Bounds on $M_{H^\pm}$ from $\bar{B} \to X_{s,d}\gamma$ decay

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based on: [MM & Matthias Steinhauser, Eur. Phys. J. C 77 (2017) 201]

SM – three families of quarks and leptons but only one doublet of scalars

Possible motivations for considering models with more scalars:

(i) Dark matter (if one of the scalars is stable due to symmetries)

(ii) Baryogenesis (if more CP violation is introduced)

(iii) part of SUSY $\leftrightarrow$ hierarchy problem

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Two-Higgs-Doublet Model (2HDM) – the simplest of such models
(impossible to satisfy (i)-(iii) simultaneously)

2HDM particle spectrum: SM with the Higgs doublet H_2 in $(1,2)_{1/2}$
and, in addition, a scalar field H_1 in $(1,2)_{-1/2}$

Physical scalars: h^0, H^0, A^0, $H^\pm$

VEVs: $v_1^2 + v_2^2 = v_{SM}^2, \quad \tan\beta = v_2/v_1.$

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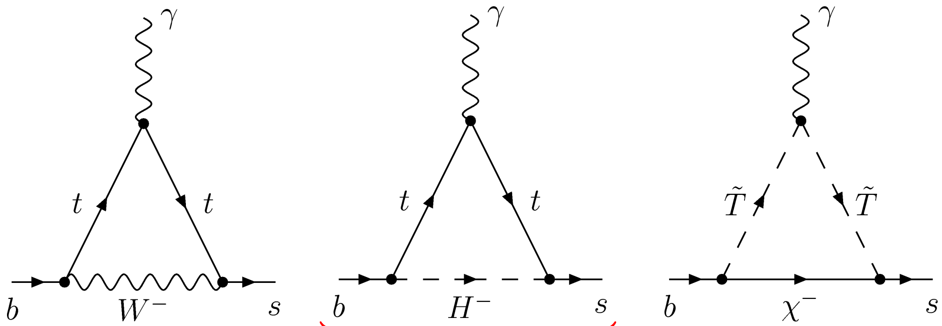
Yukawa couplings to quarks:

Models I and X: $\bar{q}\, Y_u u \widetilde{H_2} + \bar{q}\, Y_d d H_2 + \text{h.c.}$

Models II and Y: $\bar{q}\, Y_u u \widetilde{H_2} + \bar{q}\, Y_d d \widetilde{H_1} + \text{h.c.}$

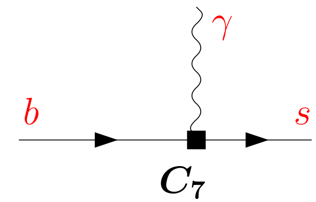
(Model I with Z_2 symmetry $H_1 \to -H_1$) $\xrightarrow{v_1 \to 0}$ **IDM**
(Inert Doublet Model)

Important constraints on beyond-SM physics come from the $b \to s\gamma$ and $b \to d\gamma$ transitions. Heavy-particle contributions to $b \to s\gamma$ are encoded in an effective low-energy local interaction:



$\Rightarrow M_{H^\pm} > \;\sim 580\,\text{GeV}$
in the 2HDM-II

$\longrightarrow$



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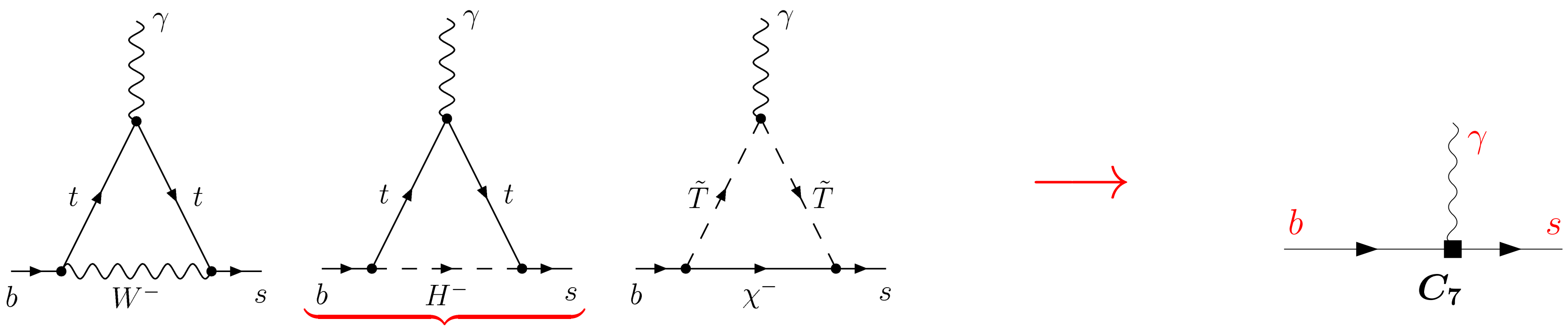

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$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$

$\mathcal{B}_{q\gamma}$, $\mathcal{B}_{c\ell\nu}$: **CP- and isospin-averaged branching ratios of $\bar{B} \to X_q\gamma$ and $\bar{B} \to X_c\ell\nu$, respectively.**

Our preferred observable: $R_\gamma = \dfrac{\mathcal{B}_{s\gamma}+\mathcal{B}_{d\gamma}}{\mathcal{B}_{c\ell\nu}} \equiv \dfrac{\mathcal{B}_{(s+d)\gamma}}{\mathcal{B}_{c\ell\nu}}$

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Eight operators Q_i matter for $\mathcal{B}_{s\gamma}^{\mathrm{SM,2HDM}}$ barring the NLO EW and CKM-suppressed effects:

$Q_{1,2}$	Q_7	Q_8	$Q_{3,4,5,6}$
current-current	photonic dipole	gluonic dipole	penguin

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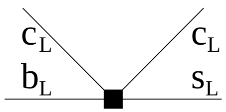

$Q_{1,2}$

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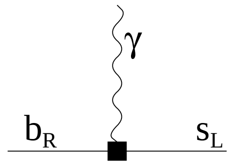
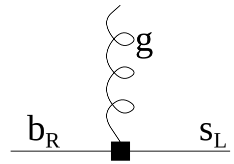


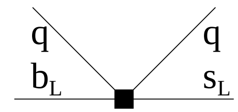
Q_7

photonic dipole



Q_8

gluonic dipole



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The Wilson coefficients $C_i(\mu_b \sim m_b)$ are calculated perturbatively (matching & mixing).

Current precision: $\mathcal{O}(\alpha_s^2)$ (NNLO).

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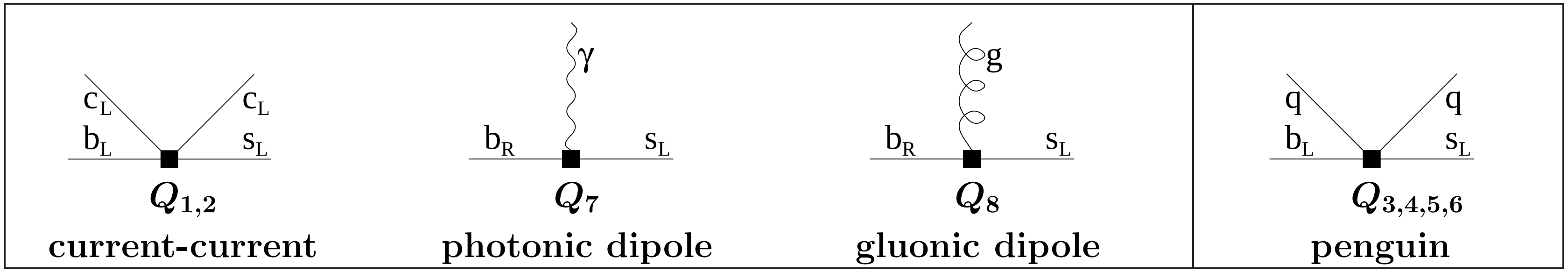

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The $\bar{B} \to X_s \gamma$ decay rate for $E_\gamma > E_0$ is a sum of the dominant perturbative contribution and a subdominant nonperturbative one $\delta\Gamma_{\rm nonp}$:

$$\Gamma(\bar{B} \to X_s \gamma) \;=\; \Gamma(b \to X_s^p \gamma) \;+\; \delta\Gamma_{\rm nonp}.$$

For $E_0 = 1.6\,{\rm GeV} \sim \frac{1}{3} m_b$, one estimates $\delta\Gamma_{\rm nonp} = (3 \pm 5)\%$.

[G. Buchalla, G. Isidori and S.-J. Rey, Nucl. Phys. B511 (1998) 594]
[M. Benzke, S.J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099]

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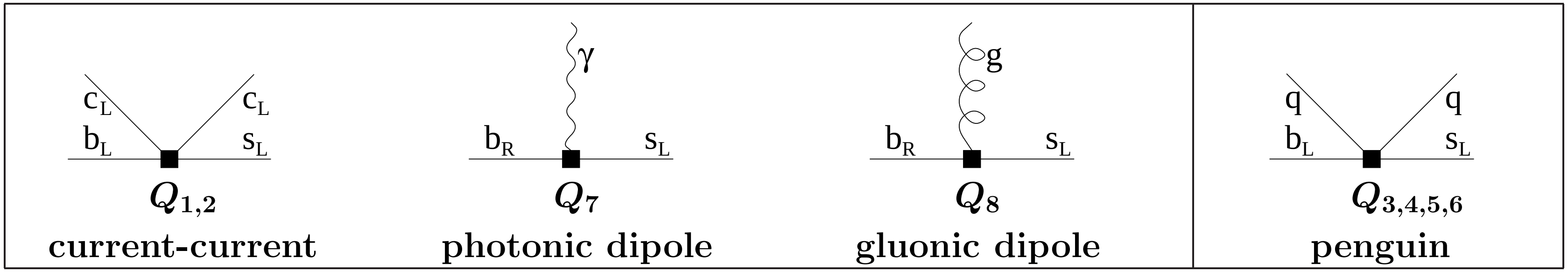

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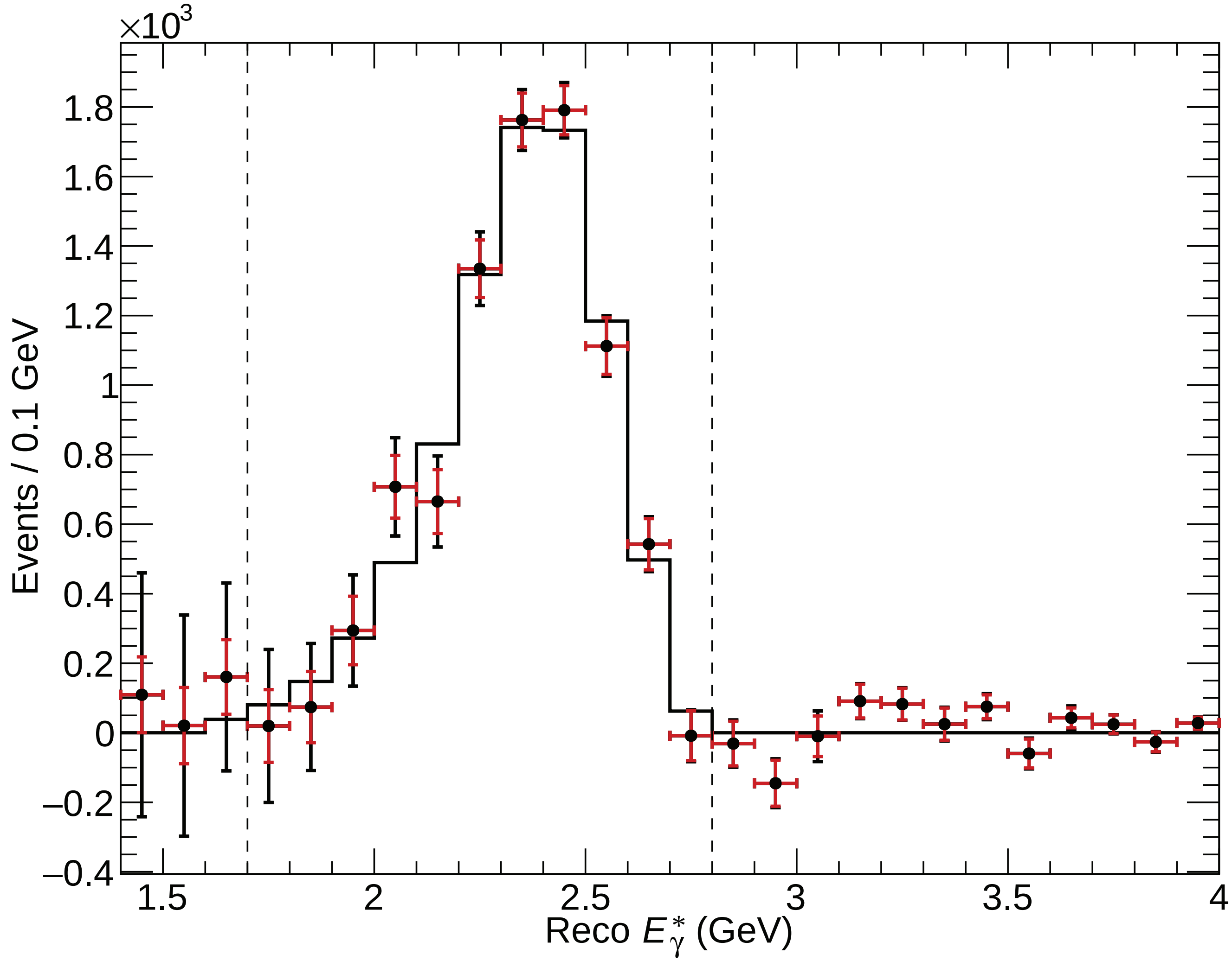
$\delta\Gamma_{\text{nonp}}$ is strongly E_0-dependent. If only Q_7 was present, we would have:

$$\left[\frac{\delta\Gamma_{\text{nonp}}}{\Gamma(b \to X_s^p \gamma)}\right]_{\text{only } C_7} = -\frac{\mu_\pi^2 + 3\mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{\alpha_s \Lambda^2}{(m_b - 2E_0)^2}, \frac{\Lambda^3}{m_b^3}\right).$$



Background-subtracted $\bar{B} \to X_{s+d}\,\gamma$ photon energy spectrum in the $\Upsilon(4S)$ rest frame, as shown in Fig. 1 of the most recent Belle analysis arXiv:1608.02344. The solid histogram has been obtained by using a shape-function model with its parameters fitted to data.

