

Fate of false vacuum at one-loop

Yutaro Shoji
University of Tokyo

arXiv:1511.04860 [hep-ph]

Collaborators: Motoi Endo, Takeo Moroi, Mihoko Nojiri (KEK)

Varia ~ Barrier
(for Japanese...)

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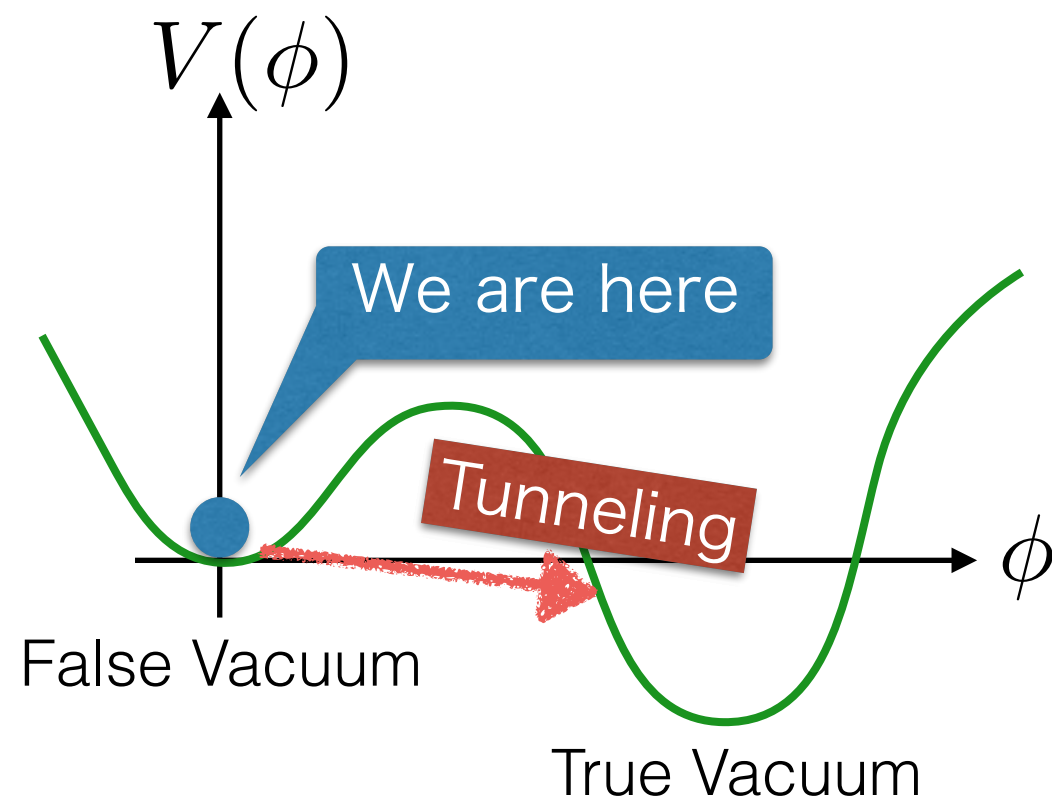
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- SM + stau system
- Summary

