RENORMALISATION-GROUP IMPROVEMENT OF MULTI-FIELD EFFECTIVE POTENTIALS

Bogumiła Świeżewska
Utrecht University

In collaboration with Tomislav Prokopec, Leonardo Chataigner, Michael G. Schmidt

Scalars2017, 2.12.2017

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WHY MULTI-FIELD?

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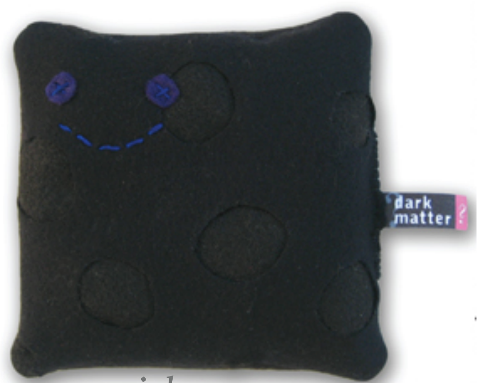

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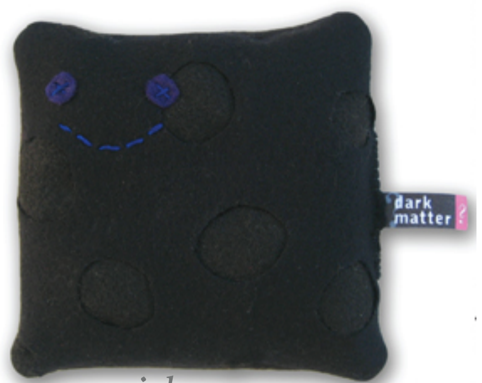

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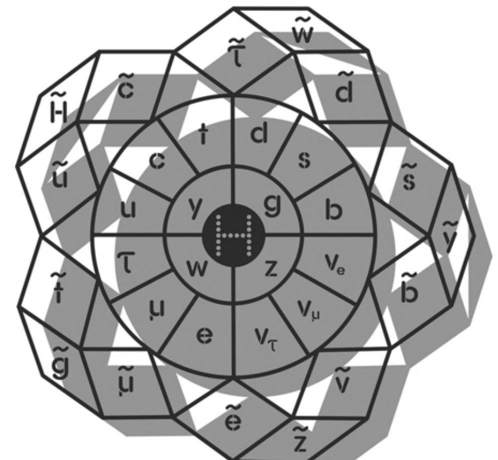

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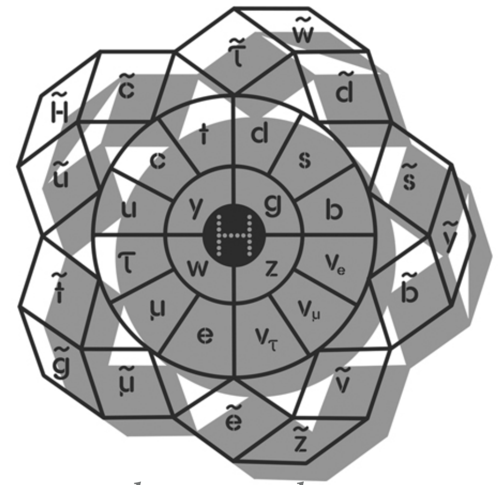
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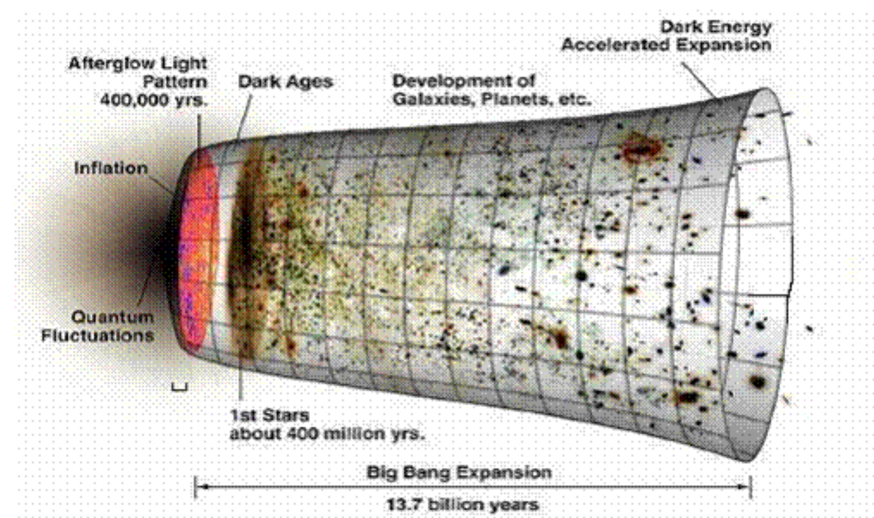

KIAS astrophysics group

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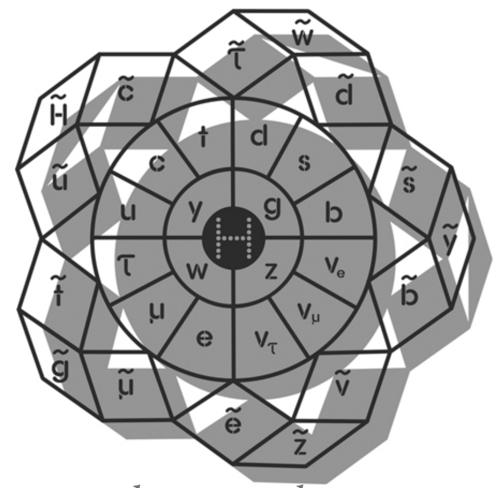

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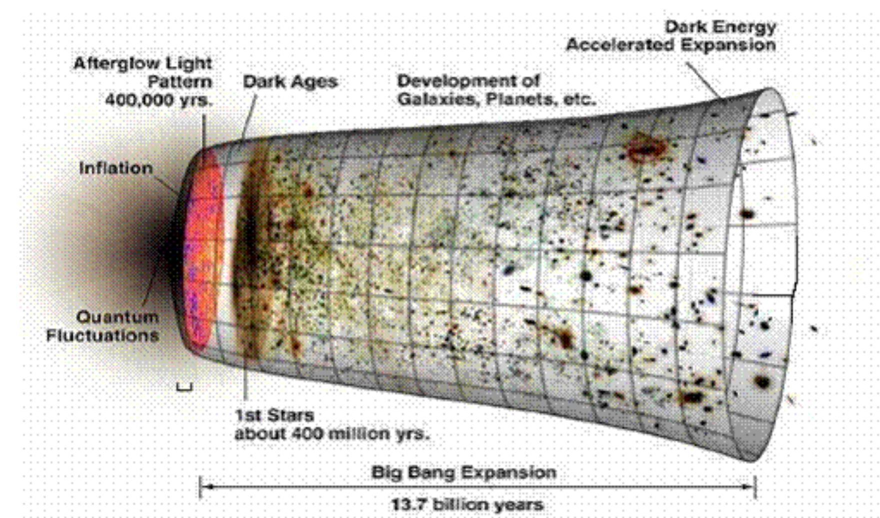