Effects of extrapolations from E_0^{exp} to 1.6 GeV can be parameterized by

$$\Delta_q \equiv \frac{\mathcal{B}_{q\gamma}(1.6)}{\mathcal{B}_{q\gamma}(E_0)} - 1$$

E_0 [GeV]	Δ_s^{BF}	$\Delta_s^{\mathrm{Belle}}$	Δ_s^{fix}	$\Delta_{s+d}^{\mathrm{fix}}$	Δ_d^{fix}
1.7	$(1.5 \pm 0.4)\%$	?	1.3%	1.5%	5.3%
1.8	$(3.4 \pm 0.6)\%$	$(3.69 \pm 1.39)\%$	3.0%	3.4%	10.5%
1.9	$(6.8 \pm 1.1)\%$	?	5.5%	6.0%	15.7%
2.0	$(11.9 \pm 2.0)\%$	?	10.0%	10.5%	22.5%

BF: O. Buchmüller and H. Flächer, PRD 73 (2006) 073008

Belle: arXiv:1608.02334, shape function-model fit to data

fix: perturbative & fixed-order HQET as in arXiv:1503.01789, arXiv:1503.01791

$$R_\gamma = \frac{\mathcal{B}_{s\gamma}+\mathcal{B}_{d\gamma}}{\mathcal{B}_{c\ell\nu}} \equiv \frac{\mathcal{B}_{(s+d)\gamma}}{\mathcal{B}_{c\ell\nu}}$$

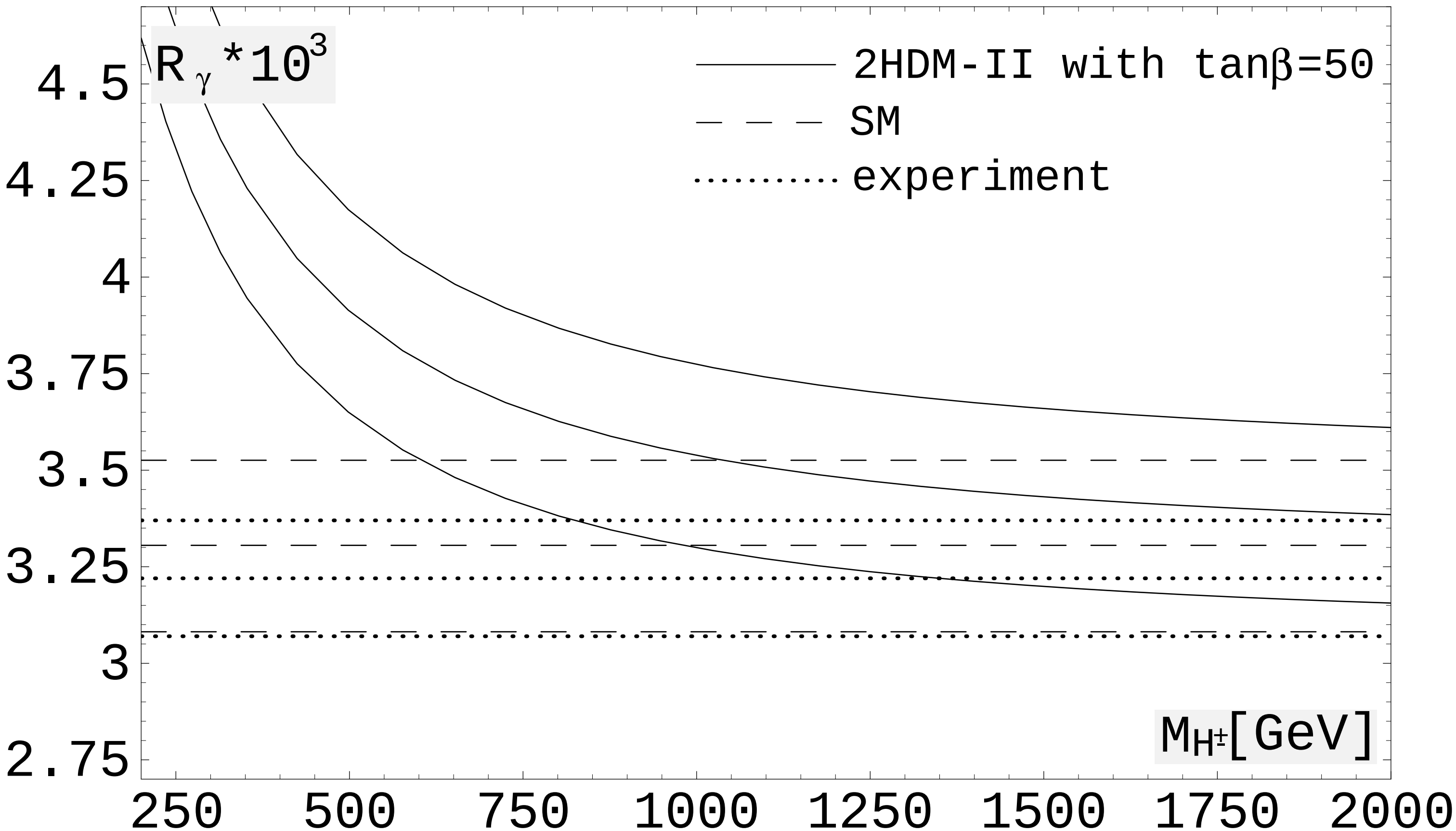
$$R_\gamma^{\rm SM}(1.6) = (331 \pm 22) \times 10^{-5}$$

MM, H. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kamiński, G. Ossola, M. Poradziński, A. Rehman, T. Schutzmeier, M. Steinhauser and J. Virto
PRL 114 (2015) 221801.

Experimental results and their naive averages:

	Babar				Belle			CLEO	w.a.	w.a.	R_γ	R_γ
E_0	incl	semi	had	aver	incl	semi	aver	incl	(E_0)	(1.6)	(E_0)	(1.6)
1.7					306(28)		306(28)		306(28)	311(28)		
					320(29)		320(29)		320(29)	326(30)	300(28)	305(28)
1.8	321(34)			321(34)	301(22)		301(22)		307(19)	318(19)		
	335(35)			335(35)	315(23)		315(23)		321(19)	333(20)	301(19)	312(19)
1.9	300(24)	329(52)	366(104)	308(22)	294(18)	351(37)	305(16)		306(13)	327(14)		
	313(25)	344(54)	381(108)	321(23)	307(19)	367(39)	319(17)		320(14)	343(15)	300(14)	322(15)
2.0	280(19)		339(79)	283(18)	279(15)		279(15)	293(46)	281(11)	315(14)		
	292(20)		353(83)	296(19)	292(15)		292(15)	306(49)	294(11)	331(14)	276(11)	310(14)

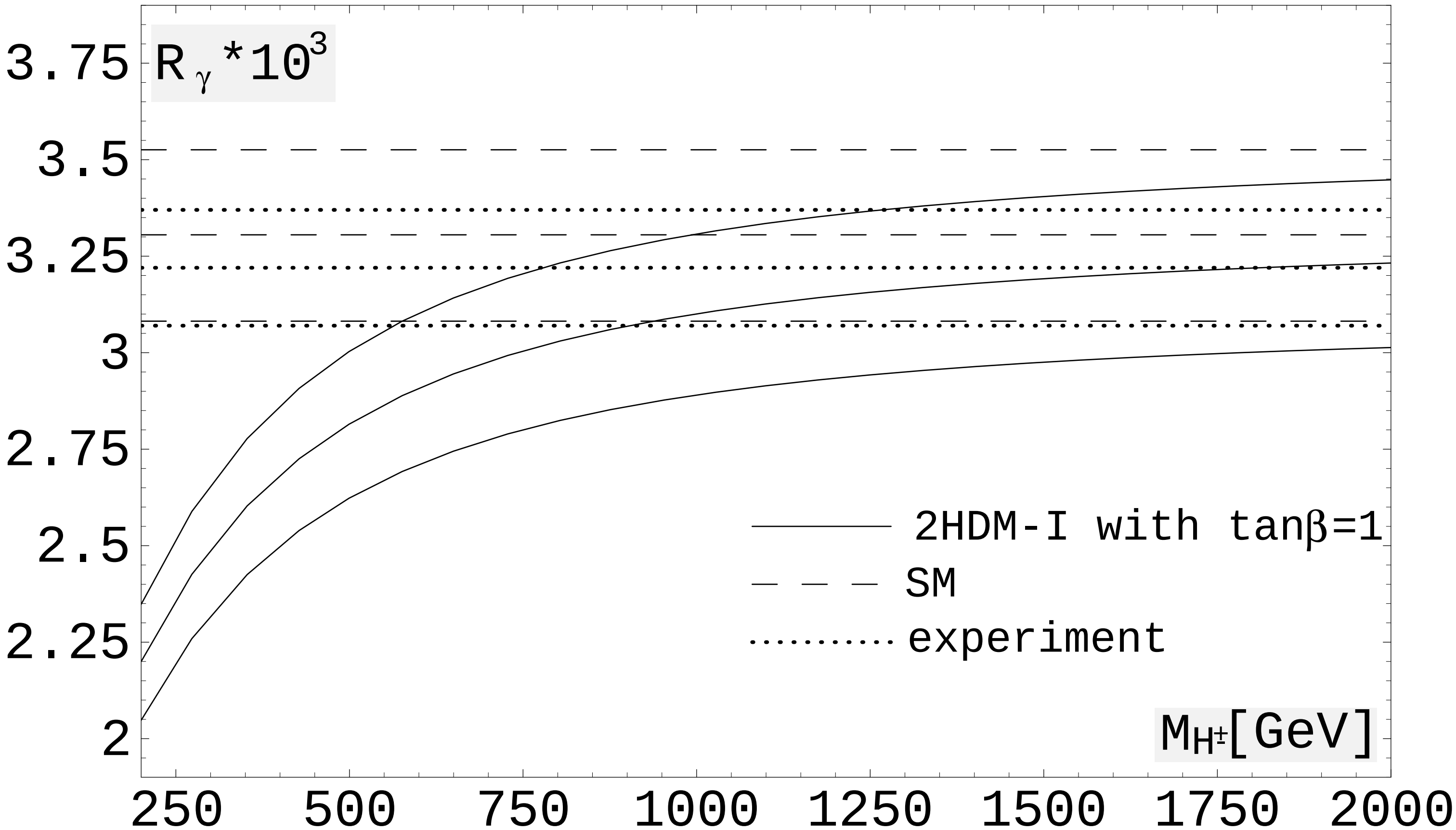
Upper rows – $\mathcal{B}_{s\gamma}$, lower rows – $\mathcal{B}_{(s+d)\gamma}$; 306(13) same as in arXiv:1612.07233v2 by HFLAV.