Introduction

Vacuum decay



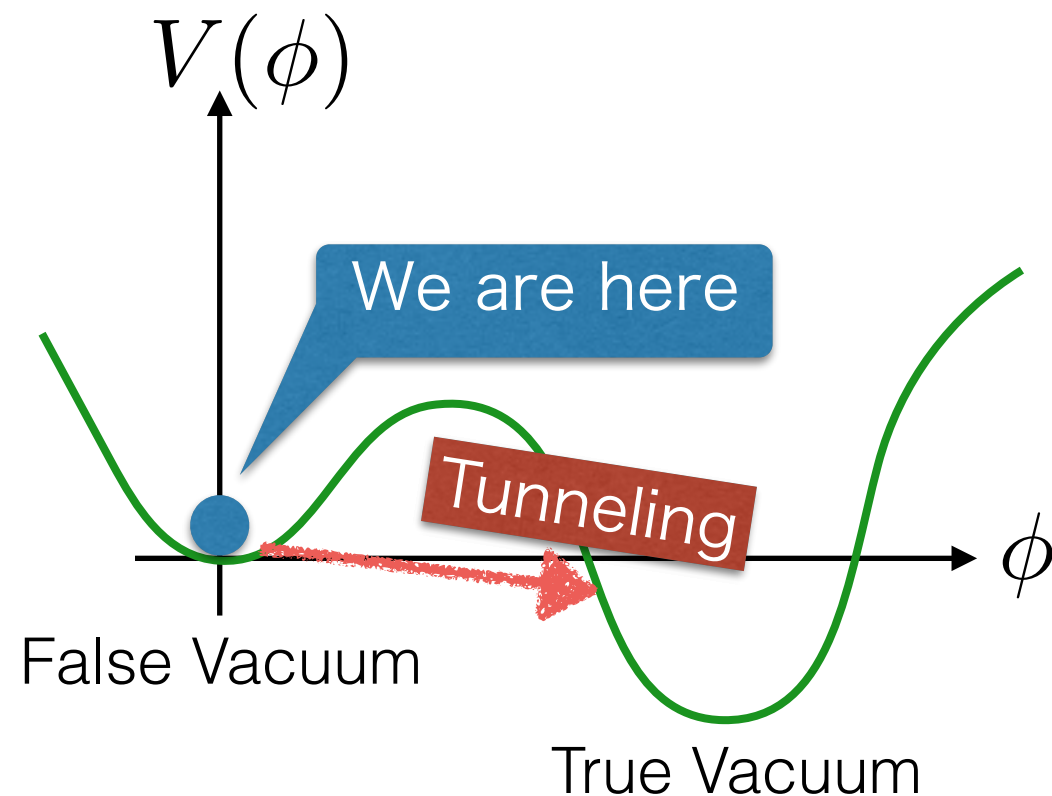
Vacuum decay

Tree level potential

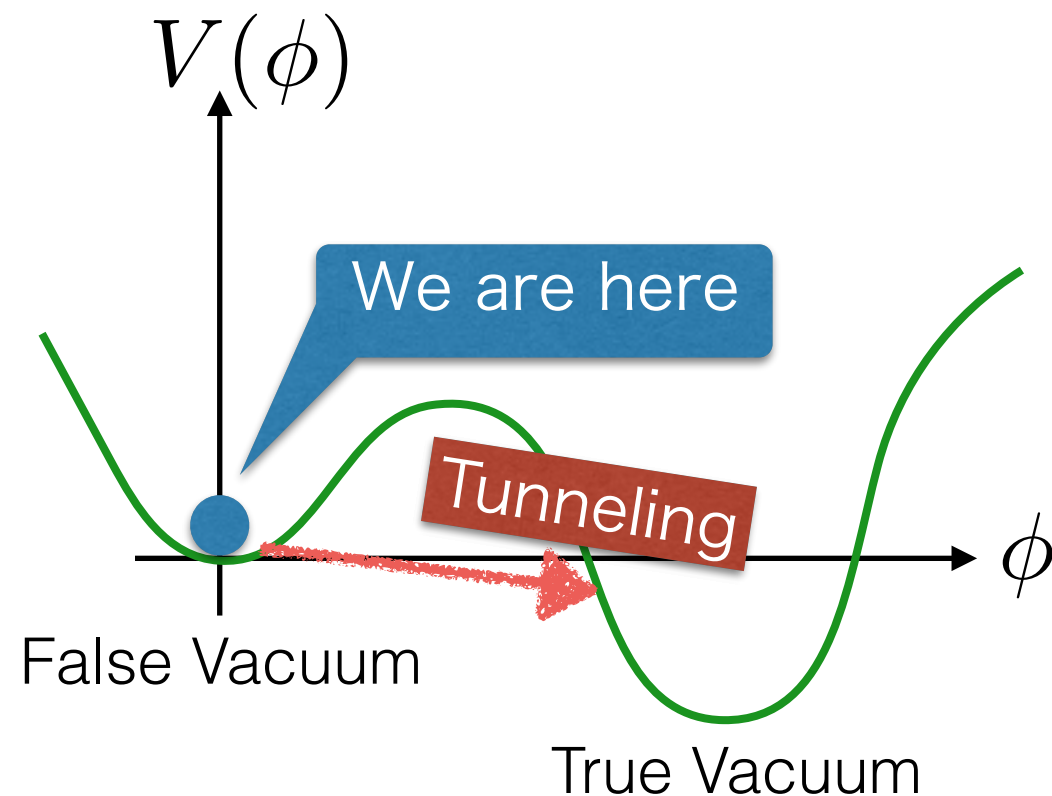
ex.) Supersymmetry

$h\tilde{t}_L\tilde{t}_R$ Higgs mass, hgg , $h\gamma\gamma$, ...

$h\tilde{\ell}_L\tilde{\ell}_R$ muon g-2, $h\gamma\gamma$, ...



