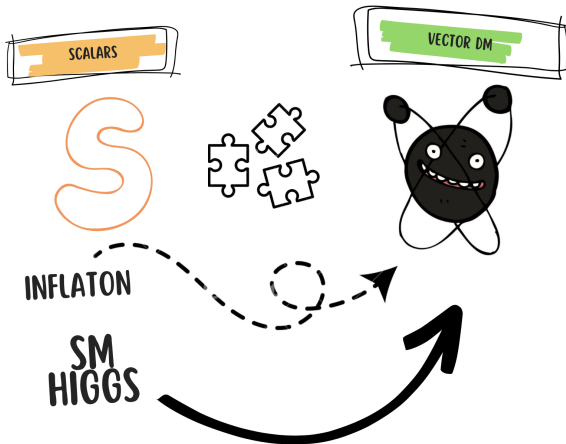


Role of scalars in gravitational production of vector dark matter

Anna Socha

University of Warsaw

based on: A. Ahmed, B. Grządkowski, AS [2005.01766](#), [2111.06065](#), [2207.11218](#)



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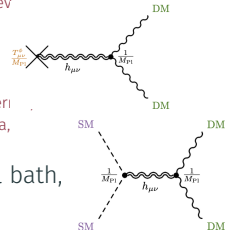
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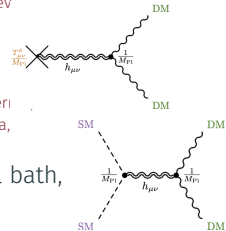
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- All these phenomena are universal and inevitable. However, they occur at slightly different moments and have distinct origins.



The action for a massive spin-1 DM spectator field X_μ in a background metric $g_{\mu\nu}$ is given by

$$S_{\text{DM}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{DM}}, \quad \mathcal{L}_{\text{DM}} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} X_{\mu\nu} X_{\alpha\beta} - \frac{1}{2} m_X^2 g^{\mu\nu} X_\mu X_\nu.$$

$\rightarrow X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$

FLRW spacetime

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2,$$

$$dt = a(\tau) d\tau$$

Gravitational coupling

only through $g^{\mu\nu}$, $\sqrt{-g}$
 without non-minimal coupling, i.e.,
 $\xi_1 R g^{\mu\nu} X_\mu X_\nu, \xi_2 R^{\mu\nu} X_\mu X_\nu$

Non-zero mass

generated e.g. via
 the Stueckelberg
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Remarks

$\Rightarrow$ X_0 does not have a kinetic term; it is an auxiliary field.

$\Rightarrow$ Massless vectors with a minimal coupling to gravity, i.e., $\xi_1 = 0 = \xi_2$ are conformally coupled to gravity.

$$X_0 = \frac{-i\vec{k} \cdot \vec{X}'}{k^2 + a^2 m_X^2}$$

**Not populated
by expansion**

EoMs for DM fields

Fourier decomposition

$$[\hat{a}_\lambda(\vec{p}), \hat{a}_\sigma^\dagger(\vec{q})] = (2\pi)^3 \delta_{\lambda,\sigma} \delta^{(3)}(\vec{p} - \vec{q})$$

$$\hat{\chi}(\tau, \vec{x}) = \sum_{\lambda=\pm, L} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \vec{\epsilon}_\lambda(\vec{k}) \{ \mathcal{X}_\lambda(\tau, k) \hat{a}_\lambda(\vec{k}) + \mathcal{X}_\lambda^*(\tau, k) \hat{a}_\lambda^\dagger(\vec{k}) \}$$

Harmonic oscillator equation, Wronskian

$$\mathcal{X}_\lambda'' + \omega_\lambda^2(\tau) \mathcal{X}_\lambda = 0,$$

$$\mathcal{X}_\lambda' \mathcal{X}_\lambda^* - \mathcal{X}_\lambda^{*'} \mathcal{X}_\lambda = -i$$

Time-dependent frequencies

Transverse modes

$$\omega_\pm^2(\tau) \equiv k^2 + a^2(\tau) m_X^2,$$

Redefined longitudinal mode*

$$\omega_L^2(\tau) \equiv k^2 + a^2(\tau) m_X^2 - \frac{k^2}{k^2 + a^2(\tau) m_X^2} \left[\frac{a''(\tau)}{a(\tau)} - \frac{3a^2(\tau) m_X^2}{k^2 + a^2(\tau) m_X^2} \left(\frac{a'(\tau)}{a(\tau)} \right)^2 \right]$$