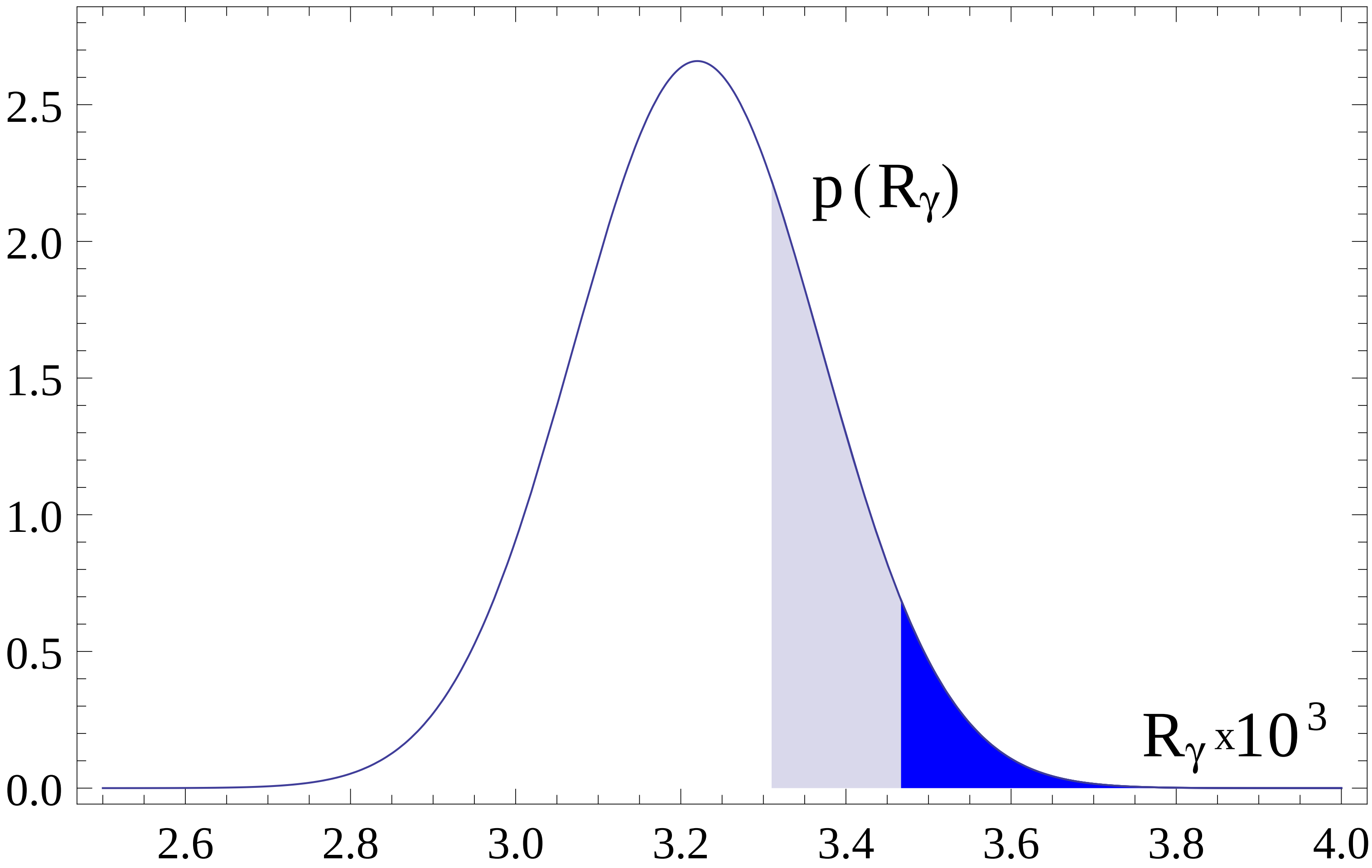
$\pm 1\sigma$ bands for $R_\gamma(1.6)$ in 2HDM-II with $\tan\beta = 50$.

The 2HDM calculation has the same precision as the SM one in arXiv:1503.1789, arXiv:1503.1791, except for the missing NLO EW corrections.

NNLO (3-loop) QCD matching conditions are from T. Hermann, MM, M. Steinhauser, arXiv:1208.2788.

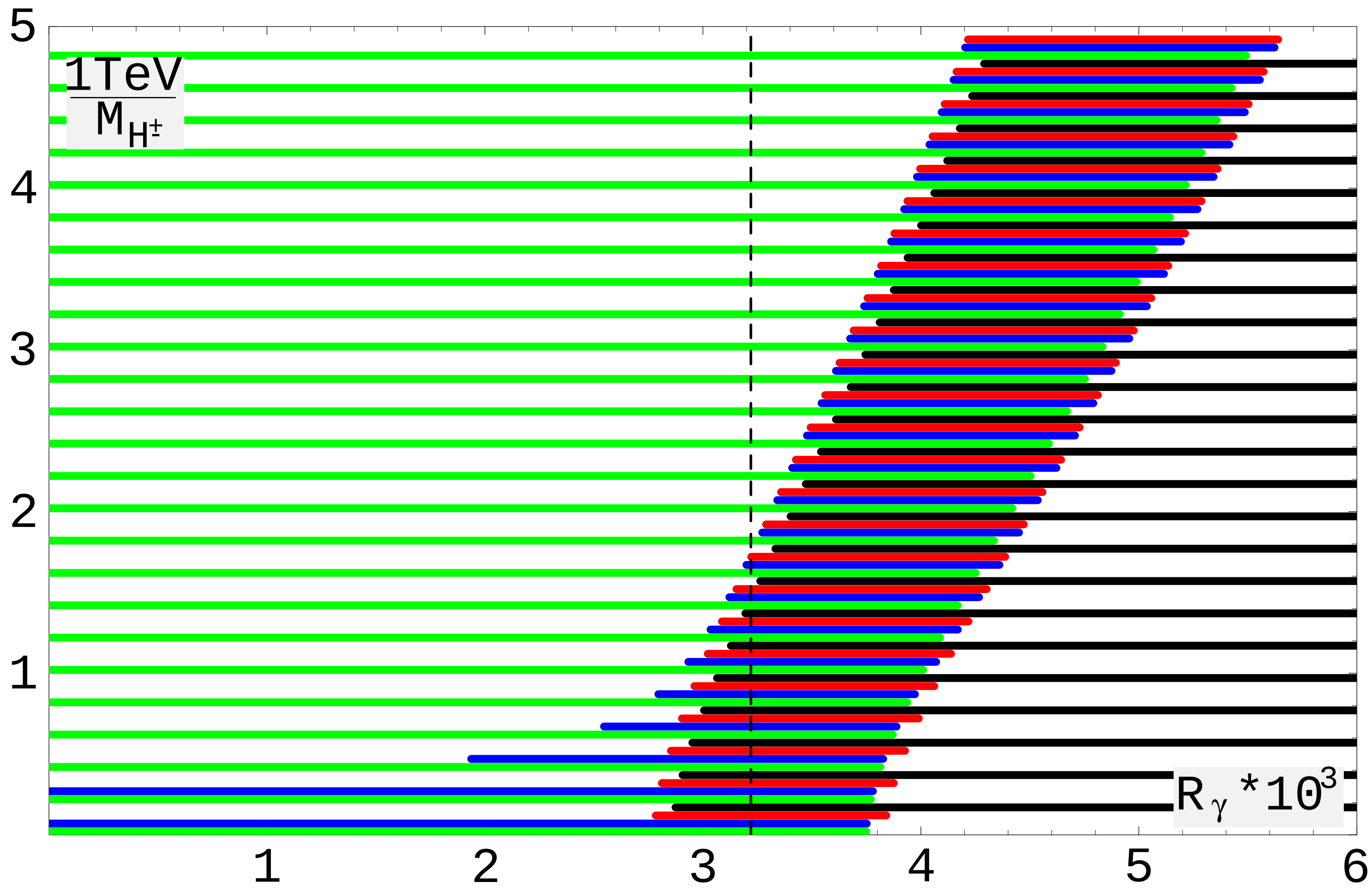


$\pm 1\sigma$ bands for $R_\gamma(1.6)$ in 2HDM-I with $\tan\beta = 1$.



Probability density for $R_\gamma^{\exp} = (3.22 \pm 0.15) \times 10^{-3}$, assuming a Gaussian distribution. The integrated probability over the dark-shaded region amounts to 5%. In the absence of theoretical uncertainties, the light-shaded region is accessible in Model-II only for $M_{H^\pm} > 1276\,\mathrm{GeV}$.

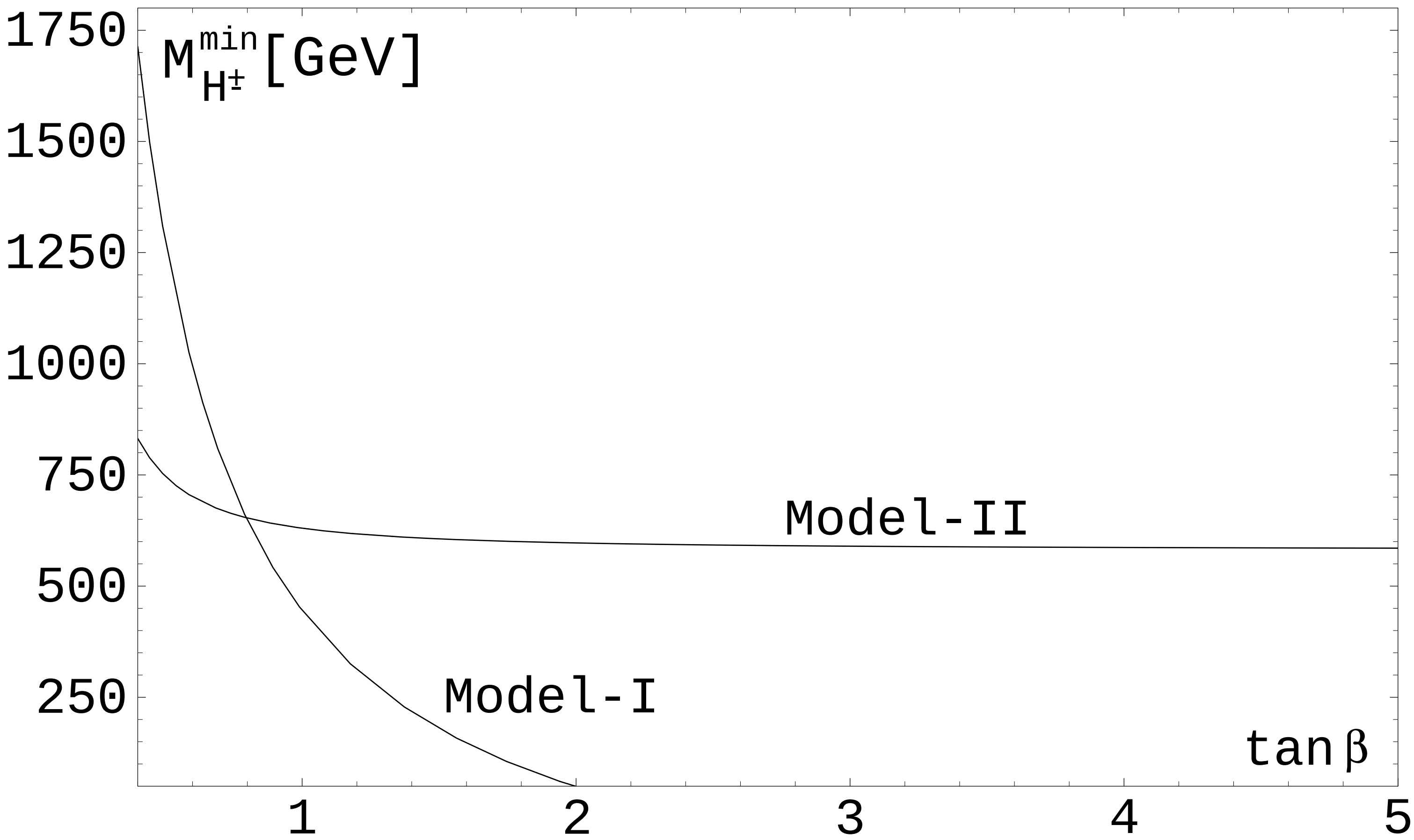
Confidence belts (95% C.L.) for 2HDM-II



Two-sided, One-sided right, One-sided left, Feldman-Cousins.