Vacuum decay



Tree level potential

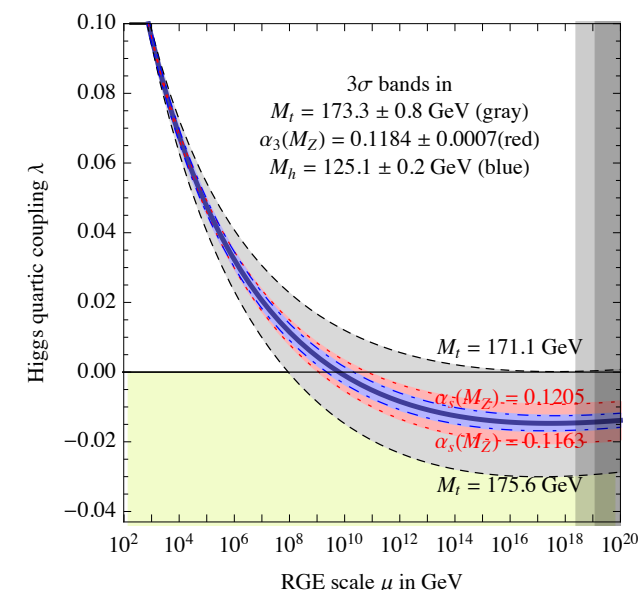
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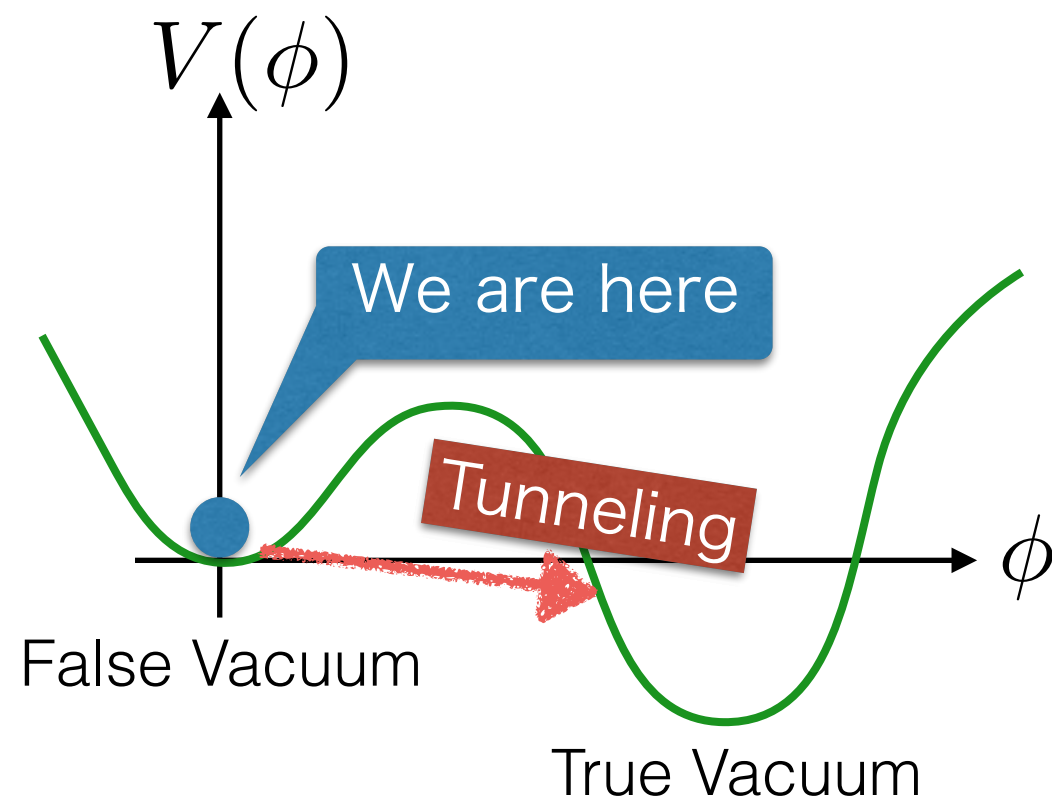
Effective potential