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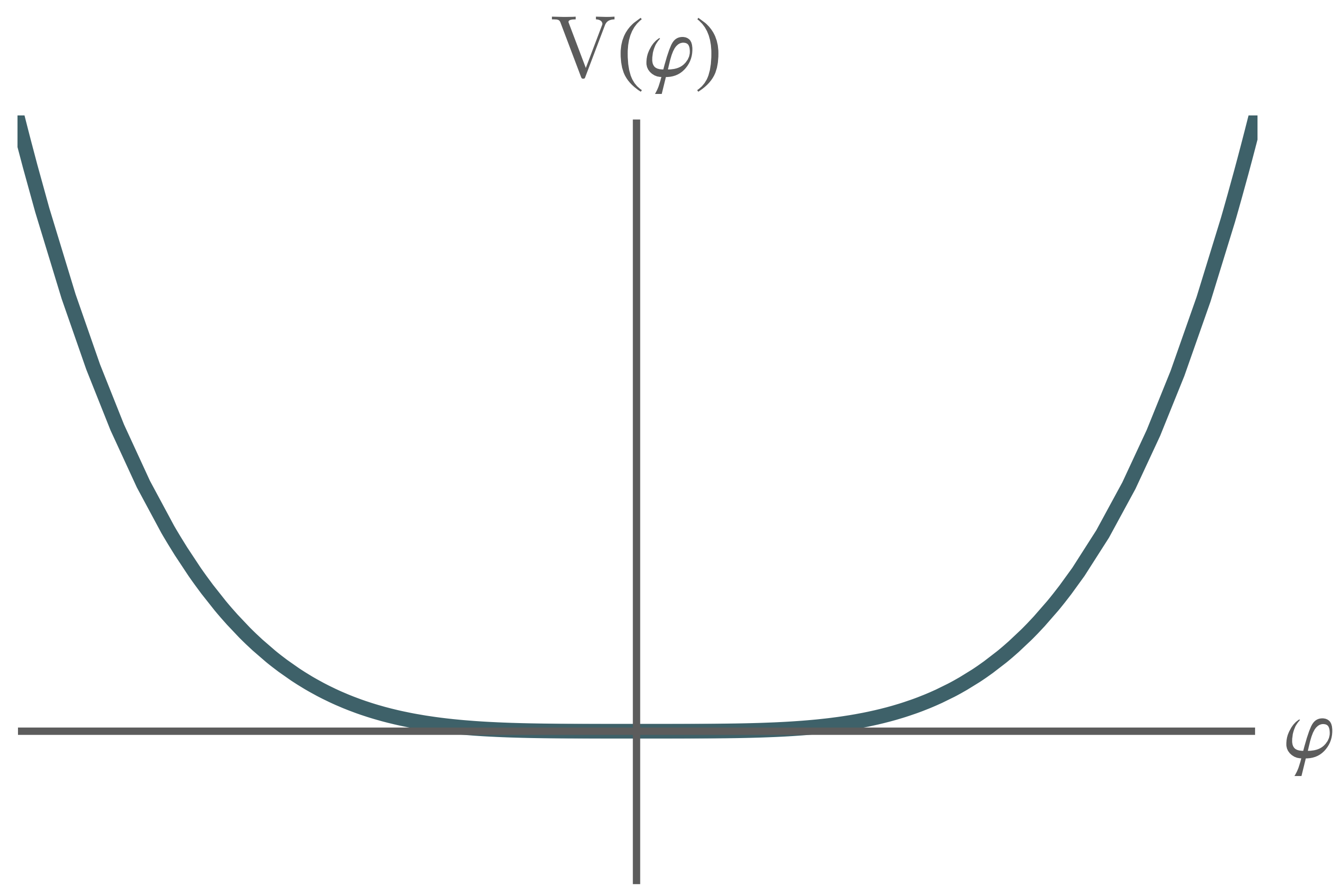
conferences.fnal.gov

WHY EFFECTIVE POTENTIAL?

- To determine the vacuum structure
- To asses stability of the vacuum state
- To determine loop-corrected couplings and masses



[S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 18888]

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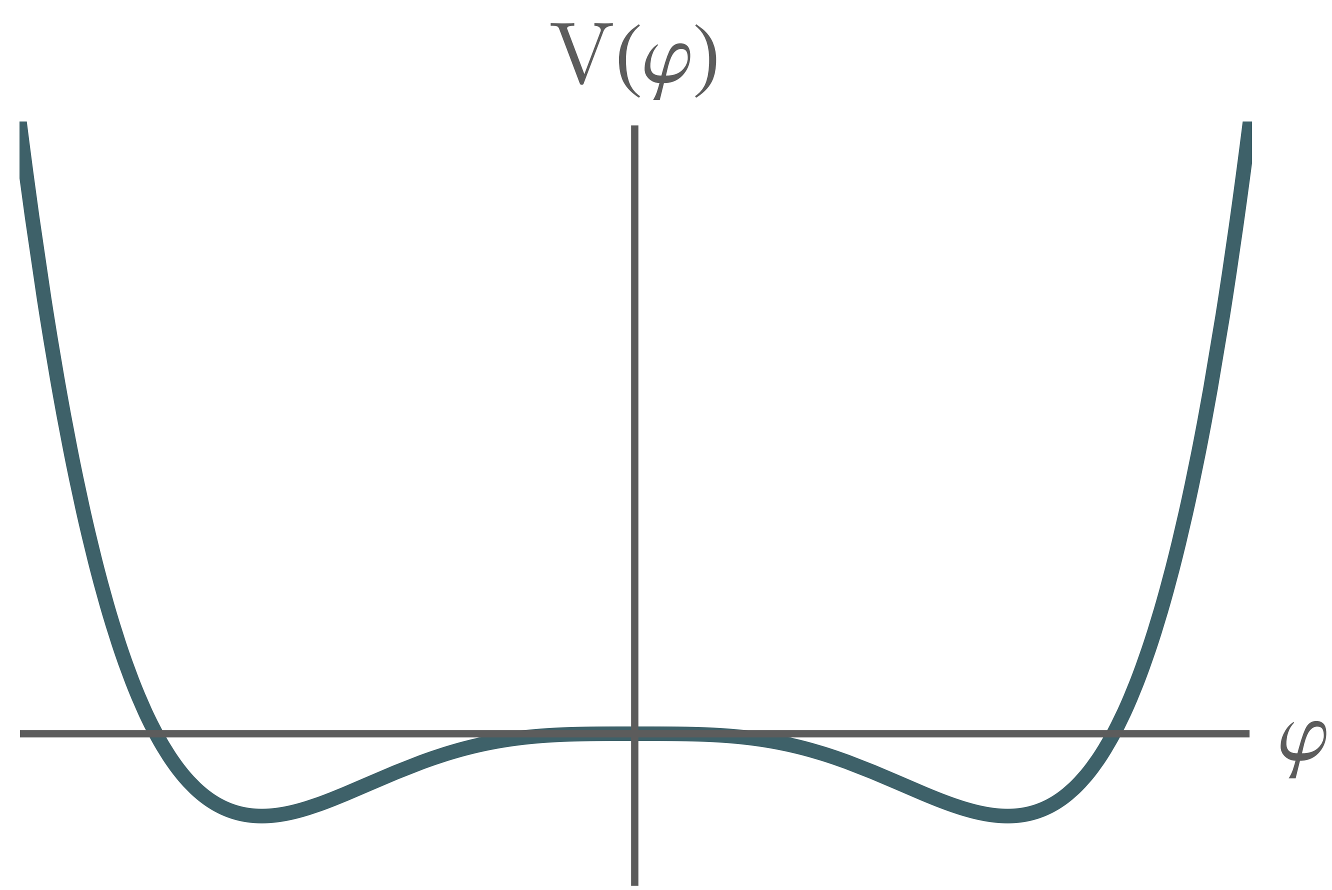


[S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 18888]

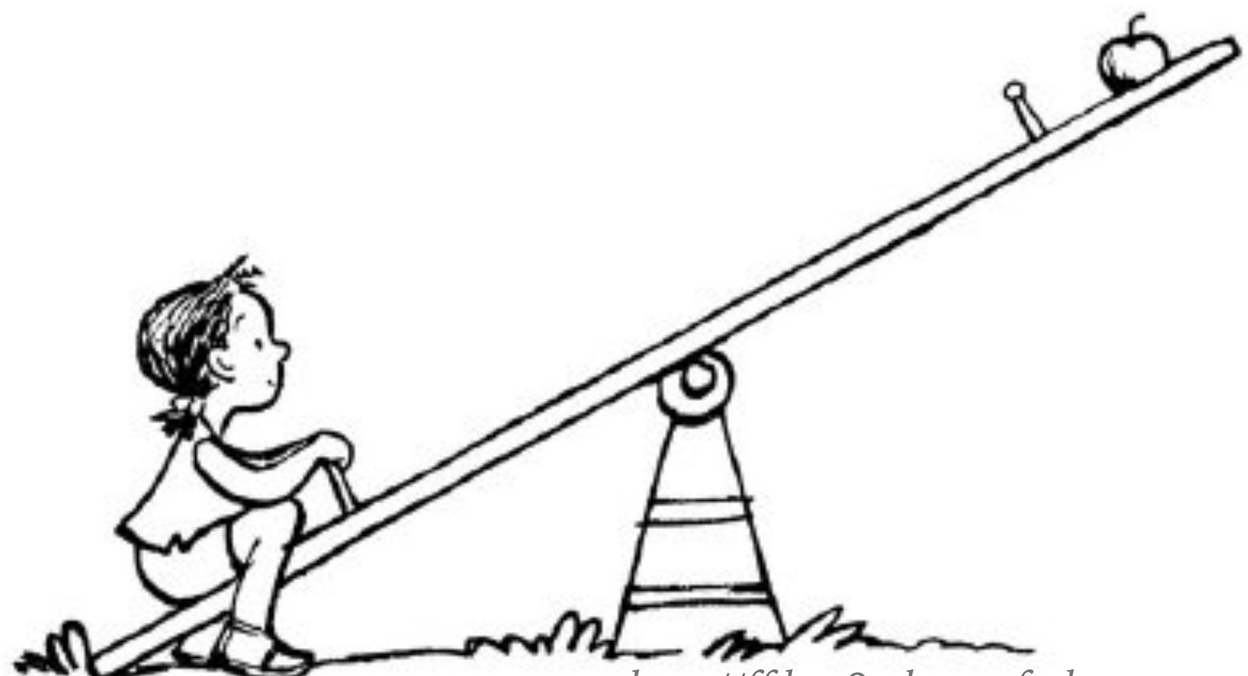
WHY RGE?