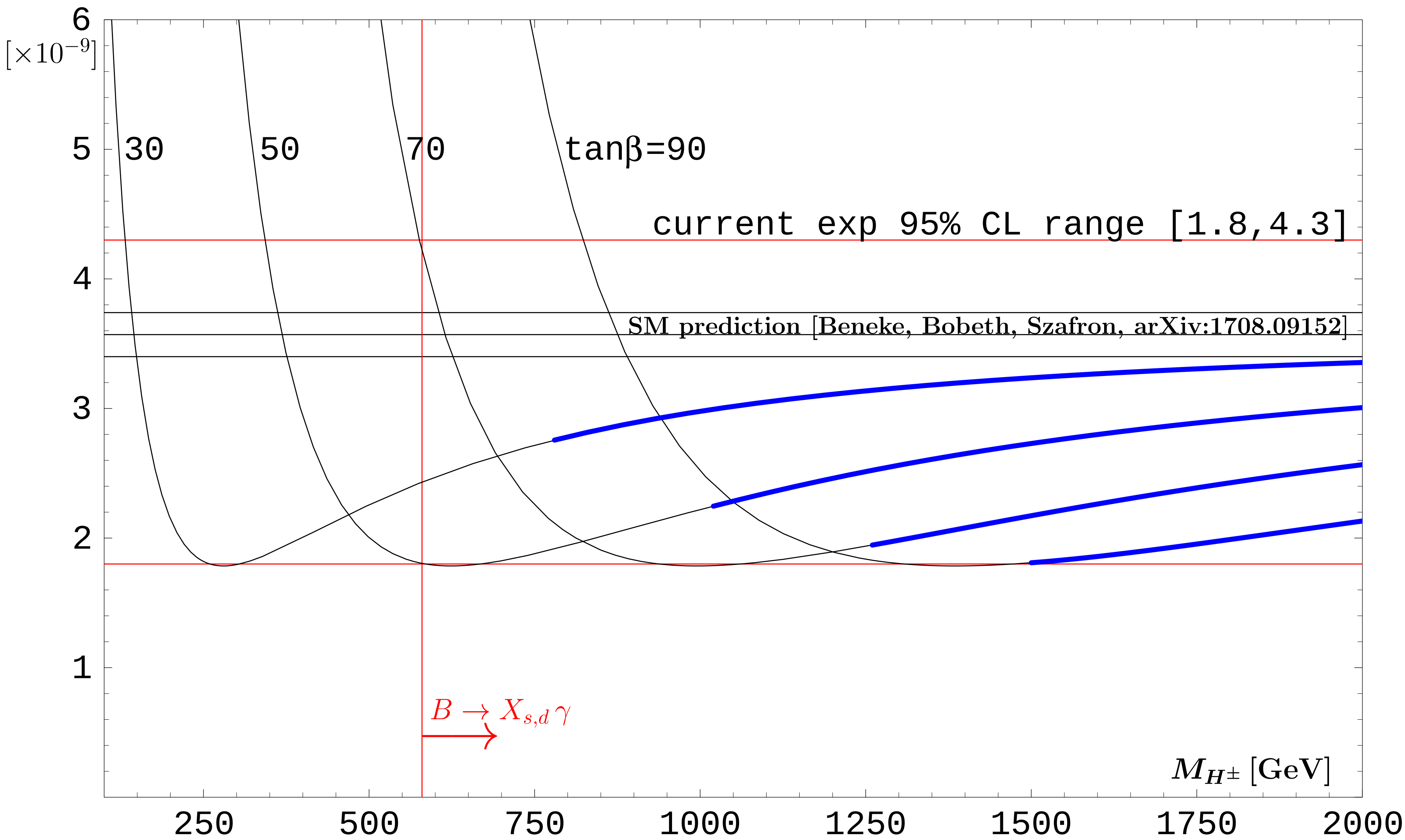
Model	$R_\gamma^{\text{exp}} \times 10^3$	95% C.L. bounds			99% C.L. bounds		
		1-sided	2-sided	FC	1-sided	2-sided	FC
	3.05 ± 0.28	307	268	268	230	208	208
I	3.12 ± 0.19	401	356	356	313	288	288
($\tan\beta = 1$)	3.22 ± 0.15	504	445	445	391	361	361
	3.05 ± 0.28	740	591	569	477	420	411
II	3.12 ± 0.19	795	645	628	528	468	461
(absolute)	3.22 ± 0.15	692	583	580	490	440	439

Bounds on $M_{H^\pm}$ [GeV] obtained using different methods.

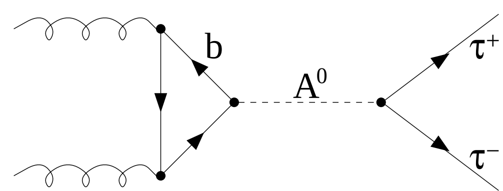


95% C.L. lower bounds on $M_{H^{\pm}}$ as functions of $\tan\beta$.

$\mathcal{B}(B_s \to \mu^+\mu^-)$ in the Two-Higgs-Doublet Model II

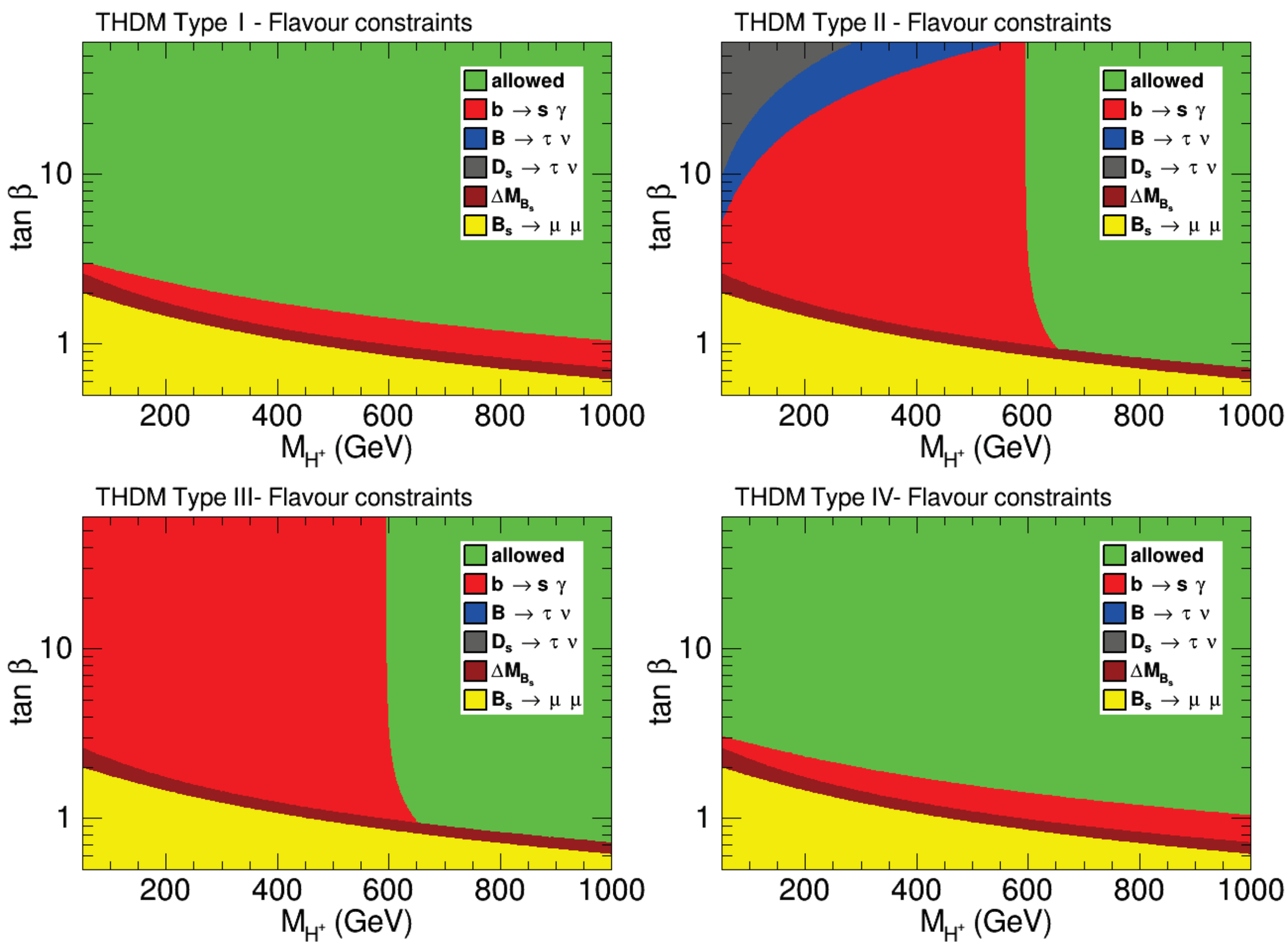


Blue lines — still allowed for $M_{H^\pm} = \sqrt{M_A^2 + M_W^2}$ after taking into account the LHC searches for $pp \to A^0 \to \tau^+\tau^-$ [ATLAS arXiv:1608.00890, CMS PAS HIG-16-037].



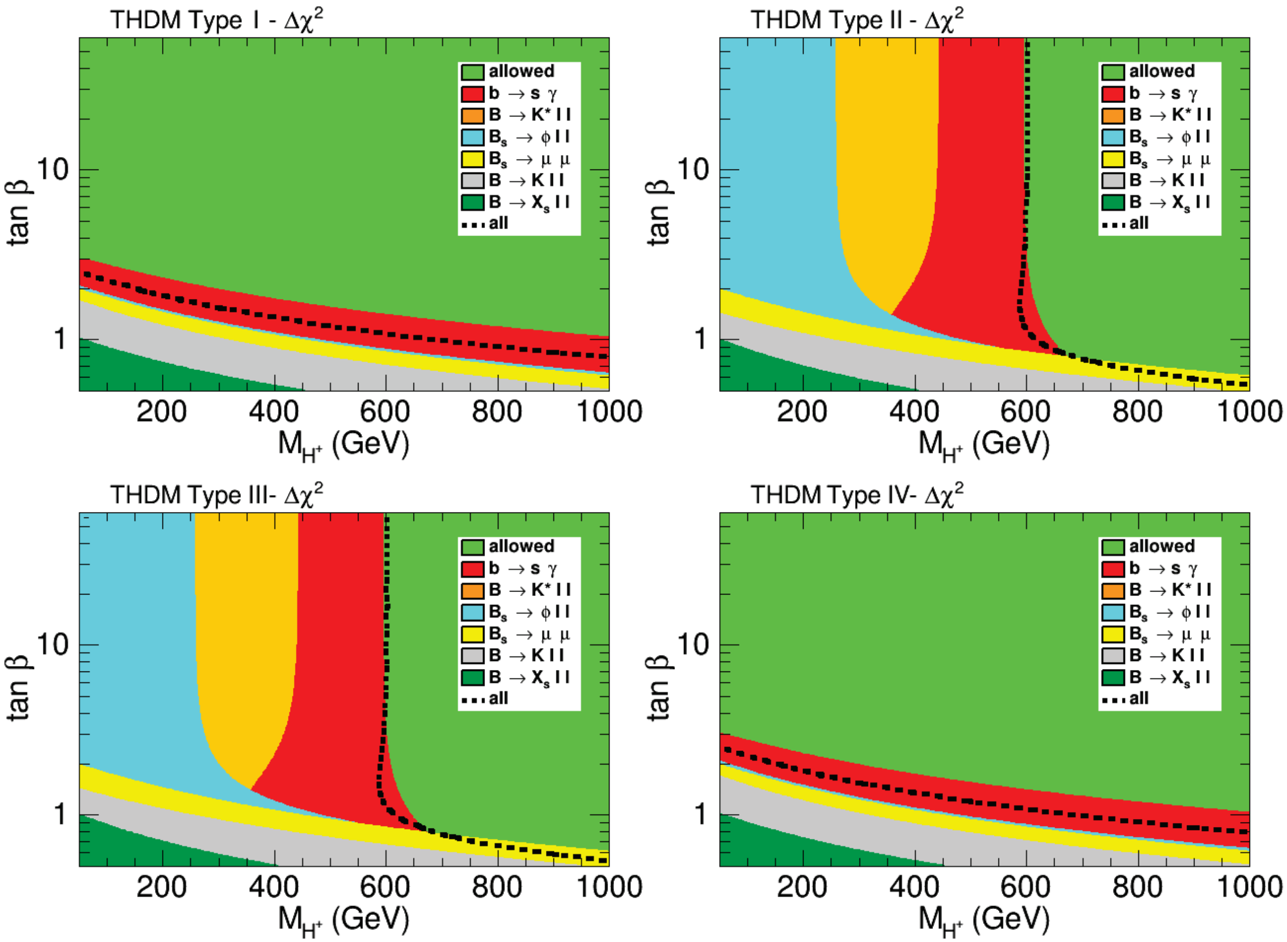
Flavour constraints

Constraints from individual (conventional) flavour observables



Contours corresponding to 95% C.L.