ex.) Standard Model



D. Buttazzo, et.al.,
1307.3536/hep-ph

Vacuum decay



Tree level potential

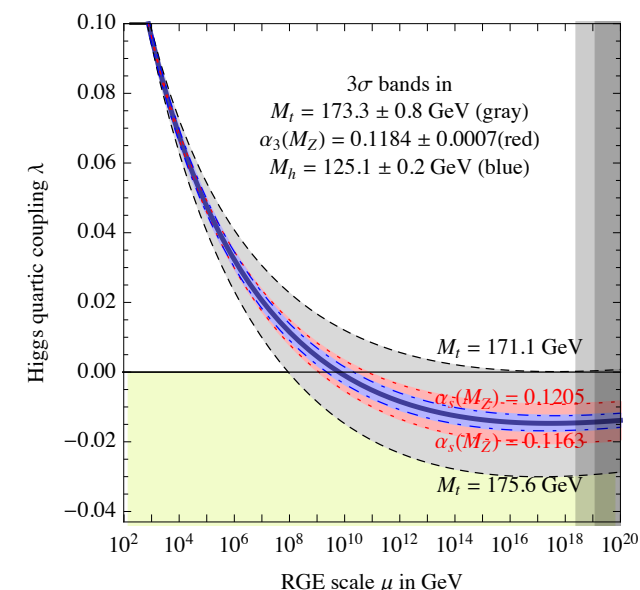
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Effective potential

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Decay rate

Bubble nucleation rate

$$\gamma = Ae^{-B}$$

classical bounce solution

Pre-exponential factor

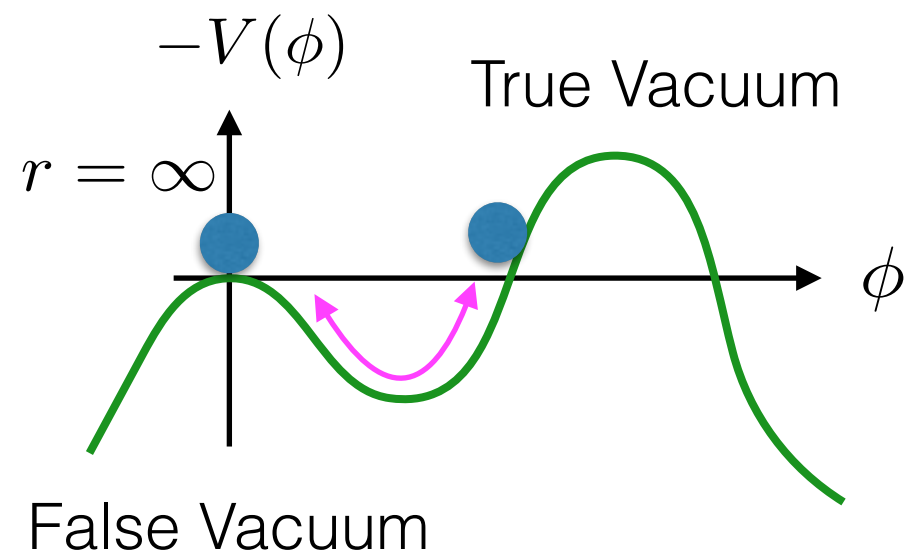
$$A \sim \mu^4$$

\ \

typical scale

1-bounce action

$$B = S(\phi_B)$$



O(4) symmetric

Toy model

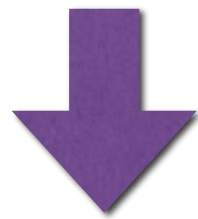
Potential

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

Toy model

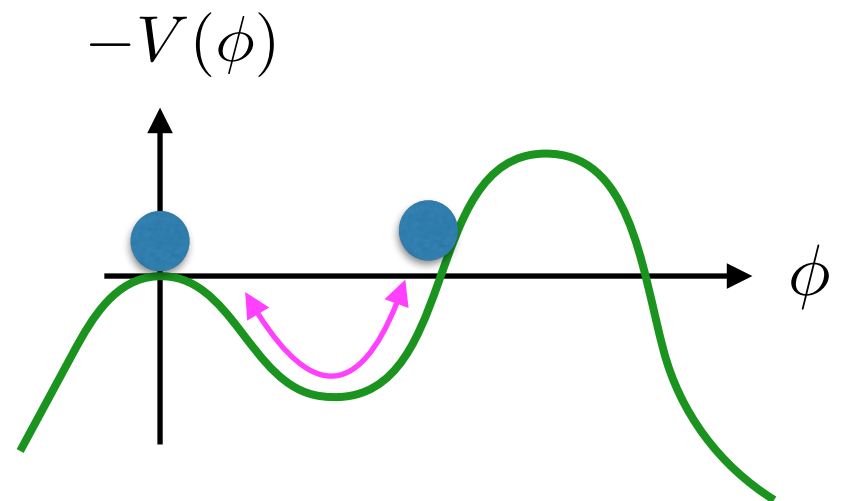
Potential

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Bounce

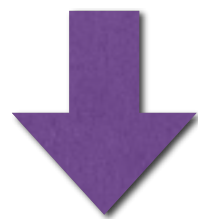
$$\partial^2 \phi = V'(\phi)$$



Toy model

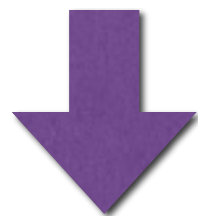
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$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$