Longitudinal mode $\mathcal{A}_L$

is not
canonically normalized

$$\mathcal{X}_L \equiv \frac{am_X}{\sqrt{k^2 + a^2 m_X^2}} \mathcal{A}_L$$

The vacuum expectation value of the total energy density of the spin-1 DM decomposes as,

$$\langle \hat{\rho}_X \rangle \equiv \langle 0 | \hat{\rho}_X | 0 \rangle = \langle \hat{\rho}_L \rangle + \langle \hat{\rho}_\pm \rangle, \quad \text{Bunch-Davies vacuum}$$

with

Transverse modes

$$\langle \hat{\rho}_\pm \rangle = \frac{1}{2a^4} \int \frac{d^3 k}{(2\pi)^3} \left\{ |\mathcal{X}'_\pm|^2 + |\mathcal{X}_\pm|^2 [k^2 + a^2 m_X^2] \right\},$$

Redefined longitudinal mode

$$\begin{aligned} \langle \hat{\rho}_L \rangle = \frac{1}{2a^4} \int \frac{d^3 k}{(2\pi)^3} \left\{ |\mathcal{X}'_L|^2 + |\mathcal{X}_L|^2 \left[k^2 + a^2 m_X^2 + \left(\frac{a'}{a} \right)^2 \frac{k^4}{(k^2 + a^2 m_X^2)^2} \right] \right. \\ \left. - (\mathcal{X}'_L \mathcal{X}_L^* + \mathcal{X}_L'^* \mathcal{X}_L) \frac{k^2}{k^2 + a^2 m_X^2} \frac{a'}{a} \right\}. \end{aligned}$$

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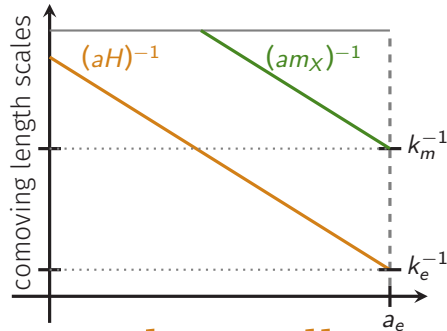
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Note that $\langle \hat{\rho}_L \rangle \gg \langle \hat{\rho}_\pm \rangle$

This could change if one considers direct X_μ coupling to the inflaton ϕ of the form $\phi X_{\mu\nu} \tilde{X}^{\mu\nu}$, see e.g. arXiv:1810.07208

Evolution of the longitudinal modes during inflation

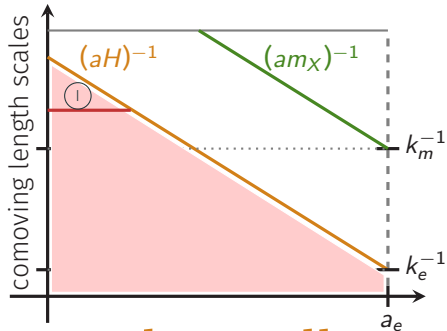


Slow-roll

$$H_I \simeq H_e,$$

$$\left(\frac{a'}{a}\right)^2 = a_I^2 H_e^2, \quad \frac{a''}{a} = 2a_I^2 H_e^2.$$

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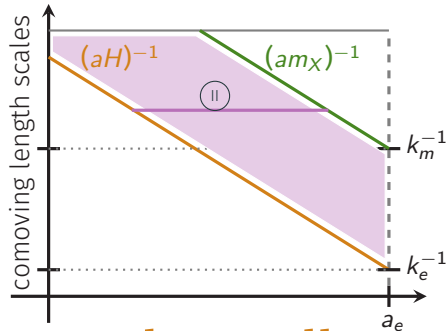
Sub-horizon