Loop expansion parametrs: $\lambda, \ \lambda \log \frac{\phi^2}{\mu^2}$

$\xrightarrow{\text{RGE}}$ Scale independence of the effective potential

RGE improved potential expansion parameter: $\bar{\lambda}\left(\log \frac{\phi^2}{\mu^2}\right)$

WHY IS THIS PROBLEMATIC?



http://ffden-2.phys.uaf.edu

[See also: M.B. Einhorn, D.R.T. Jones, Nucl. Phys. B 230 (1984) 261, C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl. Phys. B 395 (1993) 17, C. Ford, Phys. Rev. D 50 (1994) 7531, C. Ford and C. Wiesendanger, Phys. Lett. B 398 (1997) 342, M. Bando, T. Kugo, N. Maekawa and H. Nakano, Prog Theor Phys (1993) 90, J.A. Casas, V. Di Clemente, M. Quirós, Nucl.Phys. B553 (1999) 511]

RENORMALISATION GROUP EQUATION

$$\mu \frac{\mathrm{d}V}{\mathrm{d}\mu}(\mu;\lambda,\phi) = \left(\mu \frac{\partial}{\partial\mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial\lambda_i} - \frac{1}{2}\sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial\phi_a}\right) V(\mu;\lambda,\phi) = 0$$

METHOD OF CHARACTERISTICS

$$\mu \frac{\mathrm{d}V}{\mathrm{d}\mu}(\mu;\lambda,\phi) = \left(\mu\frac{\partial}{\partial\mu} + \sum_{i=1}^{N_\lambda}\beta_i\frac{\partial}{\partial\lambda_i} - \frac{1}{2}\sum_{a=1}^{N_\phi}\gamma_a\phi_a\frac{\partial}{\partial\phi_a}\right)V(\mu;\lambda,\phi) = 0$$

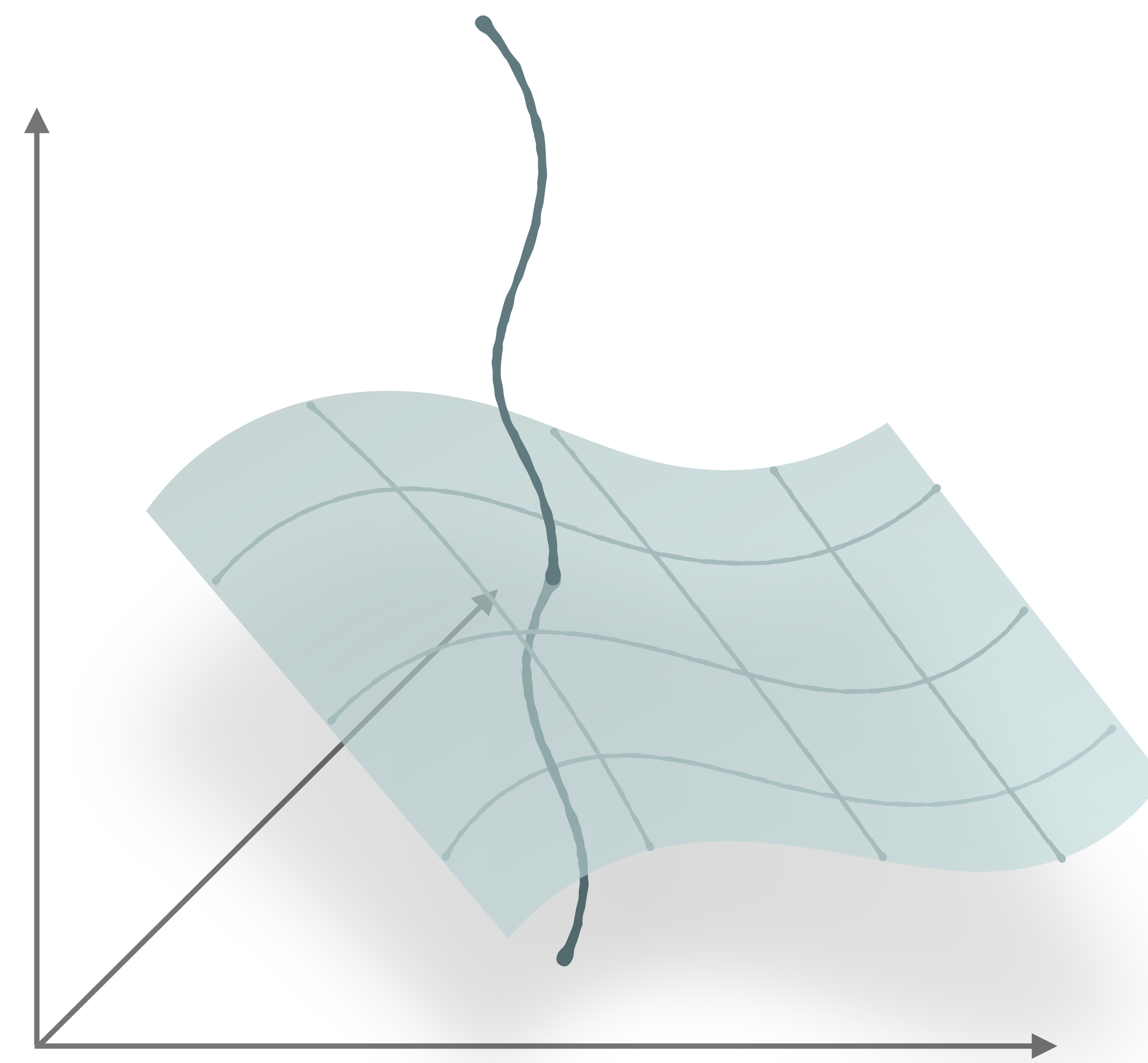
Characteristic curves

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\phi}_a(t) = -\frac{1}{2}\gamma_a(\bar{\lambda})\bar{\phi}_a(t),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) = 0$$



SINGLE-FIELD MODEL

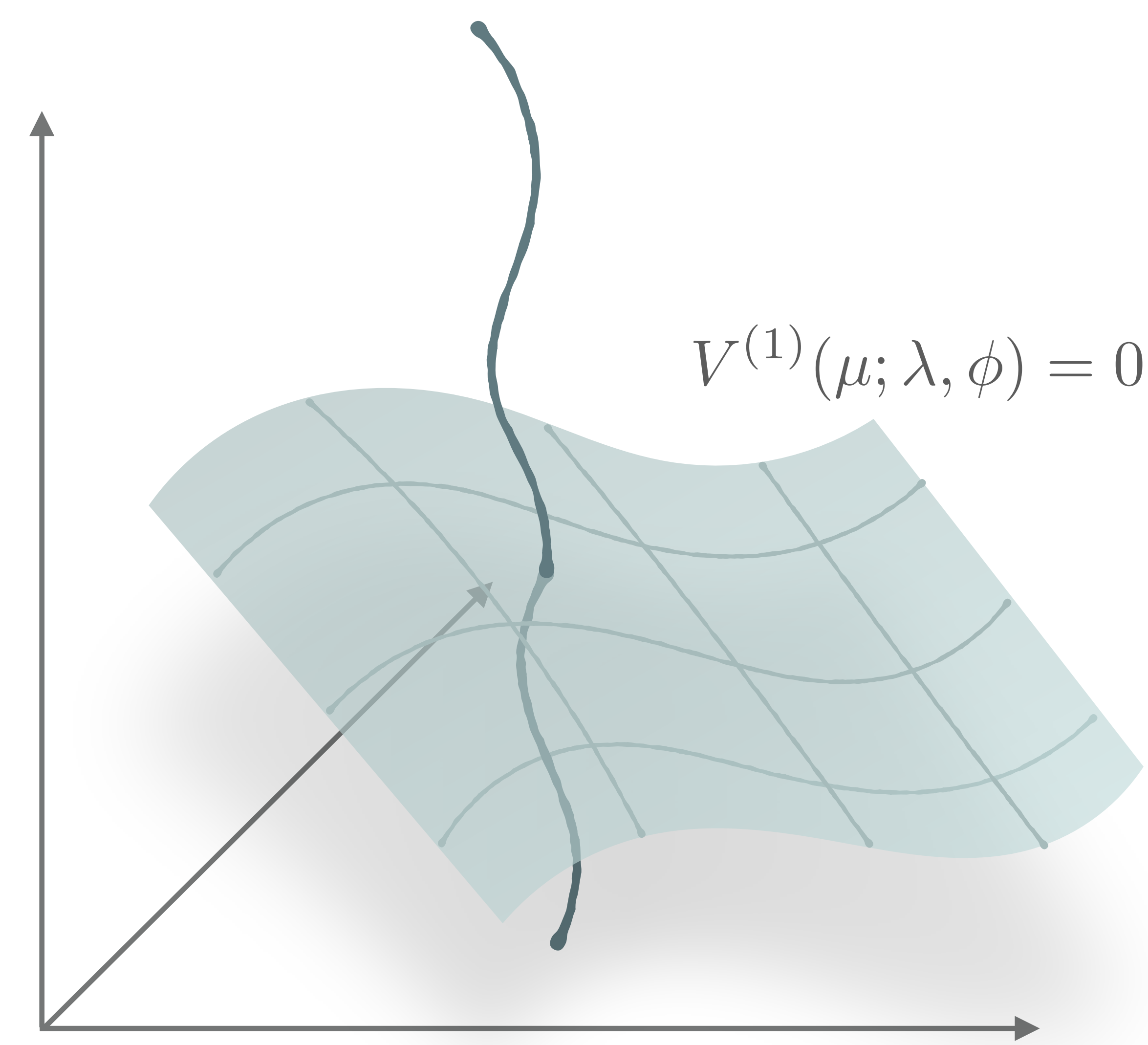
$$V_1(\mu;\lambda,\phi) = V^{(0)} + V^{(1)} = \frac{1}{4}\lambda\phi^4 + \frac{9\hbar\lambda^2\phi^4}{64\pi^2}\left[\log\frac{3\lambda\phi^2}{\mu^2} - \frac{3}{2}\right]$$