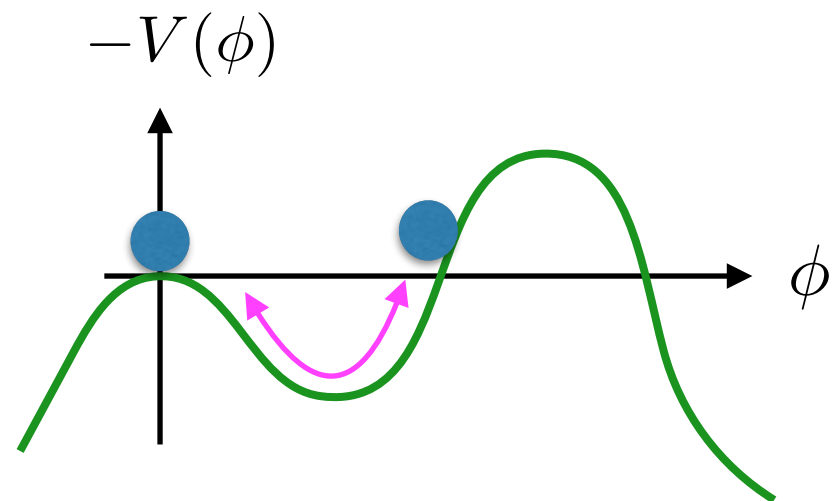
Bounce

$$\partial^2 \phi = V'(\phi)$$



Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B) \right]$$



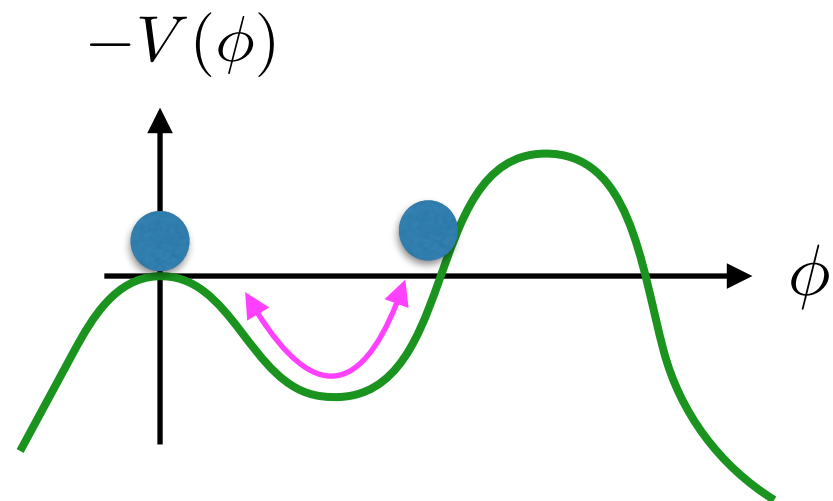
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Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B) \right]$$