$$\omega_L^2 \simeq k^2,$$

Bunch-Davies vacuum

$$\chi_L^I \simeq \frac{1}{\sqrt{2k}} e^{-ik\tau},$$

Evolution of the longitudinal modes during inflation



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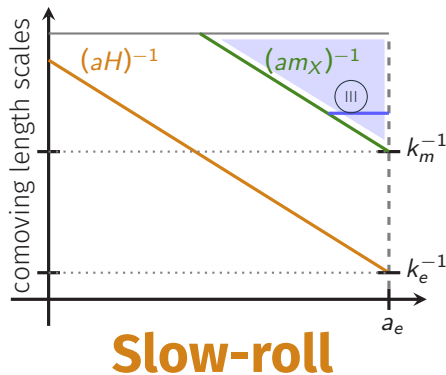
Super-horizon R

$$\omega_L^2 \simeq k^2 - 2a_I^2 H_e^2 < 0,$$

tachyonic enhancement

$$\chi_L^{\text{II}} \simeq \frac{i}{2\sqrt{k}} \frac{a_I H_e}{k} e^{-ik\tau},$$

Evolution of the longitudinal modes during inflation



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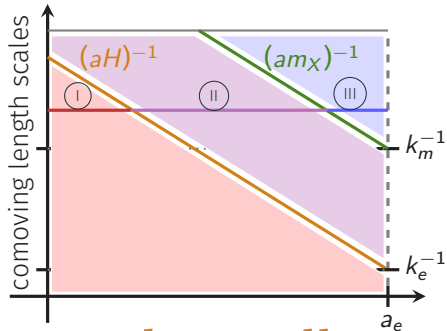
III **Super-horizon NR**

$$\omega_L^2 \simeq a_I^2 m_X^2 + k^2 \frac{H_I^2}{m_X^2},$$

constant solution

$$\chi_L^{(III)} \simeq \text{const.}$$

Evolution of the longitudinal modes during inflation



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Post-inflationary evolution

Boltzmann equations

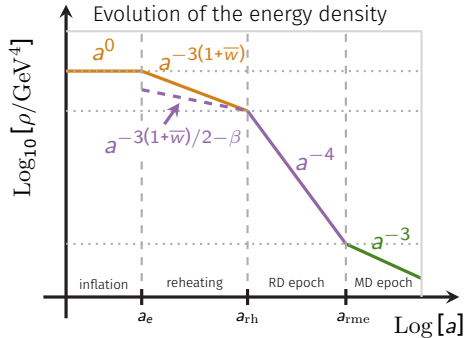
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$$\Gamma_{\phi} \propto a^{-\beta}$$

$$\dot{\rho}_{\text{SM}} + 4H\rho_{\text{SM}} = \Gamma_{\phi}\rho_{\phi}$$

Friedmann equation

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} (\rho_{\phi} + \rho_{\text{SM}})$$



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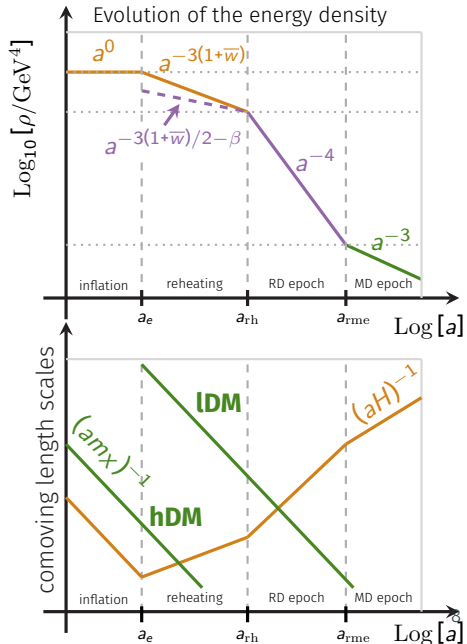
Evolution of a , H and the Hubble radius