To compute $V(\mu,\lambda,\phi)$

t_* Run to the tree-level surface

Evaluate

$$V(\mu,\lambda,\phi) = V^{(0)}(\bar{\lambda}(t_*,\lambda),\phi)$$



MULTI-FIELD MODELS

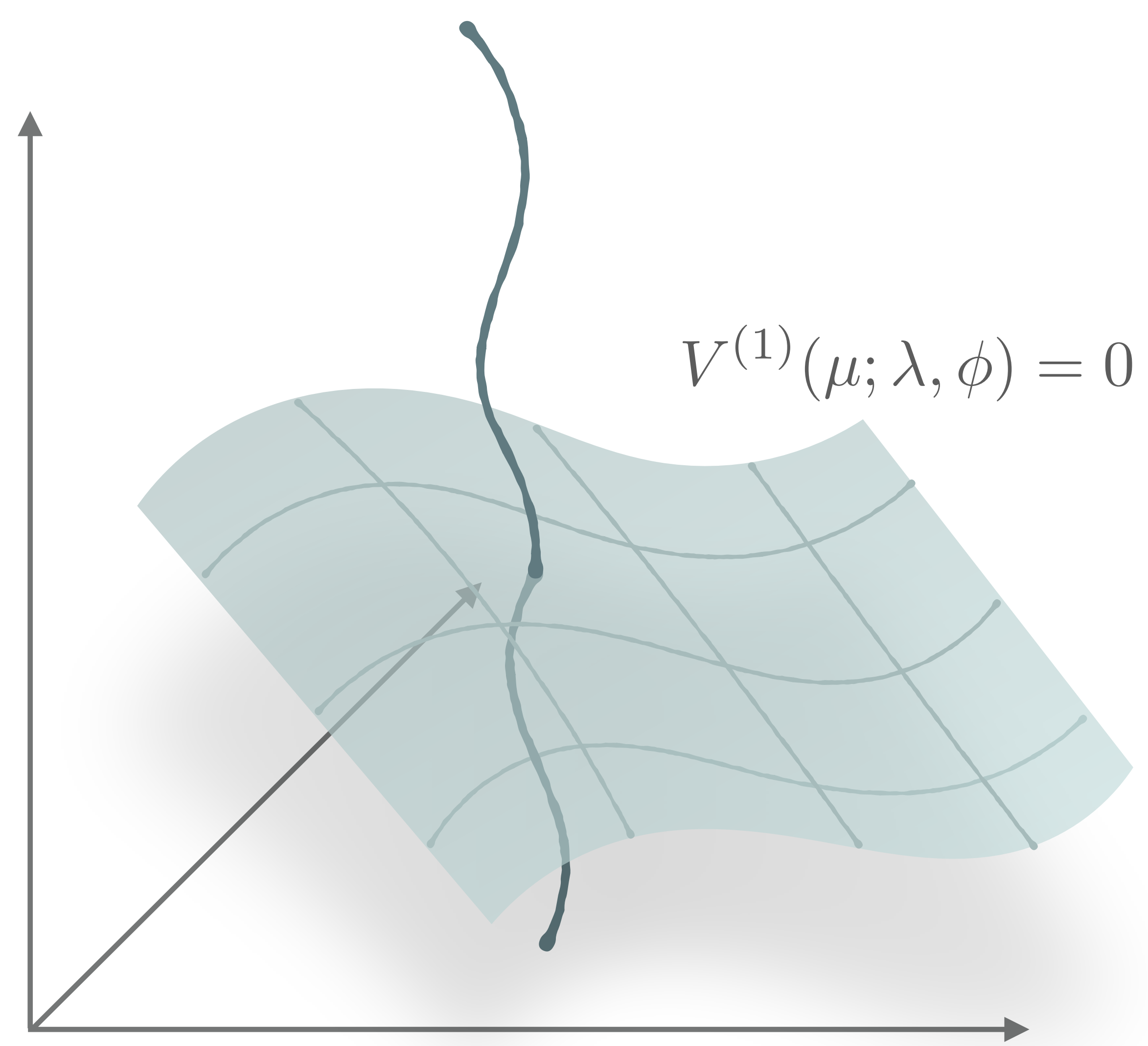
$$V^{(1)}(\mu,\lambda,\phi) = \frac{1}{64\pi^2}\sum_a n_a m_a^4(\lambda,\phi)\left[\log\frac{m_a^2(\lambda,\phi)}{\mu^2} - \chi_a\right]$$

To compute $V(\mu,\lambda,\phi)$

t_* Run to the tree-level surface

Evaluate

$$V(\mu,\lambda,\phi) = V^{(0)}(\bar\lambda(t_*),\bar\phi(t_*))$$



MULTI-FIELD MODELS

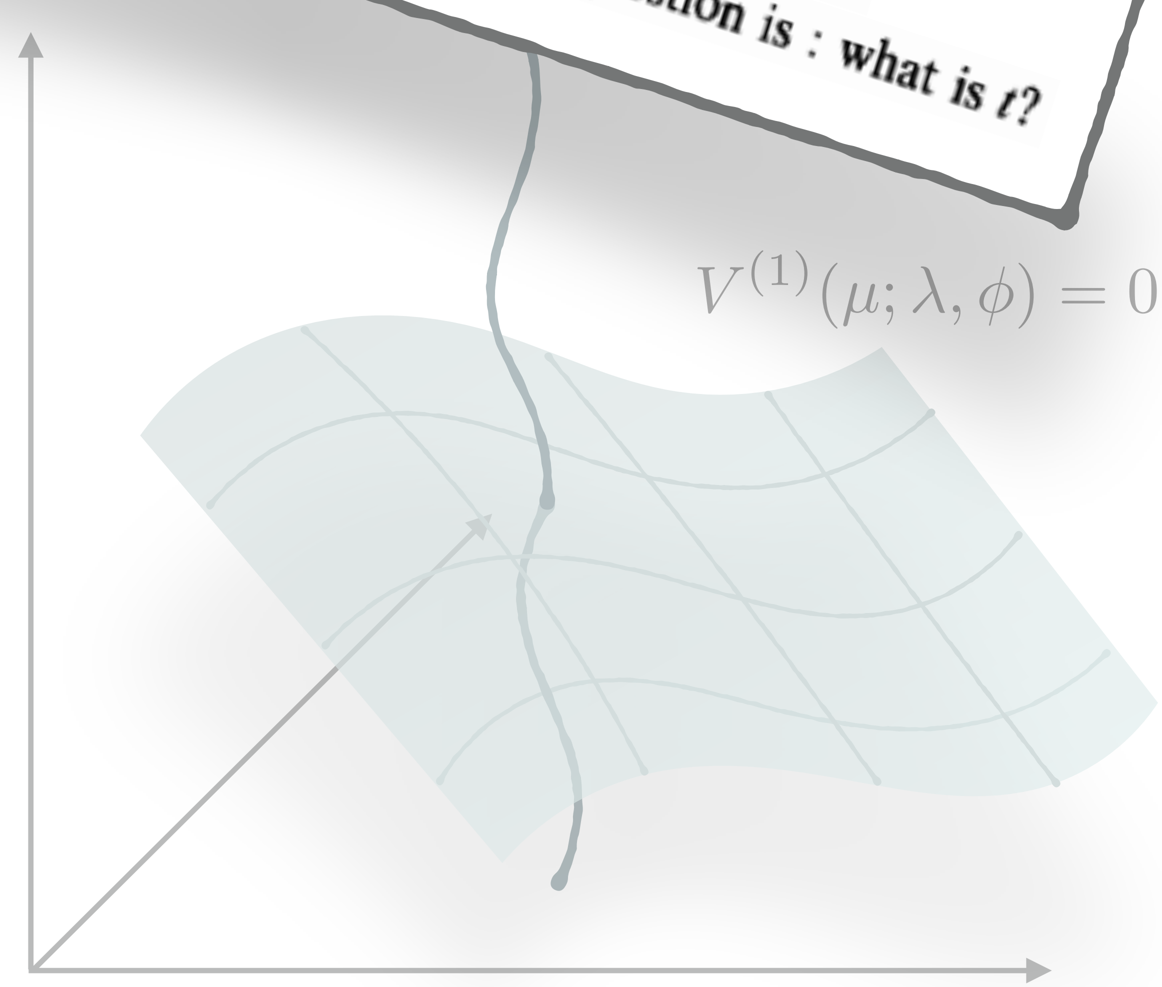
$$V^{(1)}(\mu, \lambda, \phi) = \ldots \left[\log \frac{m_a^2(\lambda, \phi)}{\mu^2} - \chi_a\right]$$