Decay rate

$$\gamma \simeq m^4 e^{-B}$$

Toy model

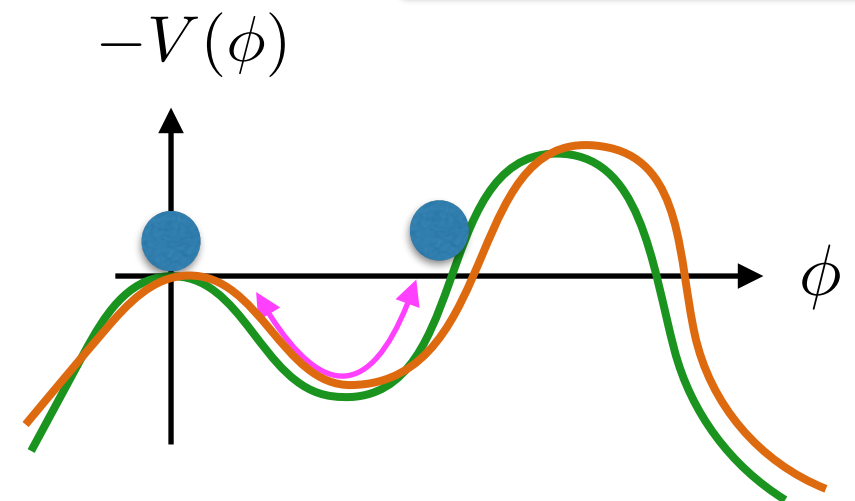
Potential

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Scale dependent

Bounce

$$\partial^2 \phi = V'(\phi)$$



Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B) \right]$$

Decay rate

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Toy model

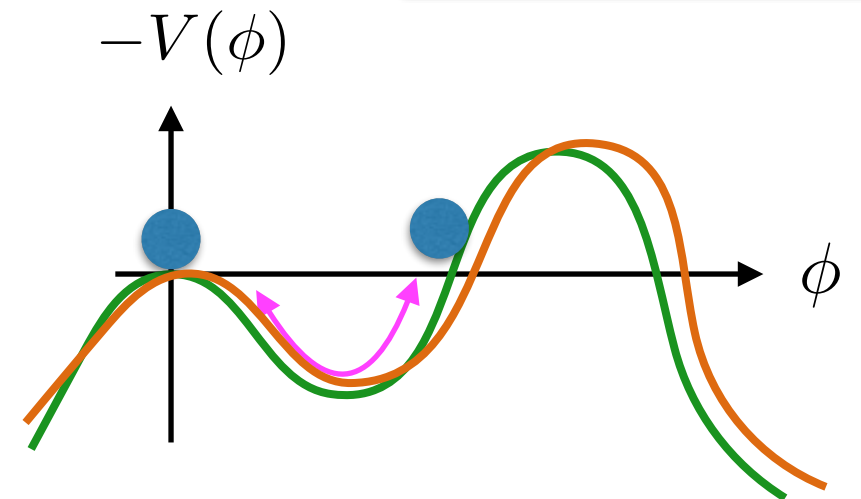
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Scale dependent

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Action

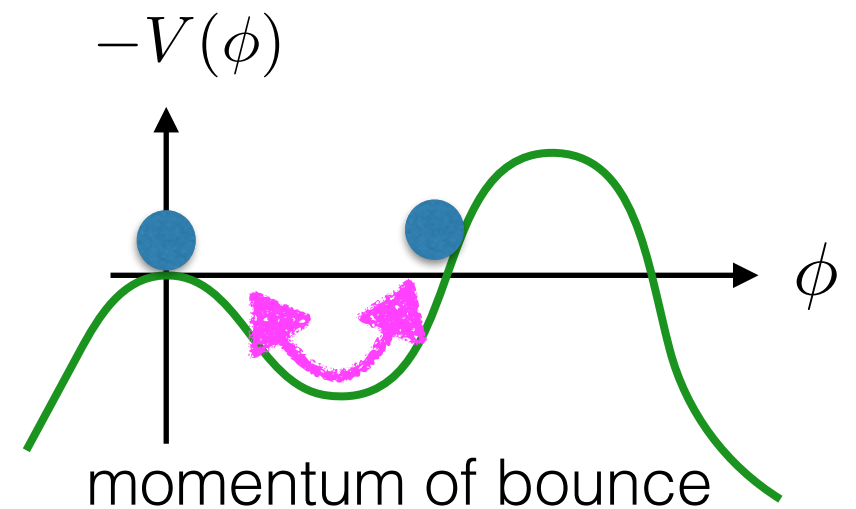
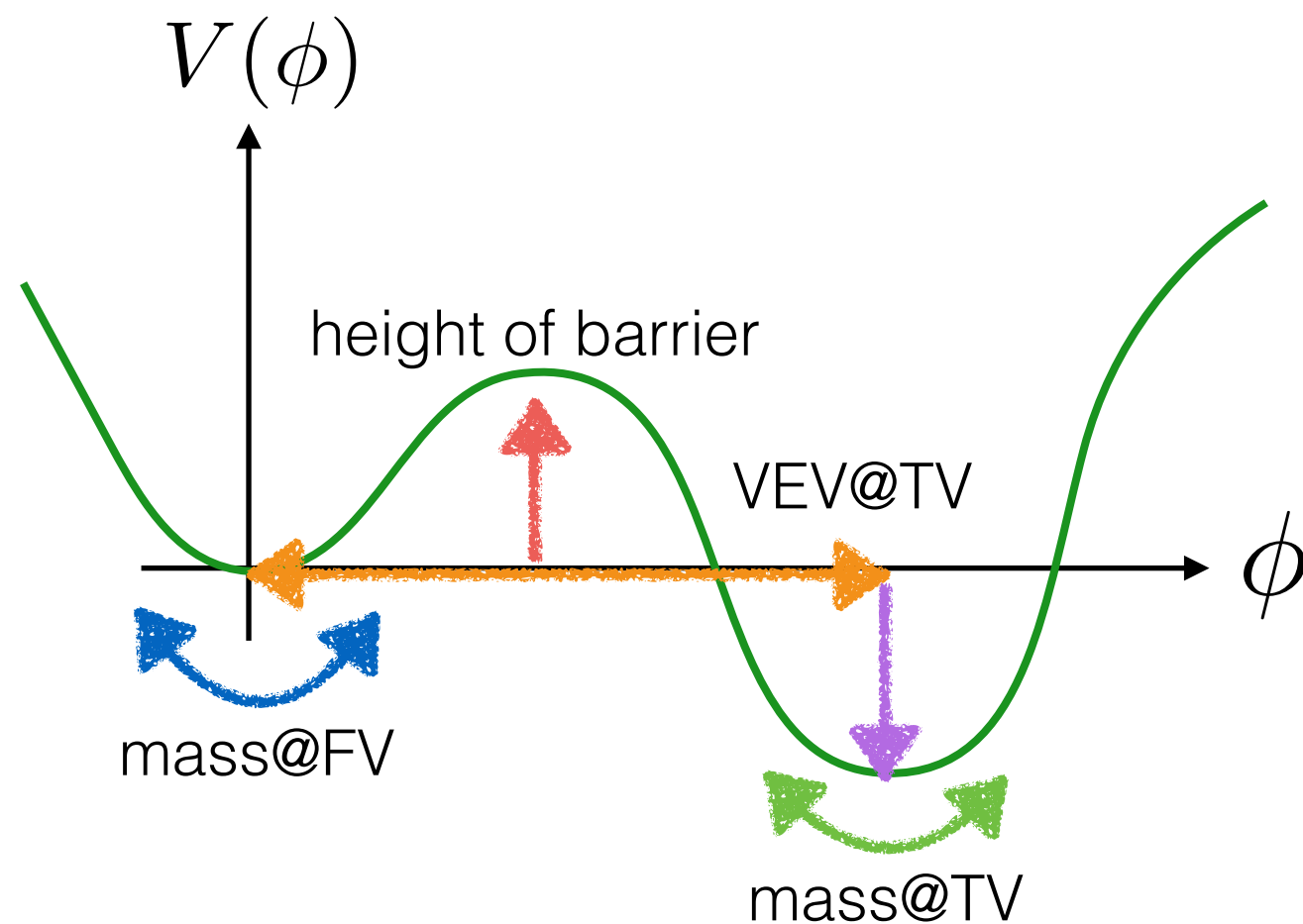
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Decay rate