Constraints from $b \to s\ell\ell$ observables



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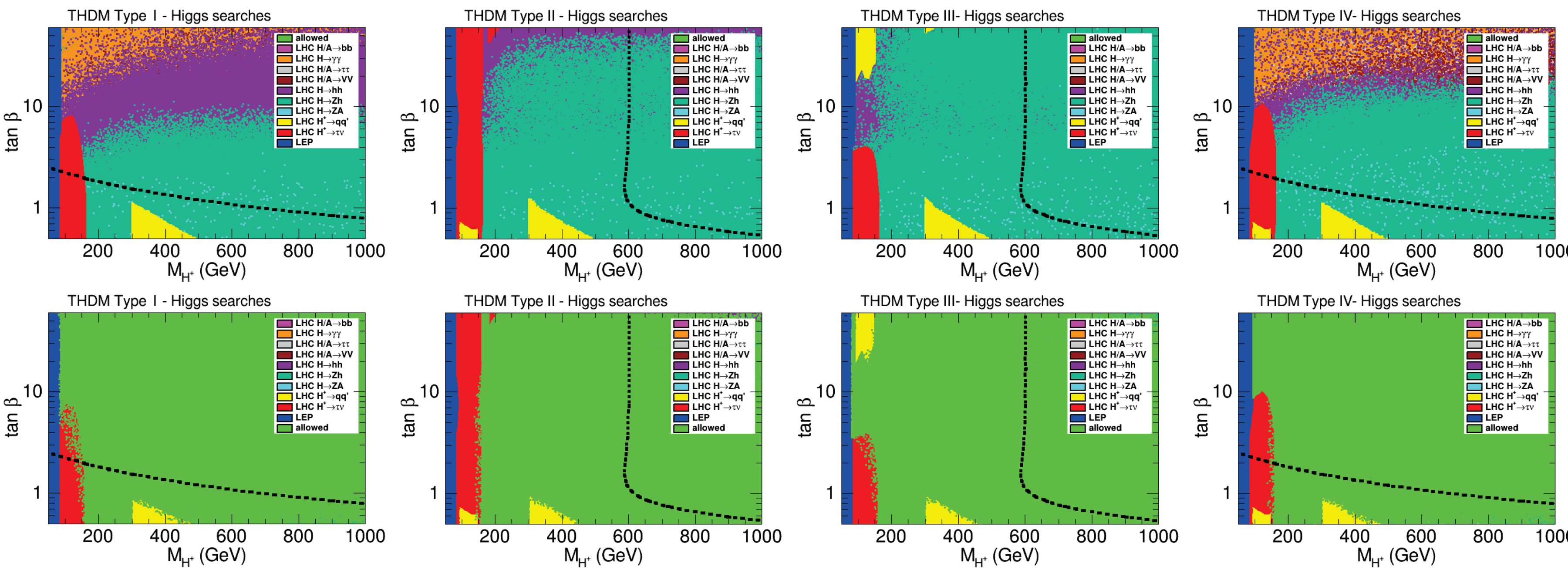


Exclusion at 95% C.L. by charged and neutral Higgs searches.
The points consistent with all collider constraints are shown in the background in the upper panels, and in the foreground in the lower panels.
The dotted line shows the combined limit from all flavour observables.

The different searches play a role and are complementary.

Still many points can escape the neutral Higgs constraints.

Summary

- **Strong constraints on $M_{H^\pm}$ in the 2HDM get imposed by measurements of the inclusive weak radiative B-meson decay branching ratio.**

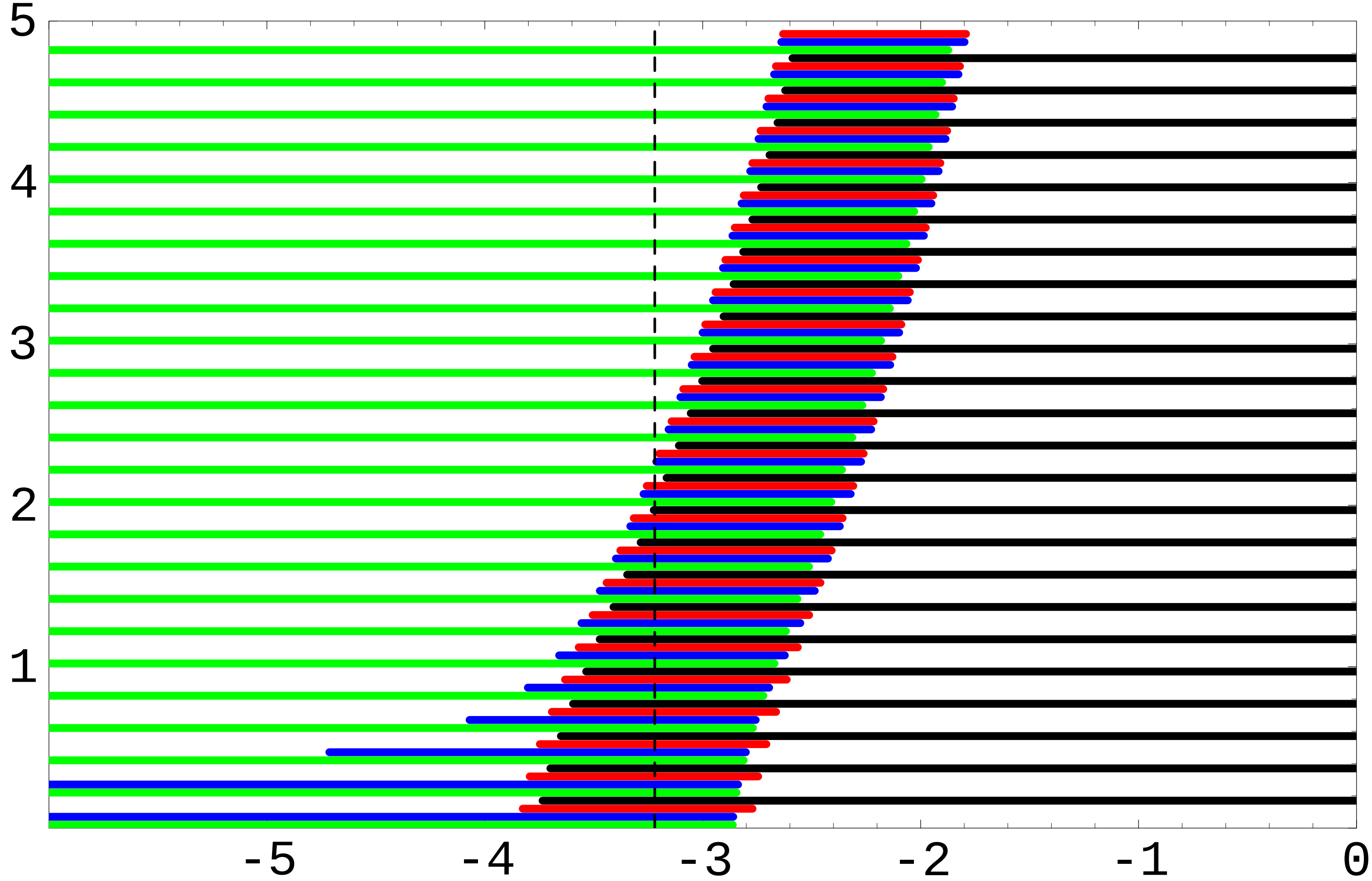
- **Although in principle straightforward, a derivation of them faces several ambiguities stemming mainly from the photon energy cutoff choice.**

- **In Model-I, the relevant constraints are obtained only for $\tan\beta \lesssim 2$.**

- **In Model-II, the absolute ($\tan\beta$-independent) 95% C.L. bounds are in the 570–800 GeV range.**

BACKUP SLIDES

Confidence belts (95% C.L) for 2HDM-I



Two-sided, One-sided right, One-sided left, Feldman-Cousins.

B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\left(\begin{smallmatrix}\text{quarks} \neq t \\ \text{\& leptons}\end{smallmatrix}\right) + N \sum_n C_n(\mu)\, Q_n$$

Q_n – **local interaction terms (operators),** C_n – **coupling constants (Wilson coefficients)**

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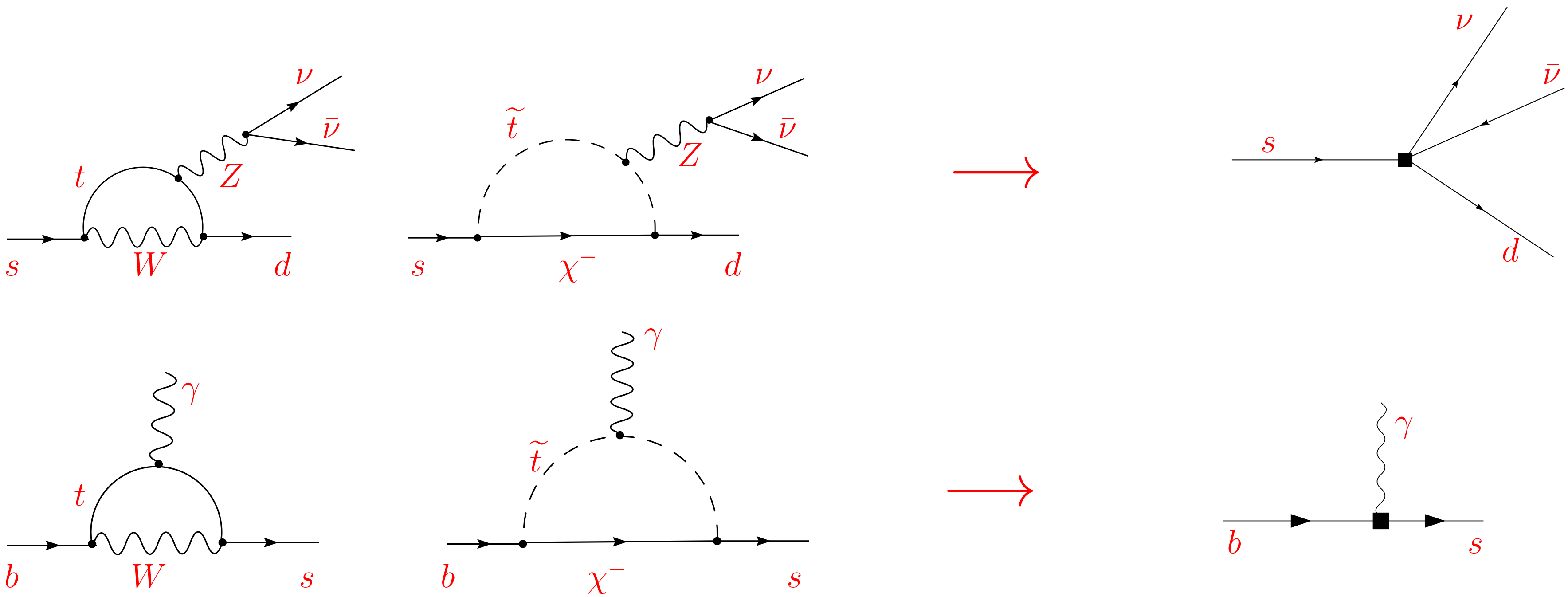
Information on the electroweak-scale physics is encoded in the values of $C_i(\mu)$, e.g.,



This is a modern version of the Fermi theory for weak interactions. It is "nonrenormalizable" in the traditional sense but actually renormalizable. It is also predictive because all the C_i are calculable, and only a finite number of them is necessary at each given order in the (external momenta)/M_W expansion.

Advantages: **Resummation of $\left(\alpha_s \ln \frac{M_W^2}{\mu^2}\right)^n$ using RGE, easier account for symmetries.**

Our ability to observe or constrain new physics depends on the accuracy of determining the SM "background". Thus, precise evaluation of $C_i(\mu)$ in the SM is particularly important.

Two steps of the Wilson coefficient calculation:

Matching: **Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green's functions.**

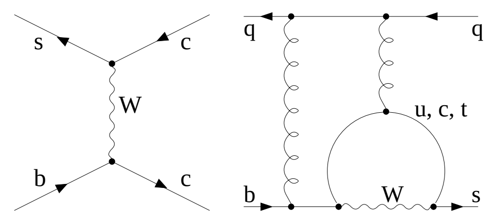
Mixing: **Deriving the effective theory Renormalization Group Equations (RGE) from the renormalization constant matrices (the operators mix under renormalization).**
Next, using the RGE to evolve C_i from μ_0 to μ ~(external momenta).