M. Sher, Electroweak Higgs potentials and vacuum stability
The form of this solution is not surprising; the only question is : what is *t*?

To compute $V(\mu, \lambda, \phi)$

t_* Run to the tree-level surface

Evaluate
$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$



MULTI-FIELD MODELS

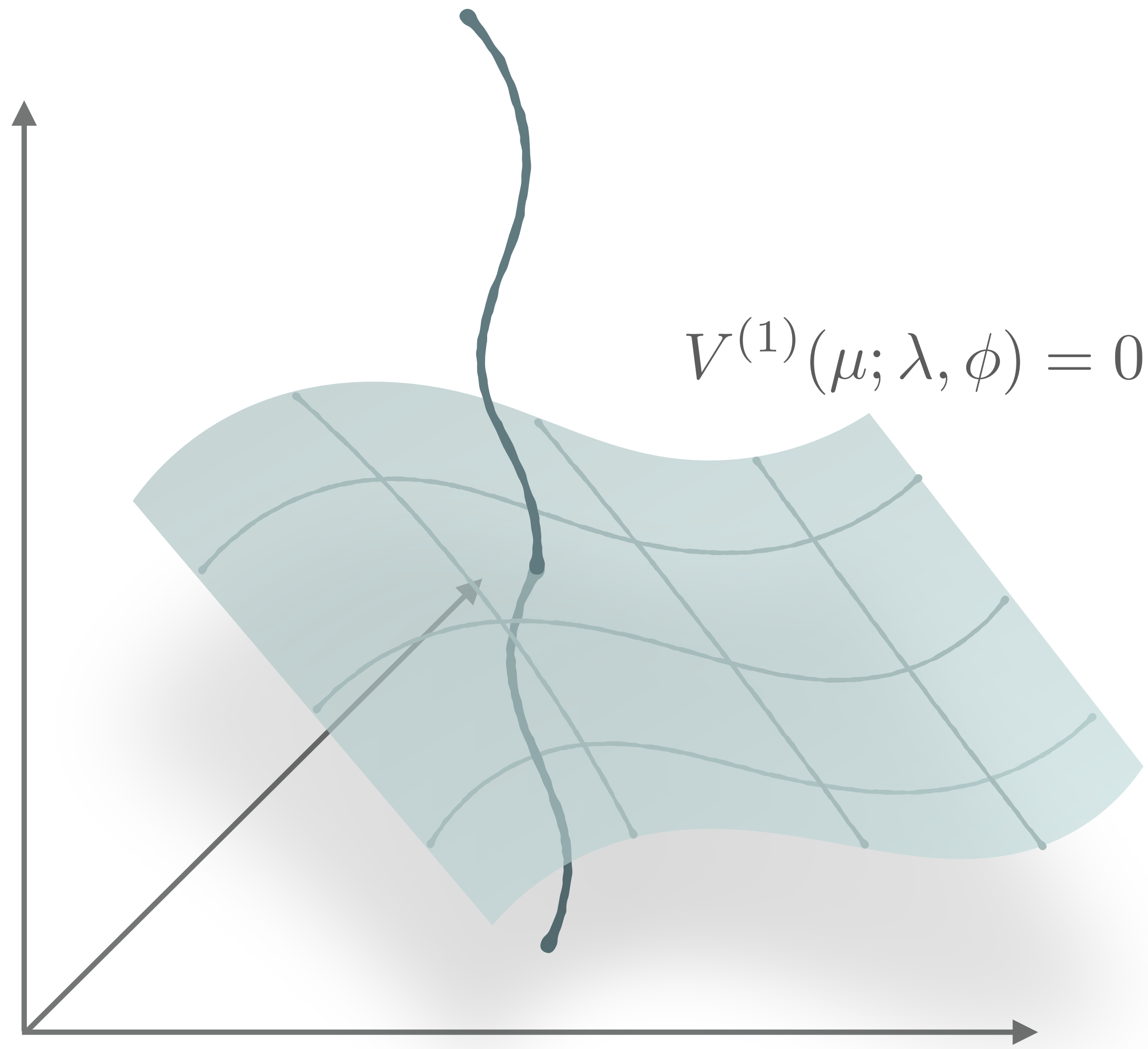
$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

What is t_*?

$$V^{(1)}(\bar{\mu}(t_*); \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

To leading order in $\hbar$

$$t_*^{(0)} = \frac{V^{(1)}(\mu, \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)}$$



VACUUM STABILITY

$$\boxed{\lim_{\phi\to\infty} V(\phi) = \,?}$$

One-loop potential unsuitable for this issue need of improvement

$$\boxed{V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))}$$

Enough to consider tree-level conditions evaluated at large scale.

RESUMMATION

The leading logarithms are not resummed. And what is?

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$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$

pivot log

μ-independent

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of the pivot log.

RESUMMATION

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$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$

pivot log

μ-independent

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of the pivot log.

If $\left|\log \frac{\mathcal{M}^2}{\mu^2}\right| \gg \max_a \left\{\left|\log \frac{m_a^2(\lambda, \phi)}{\mathcal{M}^2}\right|\right\}$ these are the dominant terms.

SUMMARY

- A method of RG improvement of effective potential with arbitrary number of scalar fields
- Achieved by running to the tree-level surface
- Can be implemented to any loop order
- Resums the pivot logarithm
- Applicable to study of positivity of the potential

RECAP

To compute $V(\mu, \lambda, \phi)$

t_*

Evaluate

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Solve numerically

$$V^{(1)}(\bar{\mu}(t_*); \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

Use approximation

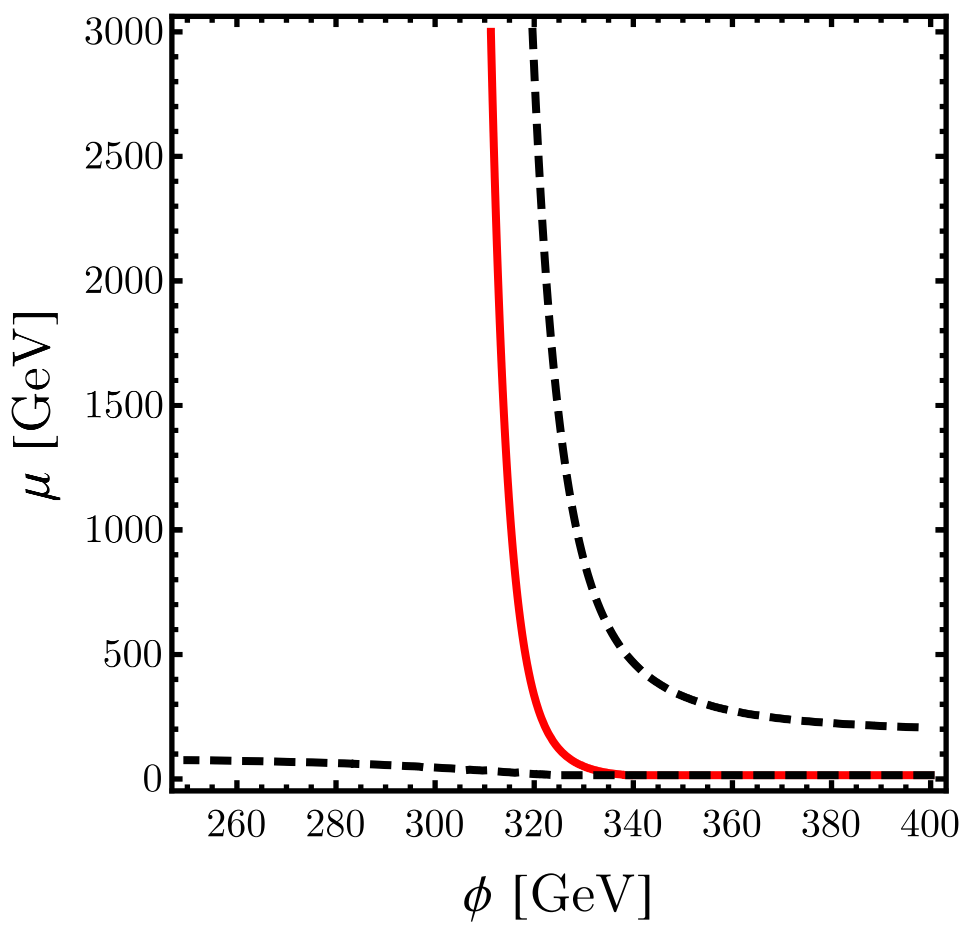
$$t_*^{(0)} = \frac{V^{(1)}(\mu, \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)}$$