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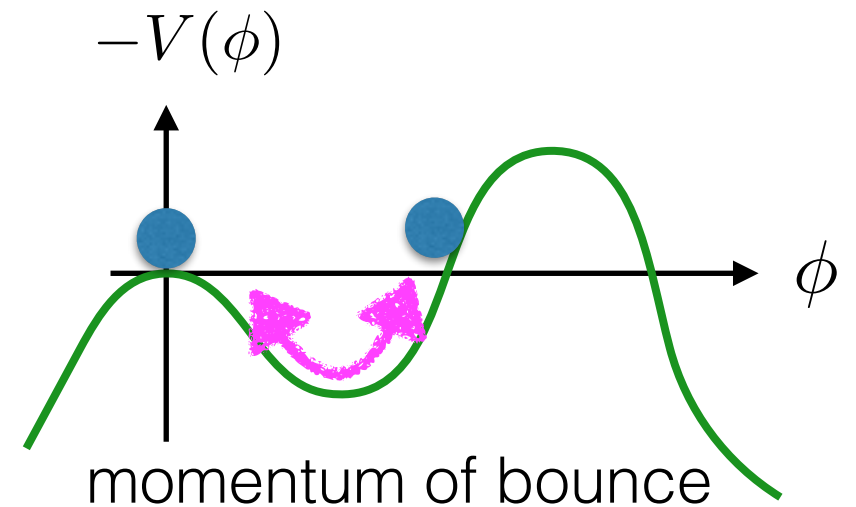
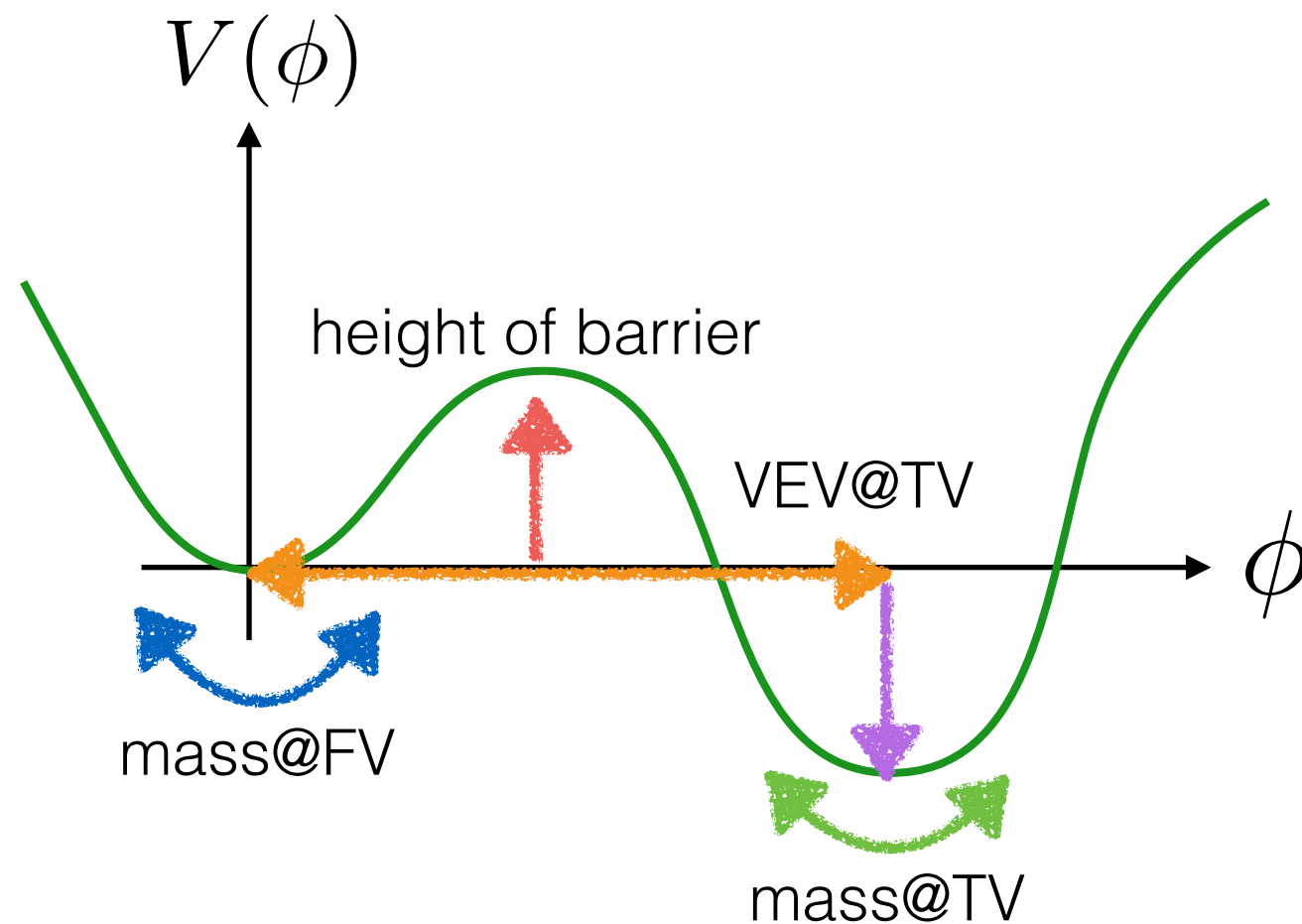
OK, use a “typical” scale!