Operator bases can chosen in a convention-dependent manner.
For example, two possible conventions for the $|\Delta B| = |\Delta S| = 1$ four-quark operators in the SM read:

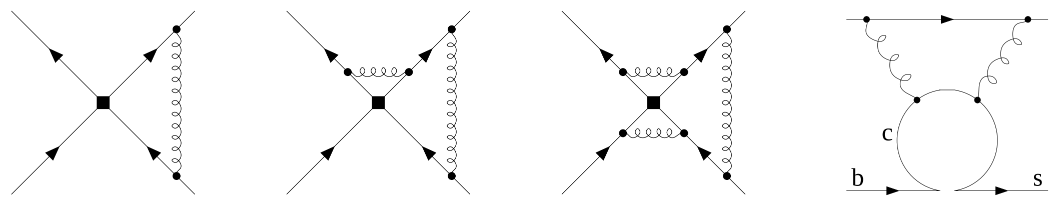
$$\begin{aligned}
Q_1 &= (\bar{s}^\alpha_L \gamma_\mu c^\beta_L)(\bar{c}^\beta_L \gamma^\mu b^\alpha_L) & P_1 &= (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)\\
Q_2 &= (\bar{s}^\alpha_L \gamma_\mu c^\alpha_L)(\bar{c}^\beta_L \gamma^\mu b^\beta_L) & P_2 &= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\\
Q_3 &= (\bar{s}^\alpha_L \gamma_\mu b^\alpha_L) \textstyle\sum_q (\bar{q}^\beta_L \gamma^\mu q^\beta_L) & P_3 &= (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q} \gamma^\mu q)\\
Q_4 &= (\bar{s}^\alpha_L \gamma_\mu b^\beta_L) \textstyle\sum_q (\bar{q}^\beta_L \gamma^\mu q^\alpha_L) & P_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma^\mu T^a q)\\
Q_5 &= (\bar{s}^\alpha_L \gamma_\mu b^\alpha_L) \textstyle\sum_q (\bar{q}^\beta_R \gamma^\mu q^\beta_R) & P_5 &= (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)\\
Q_6 &= (\bar{s}^\alpha_L \gamma_\mu b^\beta_L) \textstyle\sum_q (\bar{q}^\beta_R \gamma^\mu q^\alpha_R) & P_6 &= (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)
\end{aligned}$$

Gilman, Wise, 1979 **Chetyrkin, Münz, MM, 1996**

Matching:



Operator mixing:



Expansion in external momenta ⇒ spurious IR divergences arise.

Renormalization constant calculation using masses as IR regulators

Münz, MM, 1995	2-loop	dipole operator mixing
van Ritbergen, Vermaseren, Larin, 1997	4-loop	$\beta_{\rm QCD}$
Chetyrkin, Münz, MM, 1997	3-loop	(4-quark) $\to$ dipole
(...)		
Gambino, Gorbahn, Haisch, 2003	3-loop	(4-quark) $\to$ (quark-lepton)
Gorbahn, Haisch, 2004	3-loop	four-quark operator mixing
Czakon, 2004	4-loop	$\beta_{\rm QCD}$
Gorbahn, Haisch, MM, 2005	3-loop	dipole operator mixing
Czakon, Haisch, MM, 2006	4-loop	(4-quark) $\to$ dipole
(...)		
Luthe, Maier, Marquard, Schroder, 2017	5-loop	$\beta_{\rm QCD}$

Exact decomposition of a propagator denominator:

$$\underbrace{\frac{1}{(q+p)^2-M^2}}_{\Delta D=-2} = \underbrace{\frac{1}{q^2-m^2}}_{\Delta D=-2} + \underbrace{\frac{M^2-p^2-2qp-m^2}{q^2-m^2}\,\frac{1}{(q+p)^2-M^2}}_{\Delta D=-3}$$

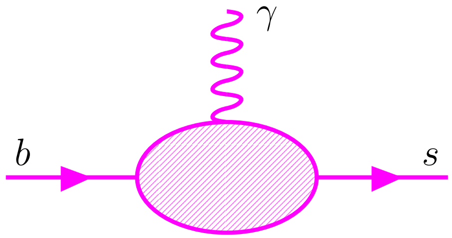
q– linear combination of loop momenta, p– linear combination of external momenta,
M– mass of the considered particle, m– IR regulator mass (arbitrary)

After applying this identity sufficiently many times, the last term can be dropped in each propagator. The only Feynman integrals to perform then are single-scale massive tadpoles.

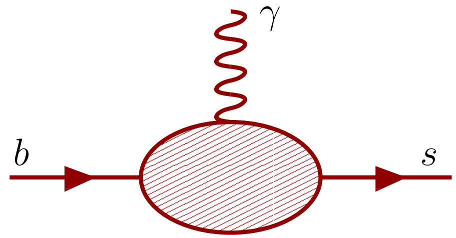
Up to three loops, explicit expressions for pole parts of all the single-scale massive tadpoles are available in terms of solved recurrences [Chetyrkin, Münz, MM, 1997] ($\leftrightarrow$ Ringberg workshop 1994).

At four loops, IBP are used for reduction to less than 20 master integrals [van Ritbergen, 1997; Schröder, 2002; Czakon, 2004] ($\leftrightarrow$ RADCOR 2002).

The matching conditions are most easily found by requiring equality of the full SM and the effective theory 1PI off-shell Green's functions that are expanded in external momenta and light masses prior to loop-momentum integration.



$=$



$$+\ \mathcal{O}\left(\frac{1}{M_W^4}\right)$$

Full EW theory
UV counterterms included

Spurious IR $\frac{1}{\epsilon^n}$ remain

Effective Theory
Loop diagrams vanish

UV $\frac{1}{\epsilon^n}$ remain

The $\frac{1}{\epsilon^n}$ poles cancel in the matching equation.

The only Feynman integrals to calculate: partly-massive tadpoles.

Algorithms for calculating 3-loop single-scale partly-massive tadpoles were developed in 1994-2000 [Chetyrkin, Kühn, Steinhauser; Avdeev, Fleischer, Mikhailov, Tarasov, Kalmykov; Broadhurst].
Full automatization in the code `MATAD` by M. Steinhauser (2000).

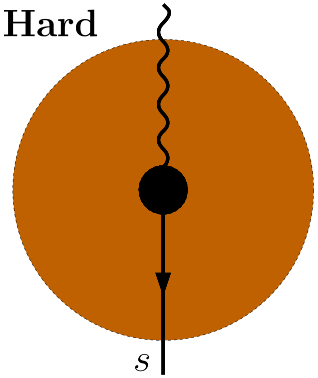
Differences among the simultaneously decoupled heavy particle masses can be taken into account by Taylor expanding around the equal-mass point. Alternatively, for large mass ratios, either asymptotic expansions or a sequence of effective theories can be applied.

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

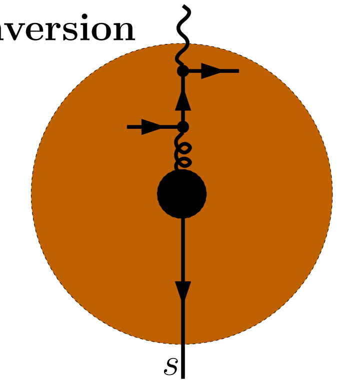
A. Without long-distance charm loops:

1. **Hard**



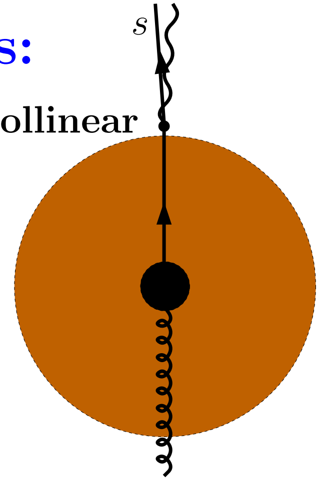
Dominant, well-controlled.

2. **Conversion**



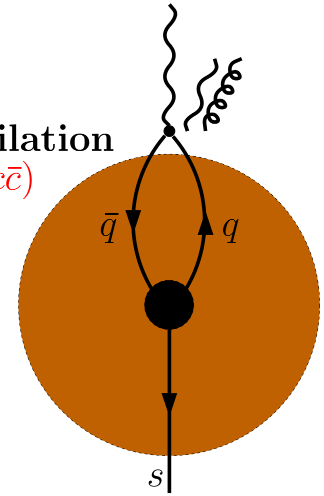
$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

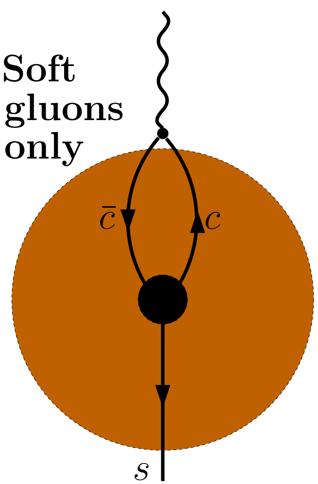
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

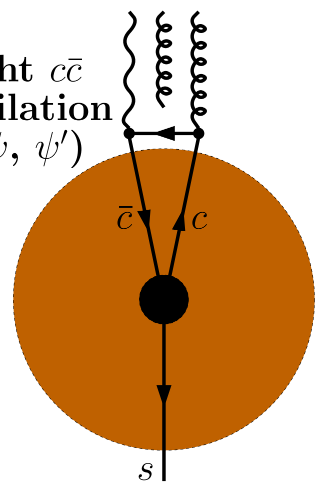
B. With long-distance charm loops:

5. **Soft gluons only**



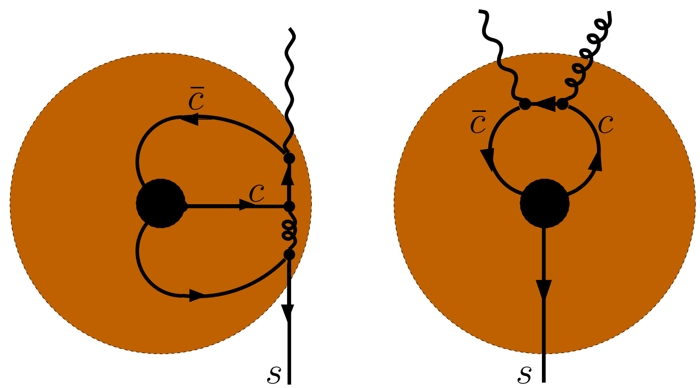
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$ $\mathcal{O}(\alpha_s \Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

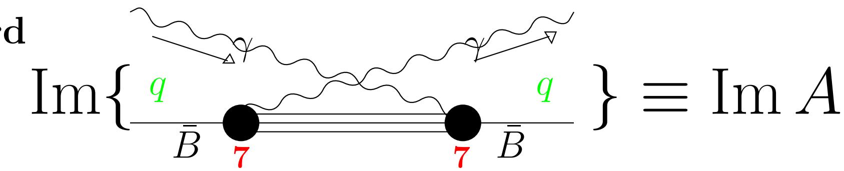
e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \;] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

The “hard” contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2(\mu_b) \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$

The “77” term in this sum is “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$**:**

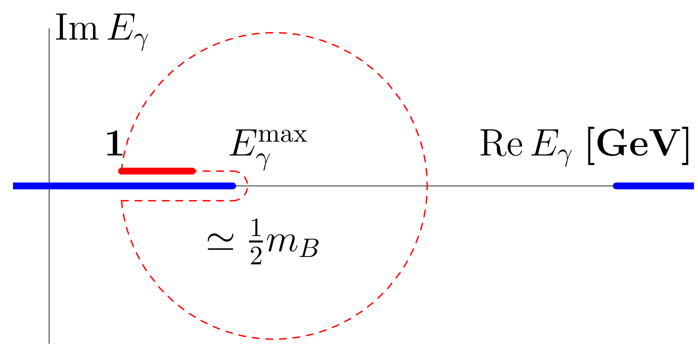


When the photons are soft enough, $m^2_{X_s} = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**

However, the $\bar{B} \to X_s \gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ**,** $\mathrm{Im}A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0) | Q^{(j)}_{\mathrm{local\ operator}} | \bar{B}(\vec{p}=0) \rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b**.**

At $(\Lambda/m_b)^0$**:** $\langle \bar{B}(\vec{p}) | \bar{b}\gamma^\mu b | \bar{B}(\vec{p}) \rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s \gamma) = \Gamma(b \to X_s^{\mathrm{parton}} \gamma) + \mathcal{O}(\Lambda/m_b).$

At $(\Lambda/m_b)^1$**: Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$**:** $\langle \bar{B}(\vec{p}) | \bar{b}_v D^\mu D_\mu b_v | \bar{B}(\vec{p}) \rangle \sim m_B\, \mu_\pi^2,$
$\langle \bar{B}(\vec{p}) | \bar{b}_v g_s G_{\mu\nu} \sigma^{\mu\nu} b_v | \bar{B}(\vec{p}) \rangle \sim m_B\, \mu_G^2,$

The HQET heavy-quark field: $b_v(x) = \frac{1}{2}(1 + \not{v}) b(x) \exp(i m_b\, v \cdot x)$ **with** $v = p/m_B$.