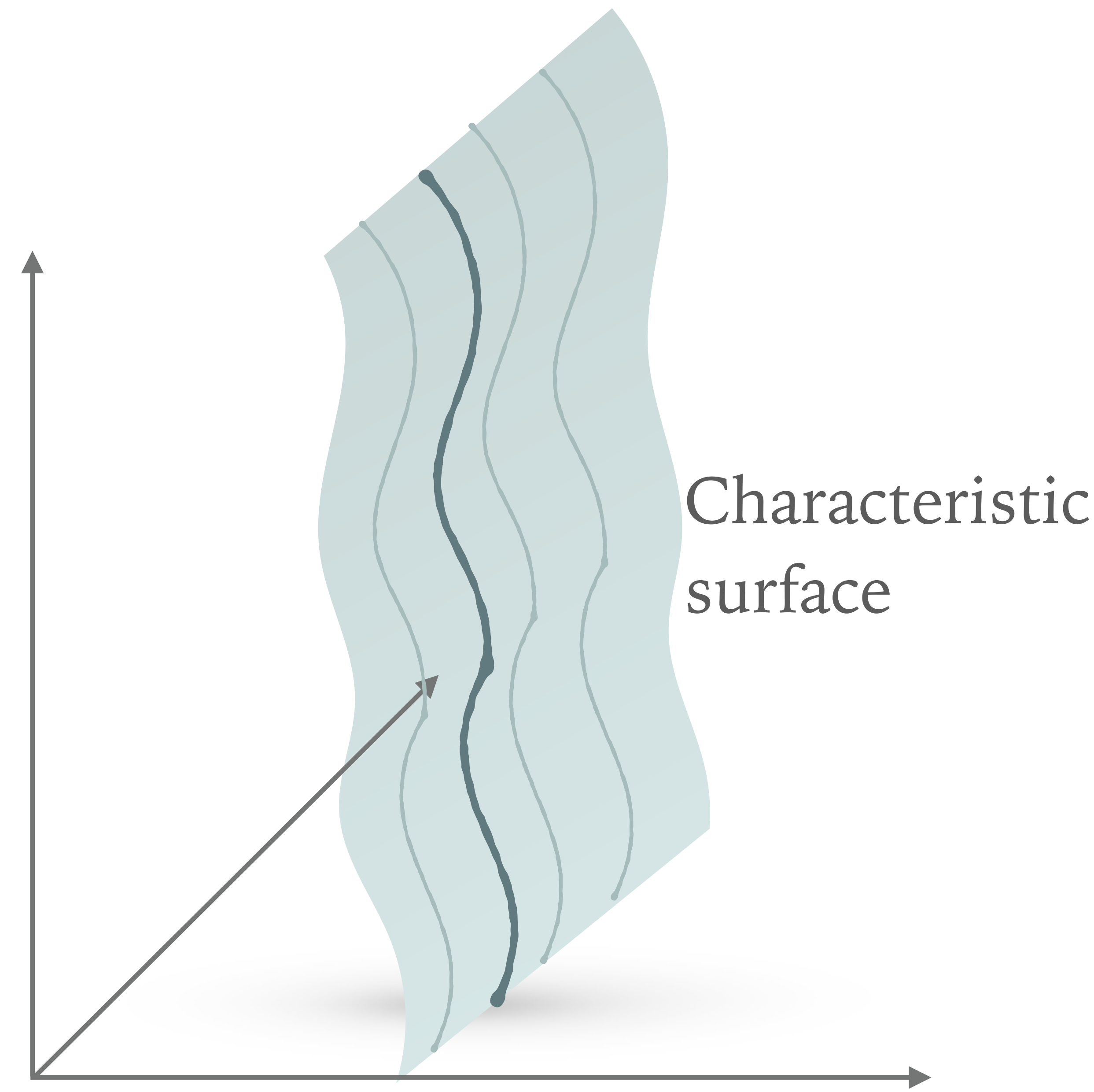
No explicit logs!