Reheating

$$H(a) = H_e(a/a_e)^{-\frac{3(1+\bar{w})}{2}}$$

RD epoch

$$H(a) = H_{\text{rh}}(a/a_{\text{rh}})^{-2}$$



DM production strongly depends on reheating dynamics

Expansion history

$$\overline{w}$$

Form of the inflaton potential $V(\phi) \sim \phi^{2n}$

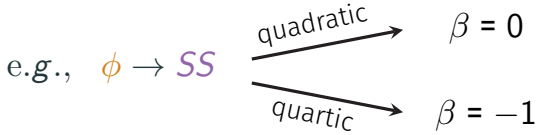
$$\overline{w} = \frac{n-1}{n+1}$$

Reheating duration

$$\gamma^2 = H_{\text{rh}}/H_e$$

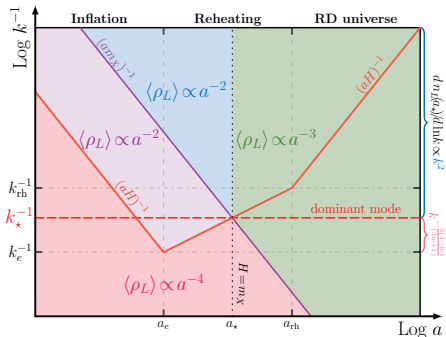
Form of the interactions

$$\Gamma_\phi \propto a^{-\beta}$$



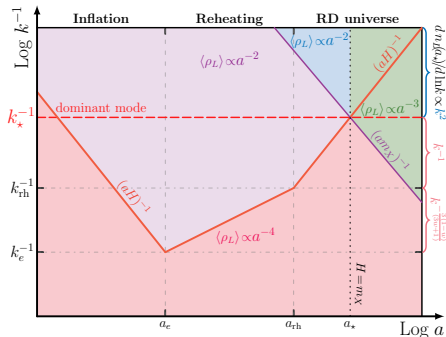
heavy DM vectors (hDM)

$$m_X > H_{\text{rh}}$$



light DM vectors (lDM)

$$m_X < H_{\text{rh}}$$



dominant mode

$$k_{\text{star}} \equiv a_{\text{star}} m_X, \quad H(a_{\text{star}}) = m_X$$

- hDM: $H_{\text{rh}} \leq m_X < H_e$,

$$\frac{d\langle n_L^{\text{hDM}}(a_*) \rangle}{d \ln k} = \frac{H_e^3}{8\pi} \begin{cases} \left(\frac{m_X}{H_e} \right)^{\frac{2}{1+w}} \left(\frac{k_e}{k} \right)^{\frac{3(1-\bar{w})}{(1+3\bar{w})}}, & k_* < k < k_e, \\ \left(\frac{m_X}{H_e} \right)^{\frac{1-3\bar{w}}{3(1+\bar{w})}} \left(\frac{k}{k_e} \right)^2, & k < k_*. \end{cases}$$

$k_e \equiv a_e H_e$

$k_* \equiv a_* m_X$

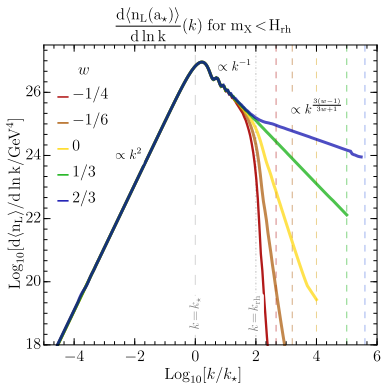
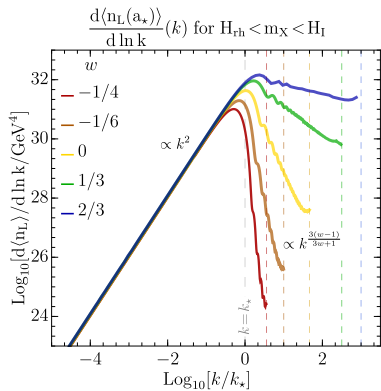
- lDM: $m_X < H_{\text{rh}}$,

$$\frac{d\langle n_L^{\text{lDM}}(a_*) \rangle}{d \ln k} = \frac{H_e^3}{8\pi} \begin{cases} \left(\frac{m_X}{H_e} \right)^{3/2} \gamma^{\frac{1-3\bar{w}}{1+\bar{w}}} \left(\frac{k_e}{k} \right)^{\frac{3(1-\bar{w})}{1+3\bar{w}}}, & k_{\text{rh}} < k < k_e, \\ \left(\frac{m_X}{H_e} \right)^{3/2} \gamma^{\frac{-1+3\bar{w}}{3(1+\bar{w})}} \frac{k_e}{k}, & k_* < k < k_{\text{rh}}, \\ \gamma^{\frac{2(1-3\bar{w})}{3(1+\bar{w})}} \left(\frac{k}{k_e} \right)^2, & k < k_*. \end{cases}$$