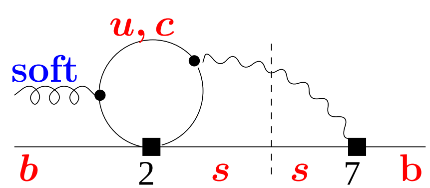
Non-perturbative effects in the presence of other operators $(Q_i \neq Q_7)$

[Benzke, Lee, Neubert, Paz, arXiv:1003.5012].

$$\frac{d}{dE_\gamma}\Gamma(\bar{B} \to X_s\gamma) = (\Gamma_{77}\text{-like term}) \; + \; \tilde{N} E_\gamma^3 \sum_{i \leq j} \mathrm{Re}\left(C_i^* C_j\right) F_{ij}(E_\gamma).$$

Remarks:

- The SCET approach is valid for large E_γ only. It is fine for $E_\gamma > E_0 \sim \frac{1}{3} m_b \simeq 1.6$ GeV. Lower cutoffs are academic anyway.
- For such E_0, non-perturbative effects in the integrated decay rate are estimated to remain within 5%. They scale like:
  - $\frac{\Lambda^2}{m_b^2}$, $\frac{\Lambda^2}{m_c^2}$ (known),
  - $\frac{\Lambda}{m_b} \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$ (negligible),
  - $\frac{\Lambda}{m_b}$, $\frac{\Lambda^2}{m_b^2}$, $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by tails of subleading shape functions ("27"),
  - $\alpha_s \frac{\Lambda}{m_b}$ to be constrained by future measurements of the isospin asymmetry ("78"),
  - $\alpha_s \frac{\Lambda}{m_b}$ but suppressed by $Q_d^2 = \frac{1}{9}$ ("88").



- Extrapolation factors? Tails of subleading functions are less important for them.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity $P(E_0)$:

$$\frac{\Gamma[b \to X_s\gamma]_{E_\gamma > E_0}}{\Gamma[b \to X_u e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{ub}}\right|^2 \frac{6\alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i(\mu_b)\, C_j(\mu_b)\, K_{ij}}_{P(E_0)}$$

Expansions of the Wilson coefficients and K_{ij} in $\widetilde{\alpha}_s \equiv \frac{\alpha_s(\mu_b)}{4\pi}$:

$$C_i(\mu_b) = C_i^{(0)} + \widetilde{\alpha}_s\, C_i^{(1)} + \widetilde{\alpha}_s^2\, C_i^{(2)} + \ldots$$

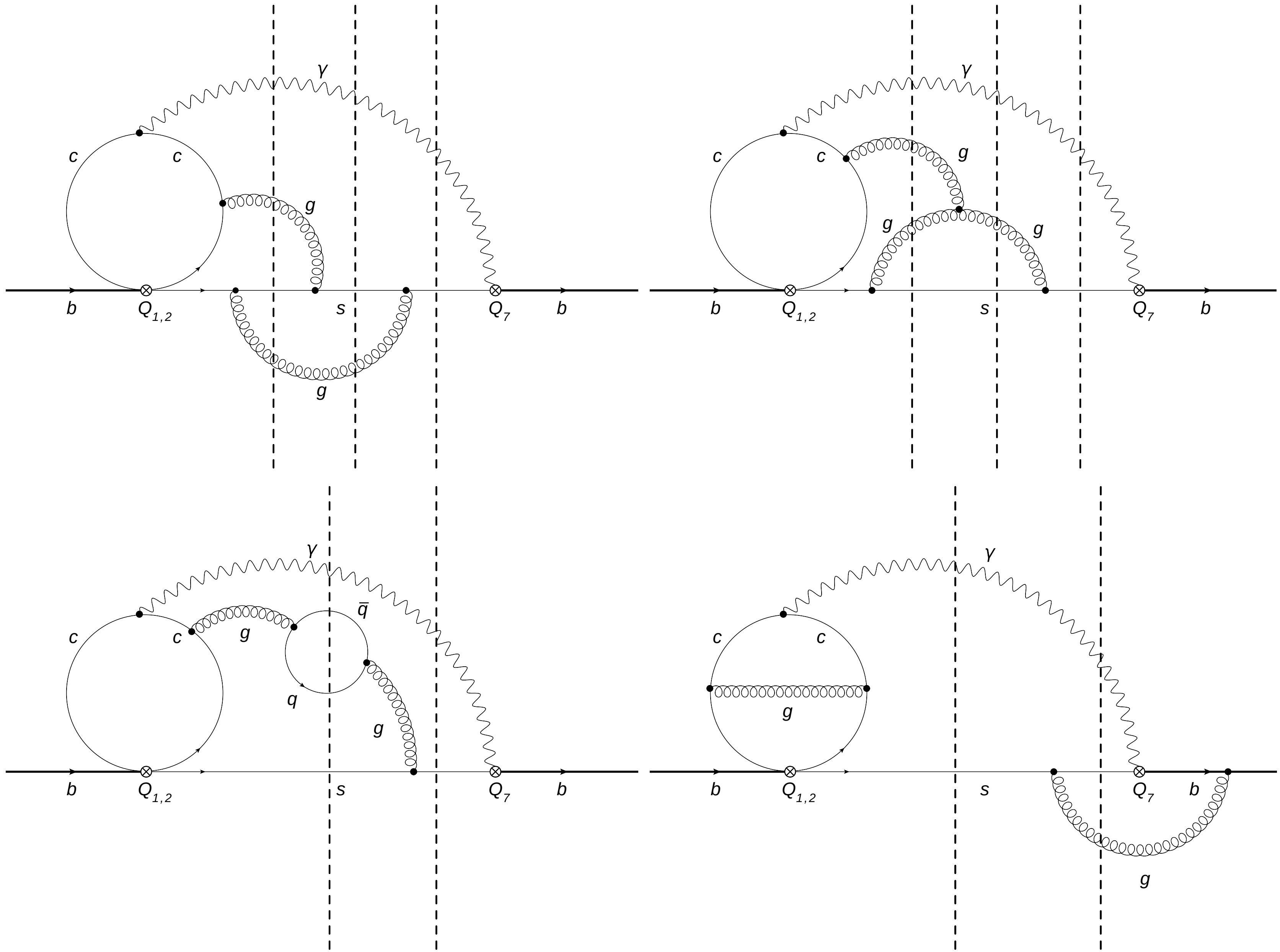
$$K_{ij} = K_{ij}^{(0)} + \widetilde{\alpha}_s\, K_{ij}^{(1)} + \widetilde{\alpha}_s^2\, K_{ij}^{(2)} + \ldots$$

Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ and $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\delta = 1 - \frac{2E_0}{m_b}$ and $z = \frac{m_c^2}{m_b^2}$.

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$ and $\delta = 1$:

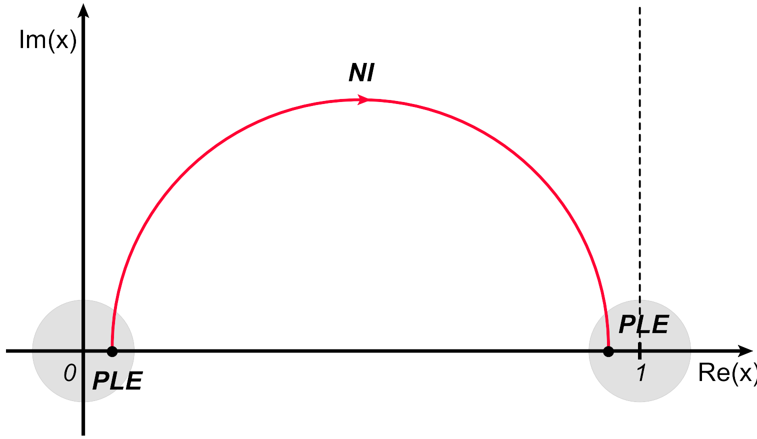
[M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, JHEP 1504 (2015) 168]



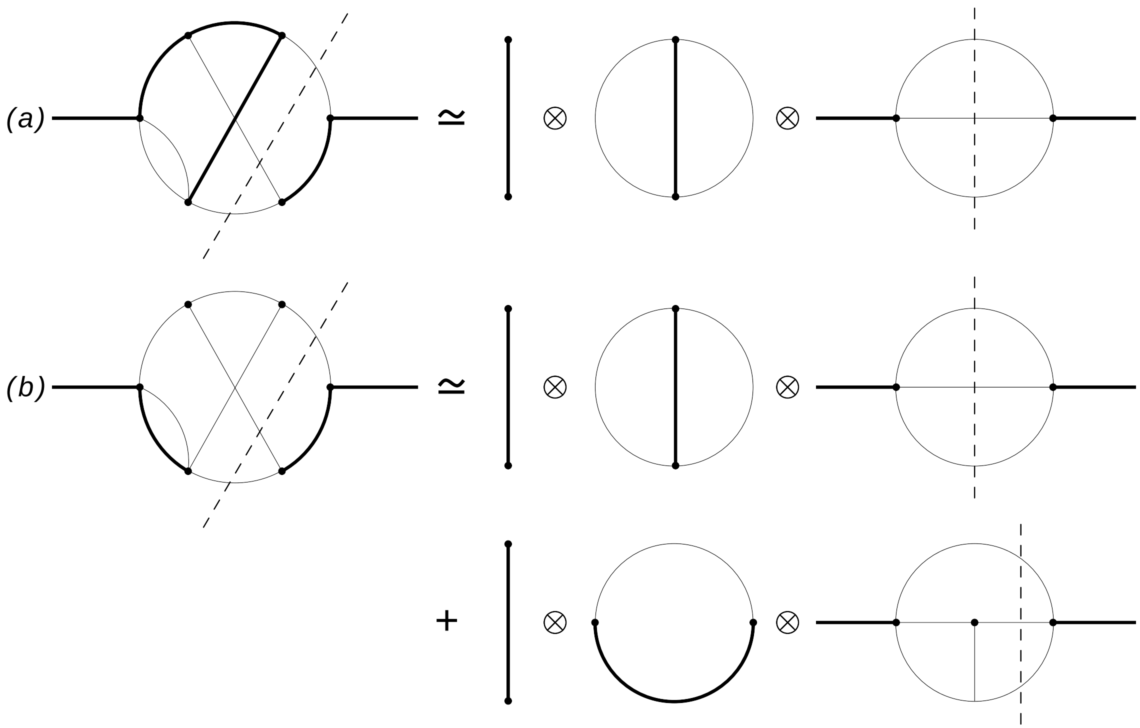
Master integrals and differential equations:

	n_D	n_{OS}	n_{eff}	$n_{massless}$
2-particle cuts	**292**	**92**	**143**	**9**
3-particle cuts	**267**	**54**	**110**	**11**
4-particle cuts	**292**	**17**	**37**	**7**
total	**851**	**163**	**290**	**27**

$$\frac{d}{dx} I_i(x) = \sum_j R_{ij}(x) I_j(x), \qquad x = \frac{p^2}{m_b^2}.$$



Boundary conditions in the vicinity of $x = 0$:



Results for the NNLO corrections:

$$K_{27}^{(2)}(z,\delta) = A_2 + F_2(z,\delta) \underbrace{- \tfrac{27}{2} f_q(z,\delta) + f_b(z) + f_c(z) + \tfrac{4}{3}\phi_{27}^{(1)}(z,\delta)\ln z}_{\text{quark loops on the gluon lines \& BLM approximation}}$$

$$+ \left[\text{terms} \sim \left(\ln\frac{\mu_b}{m_b},\ \ln^2\frac{\mu_b}{m_b},\ \ln\frac{\mu_c}{m_c}\right) \text{ or vanishing when } m_b \to m_b^{\text{pole}}\right],$$