Form preserved at higher orders difference can only come from running

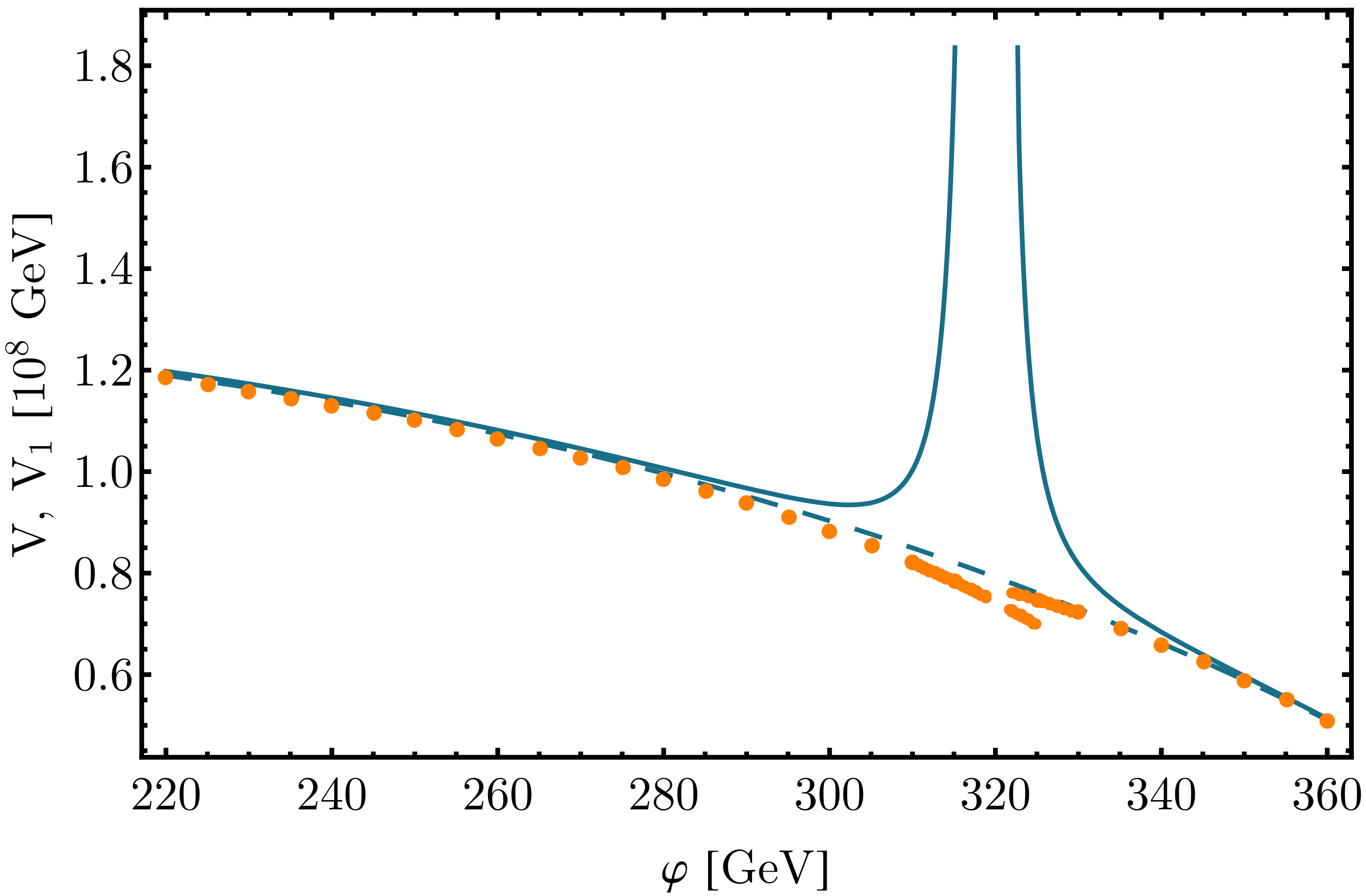
SUBTLETIES

$V^{(1)}(\mu;\lambda,\phi)=0$ is characteristic when $\mathbb{B}=0$

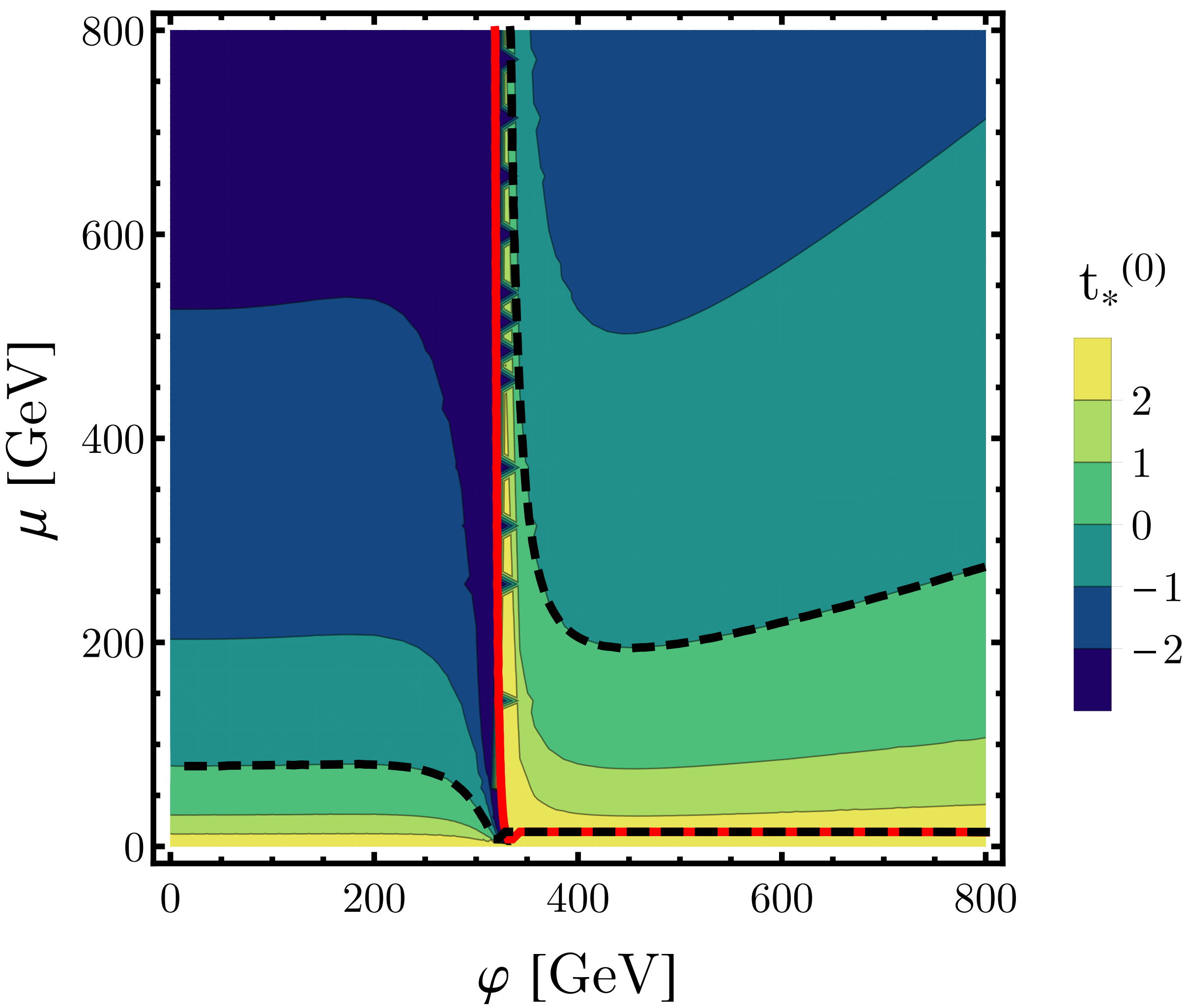




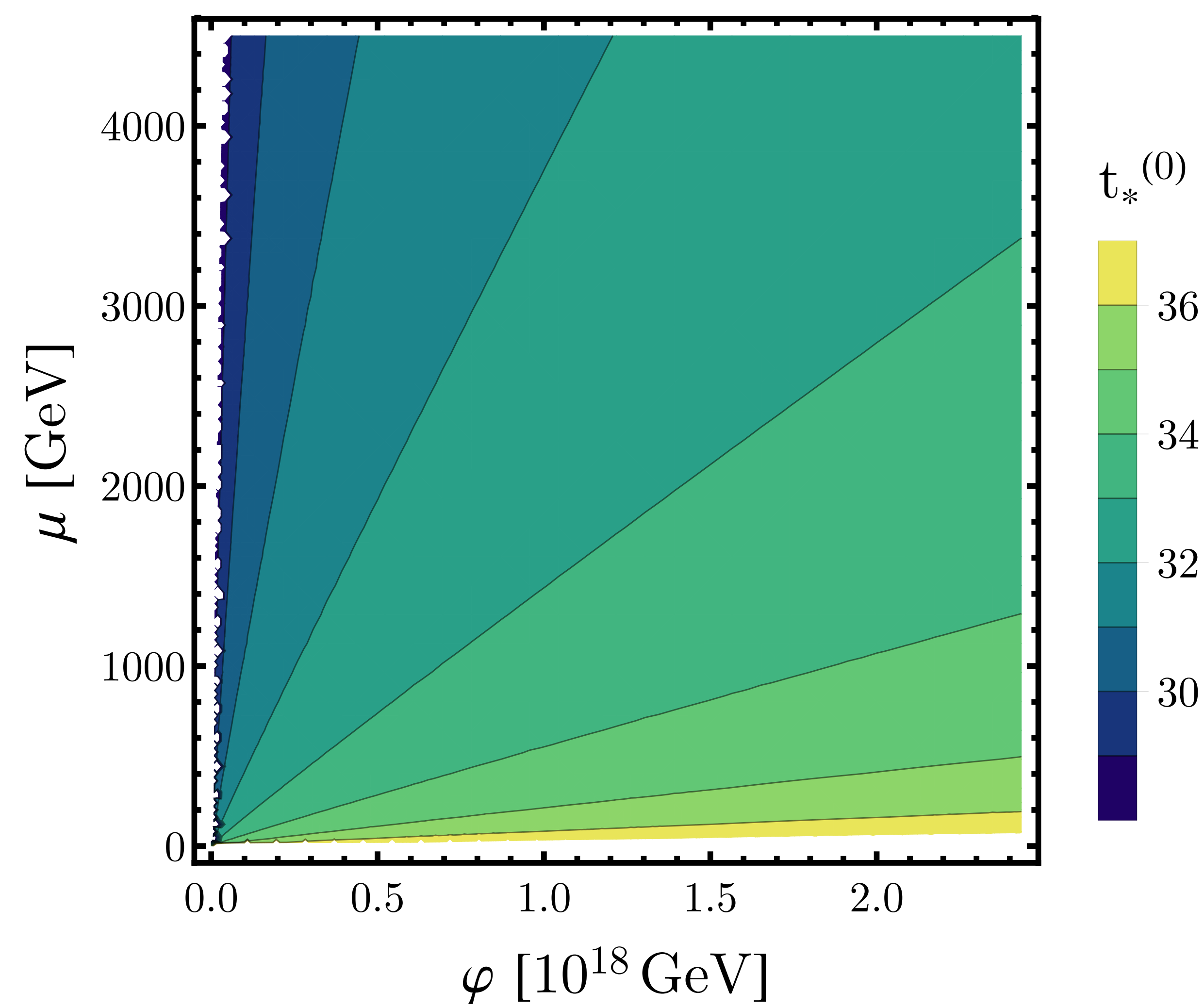
B=0 AND OTHER DETAILS



SU(2)CSM

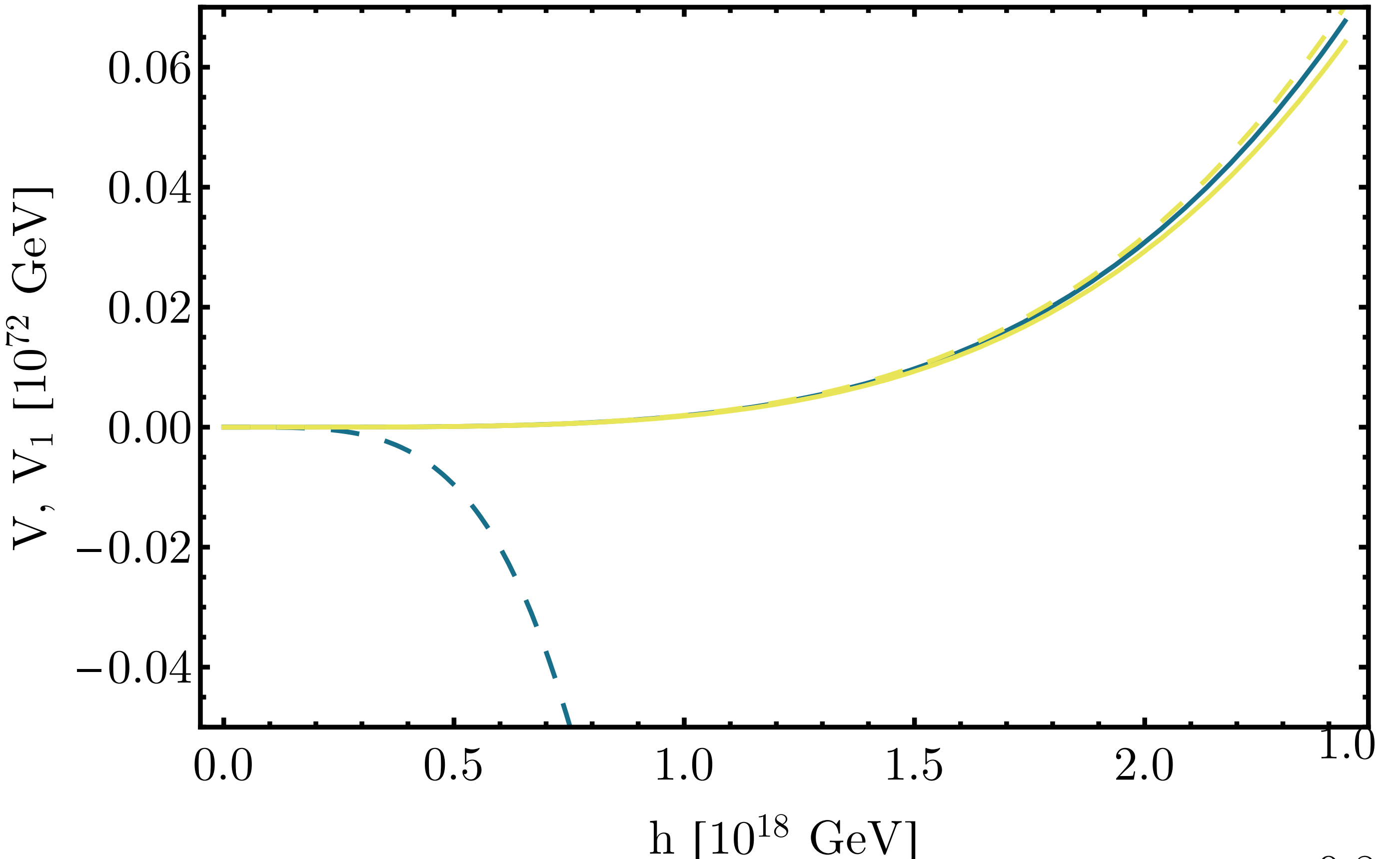


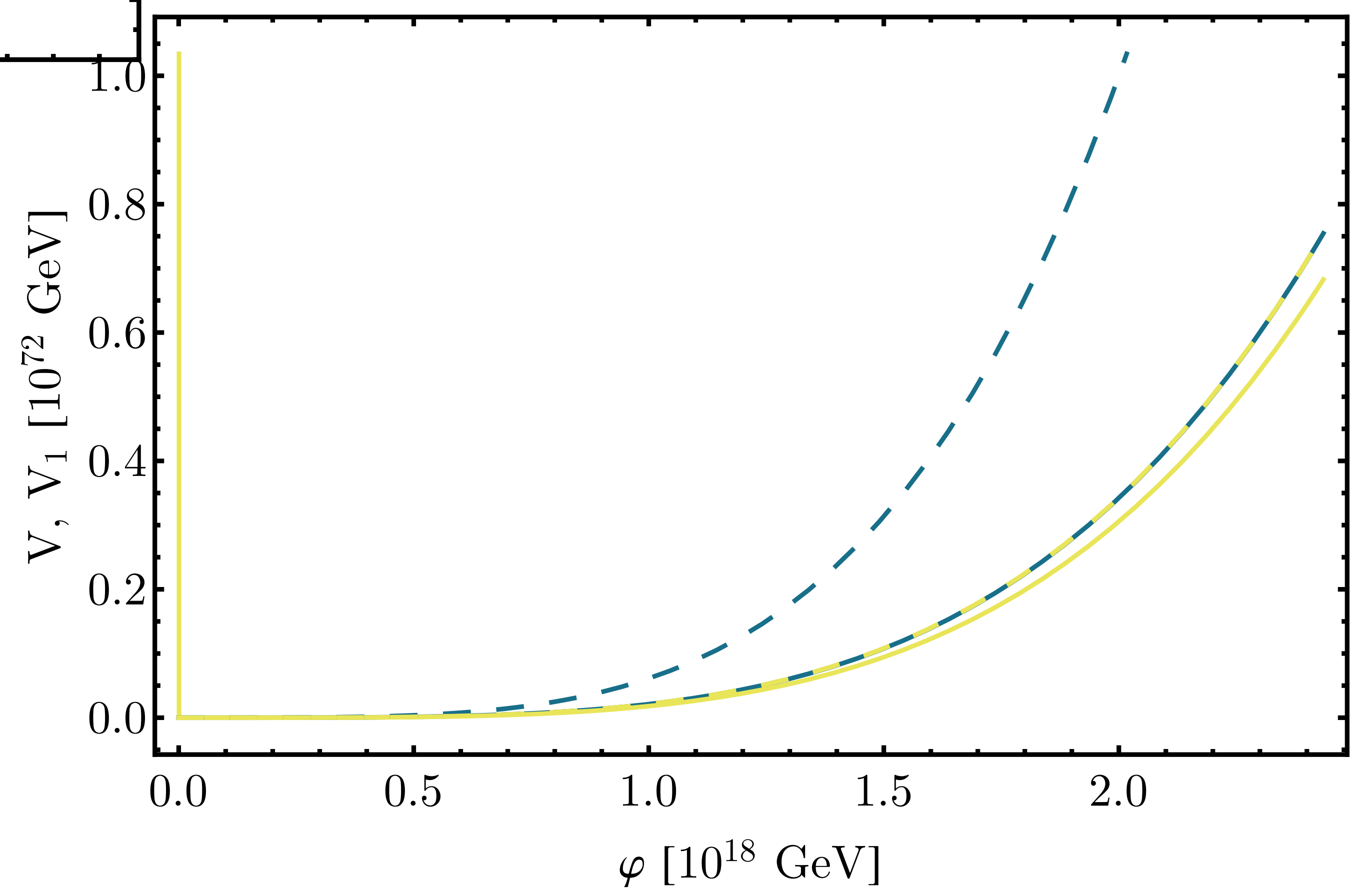
800
600
400
200
0
0
200
400
600
800
μ [GeV]
φ [GeV]
t∗(0)
2
1
0
−1
−2



4000
3000
2000
1000
0
0.0
0.5
1.0
1.5
2.0
μ [GeV]
φ [10^18 GeV]
t∗(0)
36
34
32
30

VACUUM STABILITY IN SU(2)CSM





RESUMMATION

$$t_* = t_*^{(0)} + \mathcal{O}(\hbar) = \frac{1}{2}\log\frac{\mathcal{M}^2}{\mu^2} + \frac{1}{2}\frac{\mathbb{A}\left(\lambda,\phi,\mathcal{M}\right)}{\mathbb{B}\left(\lambda,\phi\right)} + \mathcal{O}(\hbar)$$

$$V^{(0)}(\bar\lambda(t_*),\bar\phi(t_*)) = \sum_{n=0}^{\infty}\hbar^n w_n^{(n)}(\lambda,\phi)(2t_*)^n = \sum_{n=0}^{\infty}\hbar^n w_n^{(n)}(\lambda,\phi)\left(\log\frac{\mathcal{M}^2}{\mu^2}\right)^n + \cdots$$

O(N) MODEL

Computation at two-loop level — agreement with the expressions for the leading functions

The two-loop improved potential differed only slightly from one-loop improved.