reheating efficiency

$\gamma^2 \equiv H_{\text{rh}}/H_e$

$k_{\text{rh}} \equiv a_{\text{rh}} H_{\text{rh}}$



Graviton-mediated DM production

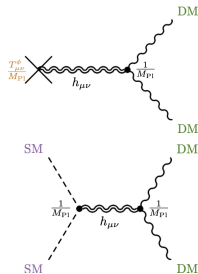
$$\mathcal{L}_{\text{int}} = \frac{h^{\mu\nu}}{M_{\text{Pl}}} \left(T_{\mu\nu}^{\text{DM}} + T_{\mu\nu}^{\phi} + T_{\mu\nu}^{\text{SM}} \right)$$

Boltzmann equation

$$\dot{n}_{\text{DM}} + 3Hn_{\text{DM}} = \mathcal{R}_{\phi} + \mathcal{S}_{\text{SM}}$$

$$\mathcal{R}_{\phi} \propto \frac{\rho_{\phi}^2}{M_{\text{Pl}}^4}$$

$$\mathcal{S}_{\text{SM}} \propto \begin{cases} T^8/M_{\text{Pl}}^4, & m_X \ll T, \\ (m_X^5 T^3/M_{\text{Pl}}^4) e^{-2m_X/T}, & m_X \gg T, \end{cases}$$



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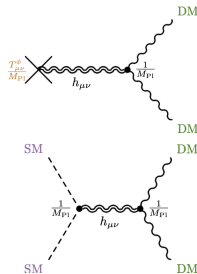
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UV sensitive

Interactions

1. Specify the inflaton evolution

The α -attractor T-model

$$V(\phi) = \Lambda^4 \tanh^{2n} \left(\frac{\phi}{M} \right)$$

R. Kallosh [et al.](#), arXiv:1306.5220

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2. Determine the reheating dynamics

Bosonic, perturbative reheating

$$\phi \rightarrow SS$$

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3. Find DM abundance

$$(\Omega_X^{\text{GP}} + \Omega_X^{\text{IG}} + \Omega_X^{\text{SMG}}) h^2$$

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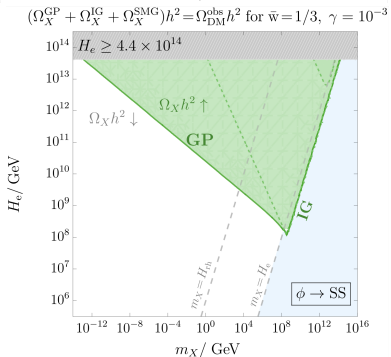
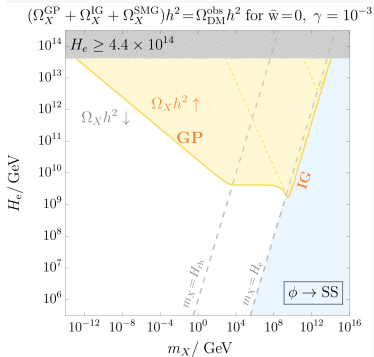
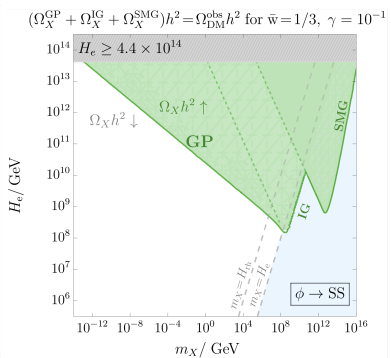
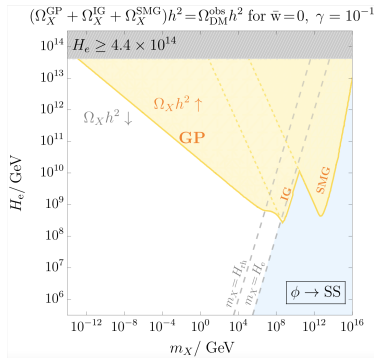
$$\phi \rightarrow SS$$