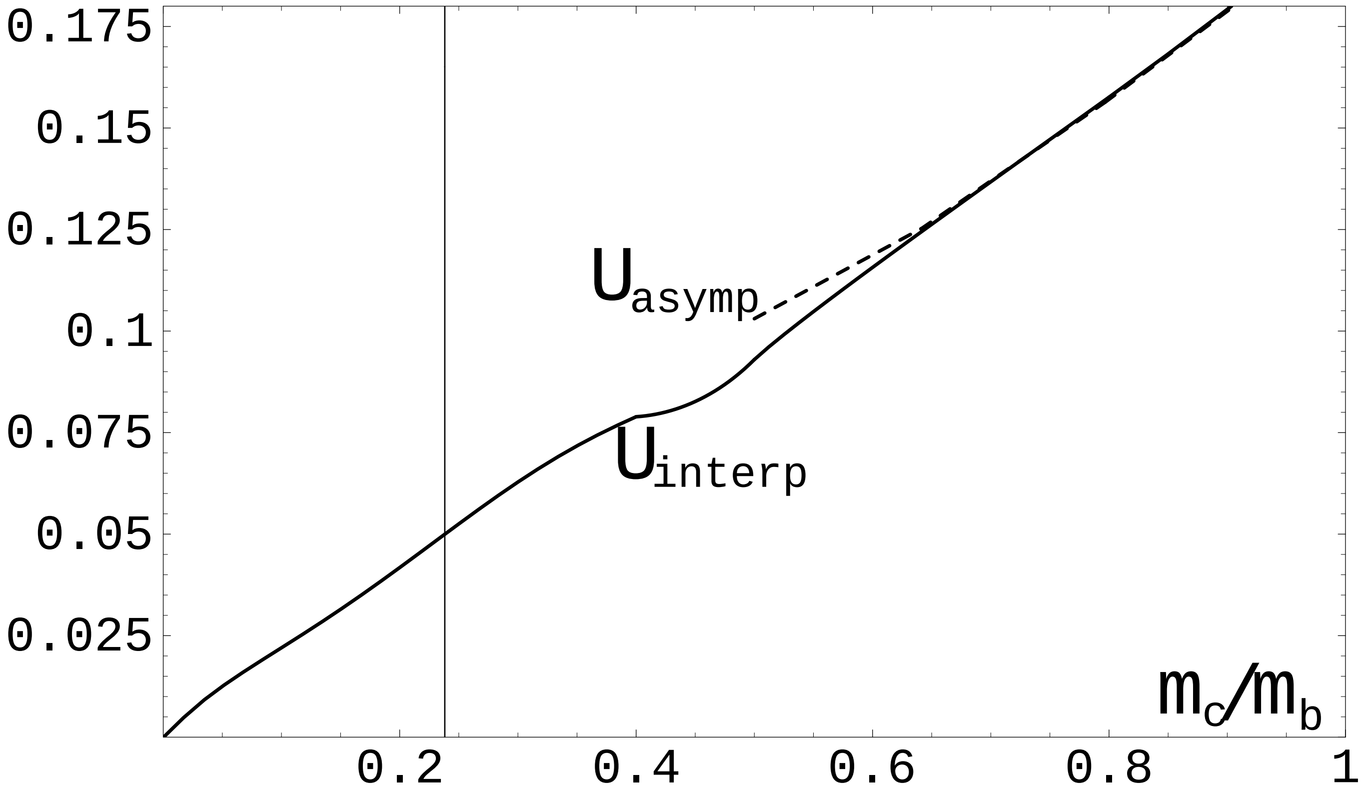
$$K_{17}^{(2)}(z,\delta) = -\tfrac{1}{6}K_{27}^{(2)}(z,\delta) + A_1 + F_1(z,\delta) + \left[\text{terms} \sim \left(\ln\frac{\mu_b}{m_b},\ \ln^2\frac{\mu_b}{m_b}\right)\right].$$

$F_i(0,1) \equiv 0$, $A_1 \simeq 22.605$, $A_2 \simeq 75.603$ from the present calculation.

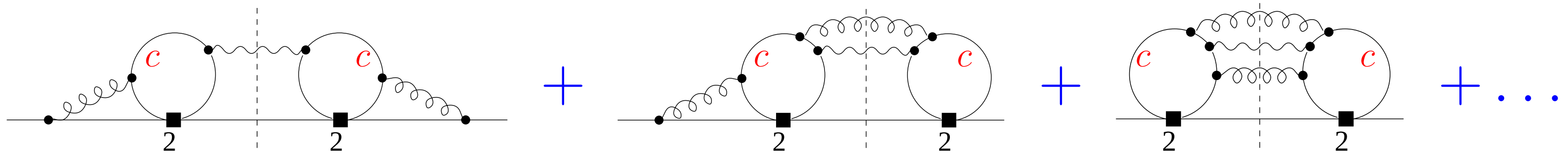
Next, we interpolate in $z = m_c^2/m_b^2$ by assuming that $F_i(z,1)$ are linear combinations of $f_q(z,1)$, $K_{27}^{(1)}(z,1)$, $z\frac{d}{dz}K_{27}^{(1)}(z,1)$ and a constant term. The known large-z behaviour of F_i [hep-ph/0609241] and the condition $F_i(0,1) \equiv 0$ fix these linear combinations in a unique manner.

Effect of the interpolated contribution on the branching ratio

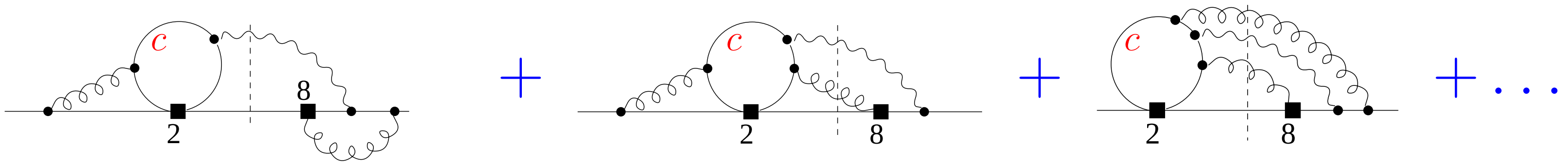
$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \frac{C_1^{(0)}(\mu_b)F_1(z,\delta)+\left(C_2^{(0)}(\mu_b)-\frac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(z,\delta)}{C_7^{(0)\mathrm{eff}}(\mu_b)}$$



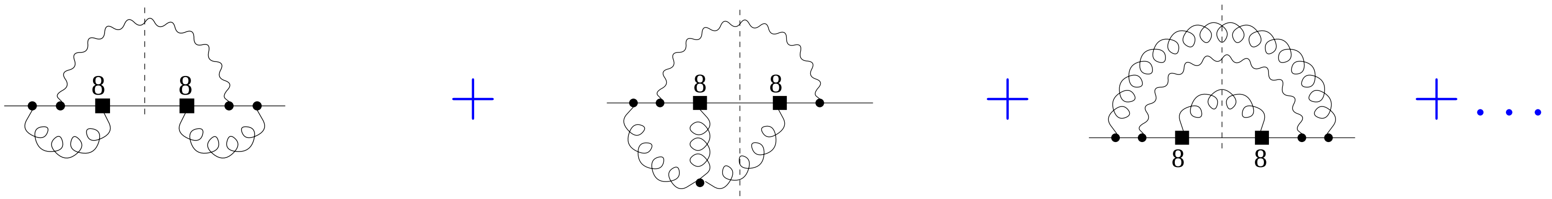
K_{22}:
(and analogous K_{11} & K_{12})



K_{28}:
(and analogous K_{18})



K_{88}:



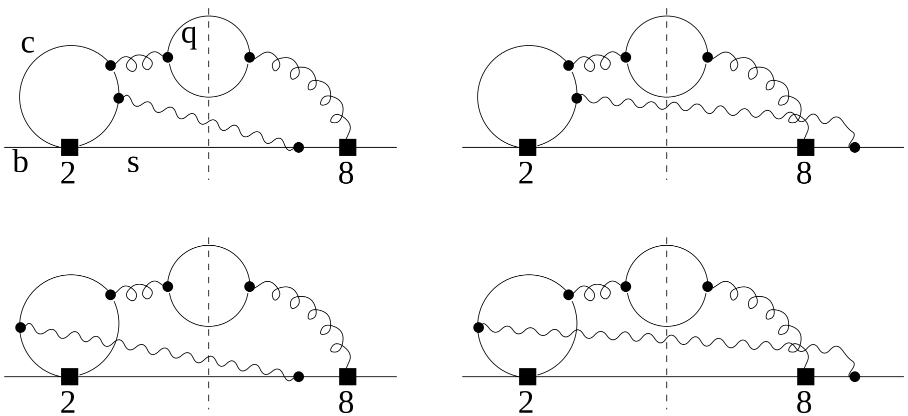
Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only: [Ligeti, Luke, Manohar, Wise, 1999], [Ferroglia, Haisch, arXiv:1009.2144], [Poradziński, MM, arXiv:1009.5685].

NLO+(NNLO BLM) corrections are not big (+3.8%).

Example:

Evaluation of the $(n > 2)$-particle cut contributions to K_{28} in the Brodsky-Lepage-Mackienzie (BLM) approximation ("naive nonabelianization", large-β_0 approximation) [Poradziński, MM, arXiv:1009.5685]:



q – **massless quark,**

N_q – **number of massless flavours (equals to 3 in practice because masses of u, d, s are neglected).**

Replacement in the final result:

$$-\tfrac{2}{3}N_q \quad \longrightarrow \quad \beta_0 = 11 - \tfrac{2}{3}(N_q + 2).$$

The diagrams have been evaluated using the method of Smith and Voloshin [hep-ph/9405204].

Non-BLM contributions to K_{ij} from quark loops on the gluon lines are quasi-completely known.
[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

Incorporating other perturbative contributions evaluated after the previous phenomenological analysis in hep-ph/0609232:

1. Four-loop mixing (current-current) $\rightarrow$ (gluonic dipole)
   M. Czakon, U. Haisch, MM, JHEP 0703 (2007) 008 [hep-ph/0612329]
2. Diagrams with massive quark loops on the gluon lines
   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]
3. Complete interference (photonic dipole)–(gluonic dipole)
   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola,
   Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]
4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole
   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   MM and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]
5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from 4-quark operators ("penguin" or CKM-suppressed)
   M. Kamiński, MM and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]
6. NLO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from interferences of the above operators with $Q_{1,2,7,8}$
   T. Huber, M. Poradziński, J. Virto, JHEP 1501 (2015) 115 [arXiv:1411.7677]

Taking into account new non-perturbative analyses:

M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters (Parametric uncertainties go down to 2.0%)

P. Gambino, C. Schwanda, Phys. Rev. D 89 (2014) 014022
A. Alberti, P. Gambino, K. J. Healey, S. Nandi, Phys. Rev. Lett. 114 (2015) 061802

Updated SM estimate for the CP- and isospin-averaged branching ratio of $\bar{B} \to X_s\gamma$ [arXiv:1503.01789, arXiv:1503.01791]:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \underbrace{\pm 0.23}_{\pm 6.9\%}) \times 10^{-4} \qquad \text{for } E_\gamma > 1.6\,\text{GeV}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, **2%** parametric

It is very close the the experimental world average:

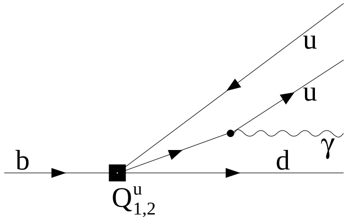
$$\mathcal{B}_{s\gamma}^{\rm exp} = (3.32 \underbrace{\pm 0.15}_{\pm 4.5\%}) \times 10^{-4} \qquad \text{[HFLAV, arXiv:1612.07233v2]}$$

Experiment agrees with the SM well within $\sim 1\sigma$.

$\Rightarrow$ Strong bound on the $H^\pm$ mass in the Two-Higgs-Doublet-Model II:

$$M_{H^\pm} > 580\,\text{GeV at } 95\%\text{C.L.} \qquad \text{[MM, M. Steinhauser, arXiv:1702.04571]}$$

$\bar{B} \to X_d\gamma$



$$\mathcal{L}_{\rm eff} \;\sim\; V_{td}^* V_{tb} \left[\sum_{i=1}^{8} C_i Q_i + \kappa_d \sum_{i=1}^{2} C_i (Q_i - Q_i^u) \right]$$

$$\kappa_d = (V_{ud}^* V_{ub})/(V_{td}^* V_{tb}) \;=\; \left(0.007^{+0.015}_{-0.011}\right) + i\left(-0.404^{+0.012}_{-0.014}\right)$$

$$\left.\begin{array}{l} \mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5} \\ \mathcal{B}_{d\gamma}^{\rm exp} = (1.41 \pm 0.57) \times 10^{-5} \end{array}\right\} \text{for } E_0 = 1.6\,\text{GeV}$$

- $\mathcal{B}_{d\gamma}^{\rm SM}$ is rough: m_b/m_q varied between $10 \sim m_B/m_K$ and $50 \sim m_B/m_\pi$ $\Rightarrow$ 2% to 11% of $\mathcal{B}_{d\gamma}$.
- Fragmentation functions give a similar range [H. M. Asatrian and C. Greub, arXiv:1305.6464].
- Collinear logarithms and isolated photons

The ratio R_γ

$$R_\gamma^{\rm SM} \equiv \left(\mathcal{B}_{s\gamma}^{\rm SM} + \mathcal{B}_{d\gamma}^{\rm SM}\right)/\mathcal{B}_{c\ell\nu} = (3.31 \pm 0.22) \times 10^{-3}$$

Generic (but CP-conserving) beyond-SM effects:

$$\mathcal{B}_{s\gamma} \times 10^4 = (3.36 \pm 0.23) - 8.22\,\Delta C_7 - 1.99\,\Delta C_8,$$
$$R_\gamma \times 10^3 = (3.31 \pm 0.22) - 8.05\,\Delta C_7 - 1.94\,\Delta C_8.$$