|^2 \tag{2}$$

Vacuum expectation values:

$$\langle H \rangle = \frac{v_{SM}}{\sqrt{2}}, \qquad \langle S \rangle = \frac{v_x}{\sqrt{2}} \tag{3}$$

$U(1)_X$ vector gauge boson V_μ

- $D_\mu = \partial_\mu + i g_x V_\mu$
- Stability condition - no mixing of $U(1)_X$ with $U(1)_Y$ $\quad \xcancel{B_{\mu\nu} V^{\mu\nu}}$

  $\mathcal{Z}_2 : V_\mu \to -V_\mu, \qquad S \to S^*, \qquad S = \phi e^{i\sigma} \colon \phi \to \phi,\ \sigma \to -\sigma$
- V_μ acquires mass due to the Higgs mechanism in the hidden sector

$$M_{Z'} = g_x v_x \tag{4}$$

Scalar mixing

$$S = \frac{1}{\sqrt{2}}(v_x + \phi_S + i\sigma_S) \quad , \quad H^0 = \frac{1}{\sqrt{2}}(v + \phi_H + i\sigma_H), \quad \text{where} \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{5}$$

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v^2 & \kappa v v_x \\ \kappa v v_x & 2\lambda_S v_x^2 \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_H \\ \phi_S \end{pmatrix} \tag{6}$$

$M_{h_1} = 125$ GeV - observed Higgs particle

Higgs couplings

$$\mathcal{L} \supset \frac{h_1 \cos\alpha + h_2 \sin\alpha}{v} \left(2 M_W W_\mu^+ W^{\mu -} + M_Z^2 Z_\mu Z^\mu - \sum_f m_f \bar{f} f \right) \tag{7}$$

Resonance with the second scalar

$$\frac{\sigma_{\rm self}}{M_{Z'}} = g_x^4 \frac{M_{Z'}}{8\pi M_{h_2}^4} \frac{\cos^4\alpha}{\delta^2+\gamma^2},$$

$$\delta = (4M_{Z'}^2 - M_{h_2}^2)/M_{h_2}^2, \qquad \gamma = \Gamma_{h_2}/M_{h_2}$$

Mixing angle α

$\langle\sigma v\rangle \sim (\cos\alpha\sin\alpha)^2 \longleftarrow$ **suppressed with the small mixing angle** $\cos\alpha \approx 1$

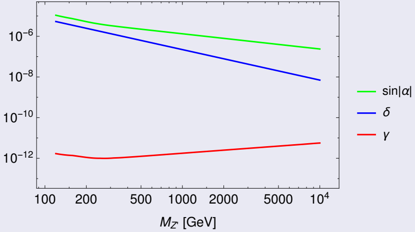
Perturbativity limits

$M_{Z'} = g_x v_x$, $M_{h_2} = \sqrt{2\lambda_S}v_x$, $2M_{Z'} \approx M_{h_2} \implies \lambda_S < 4\pi$, $g_x < \sqrt{2\pi}$

$\sigma_{\rm self}/M_{Z'} = 1$ **cm**^{2}/g

Unphysical pole $2M_{Z'} > M_{h_2}$

Γ_{h_2} suppressed by α^2 (only SM decays)



Physical pole $2M_{Z'} < M_{h_2}$

Γ_{h_2} suppressed by $\sqrt{|\delta|}$ (phase space)