3. Find DM abundance

$$(\Omega_X^{\text{GP}} + \Omega_X^{\text{IG}} + \Omega_X^{\text{SMG}}) h^2$$

4. Compare with the observed value

$$\Omega_X^{\text{obs}} h^2 \simeq 0.1198 \pm 0.0012$$



Summary

- Purely Gravitational DM model could be considered as a minimal, albeit viable scenario for the dark sector.
- It has been established that the spectrum of dark vectors produced gravitationally has a peak structure. In particular, it is centered around a characteristic comoving momentum k_* .
- We have demonstrated that accounting for the finite duration of reheating has a significant impact on the production of heavy DM vectors, i.e., with mass $m_X > H_{\text{rh}}$.
- VDM particles with mass $m_X < H_e$ are abundantly produced in the inflationary era due to the tachyonic enhancement of the longitudinal momentum modes.
- Superheavy DM vectors ($m_X > H_e$) can be created thermally by the SM bath in an (almost) instantaneous reheating scenario.

Thank you for your attention!

Back-up slides

The isocurvature constraints are suppressed if

$$k_{\star} \ll k_{\text{CMB}} \approx 0.05 \text{Mpc}^{-1}.$$

$$k_{\star} \approx 1400 \text{pc}^{-1} \sqrt{\frac{m_X}{10^{-14} \text{GeV}}} \begin{cases} \left(\frac{H_{\text{rh}}}{m_X} \right)^{\frac{1-3w}{6(1+w)}}, & H_{\text{rh}} \leq m_X < H_e, \\ 1, & H_{\text{mre}} \leq m_X < H_{\text{rh}}. \end{cases}$$

IDM vectors with mass $m_X \geq 10^{-14} \text{GeV}$ are safe

$$\left(\frac{H_{\text{rh}}}{m_X} \right)^{\frac{1-3w}{6(1+w)}} = \begin{cases} \left(\frac{m_X}{H_{\text{rh}}} \right)^{[0, 1/6]} \geq 1, & w = [1/3, 1), \\ \left(\frac{H_{\text{rh}}}{m_X} \right)^{(1/2, 0)} < 1, & w = (-1/3, 1/3). \end{cases}$$

lower bound on
the reheating scale

$$m_X \geq H_{\text{rh}} \geq 10^{-14} \text{GeV} \left(\frac{10^{-14} \text{GeV}}{m_X} \right)^{\frac{2(1+3w)}{(1-3w)}}, \quad w = (-1/3, 1/3).$$

Time-averaged Boltzmann equations

Expansion

$$\dot{\rho}_{\phi} + 3(1 + \overline{w})H\rho_{\phi} = -\langle\Gamma_{\phi}\rangle\rho_{\phi}$$

$$\dot{\rho}_{\mathcal{R}} + 4H\rho_{\mathcal{R}} = \langle\Gamma_{\phi}^{\mathcal{R}}\rangle\rho_{\phi}$$

$$\rightarrow \overline{w} \equiv \langle p_{\phi} \rangle / \langle \rho_{\phi} \rangle$$

$$H^2 = \frac{\rho_{\phi} + \rho_{\mathcal{R}}}{3M_{pl}^2} \leftarrow$$

Interactions

Time-dependent decay rate

$$\langle\Gamma_{\phi}^{\mathcal{R}}\rangle \simeq \langle\Gamma_{\phi}\rangle = \Gamma_{\phi}^e \left(\frac{a_e}{a}\right)^{\beta}$$

constant parameter

Non-instantaneous reheating

$$\rho_\phi(a) \stackrel{H \gg \Gamma_\phi}{\simeq} 3M_{\text{Pl}}^2 H_e^2 \left(\frac{a_e}{a}\right)^{3(1+\bar{w})}$$

$$\rho_{\mathcal{R}}(a) = \frac{6M_{\text{Pl}}^2 H_e \Gamma_\phi^e}{5-3\bar{w}-2\beta} \left[\left(\frac{a_e}{a}\right)^{\beta+3(1+\bar{w})/2} - \left(\frac{a_e}{a}\right)^4 \right]$$