Renormalization scale



OK, use a “typical” scale!

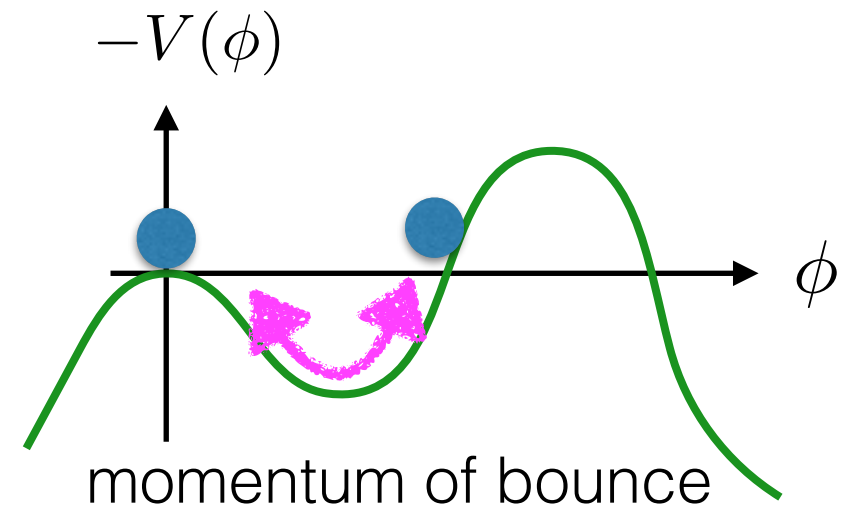
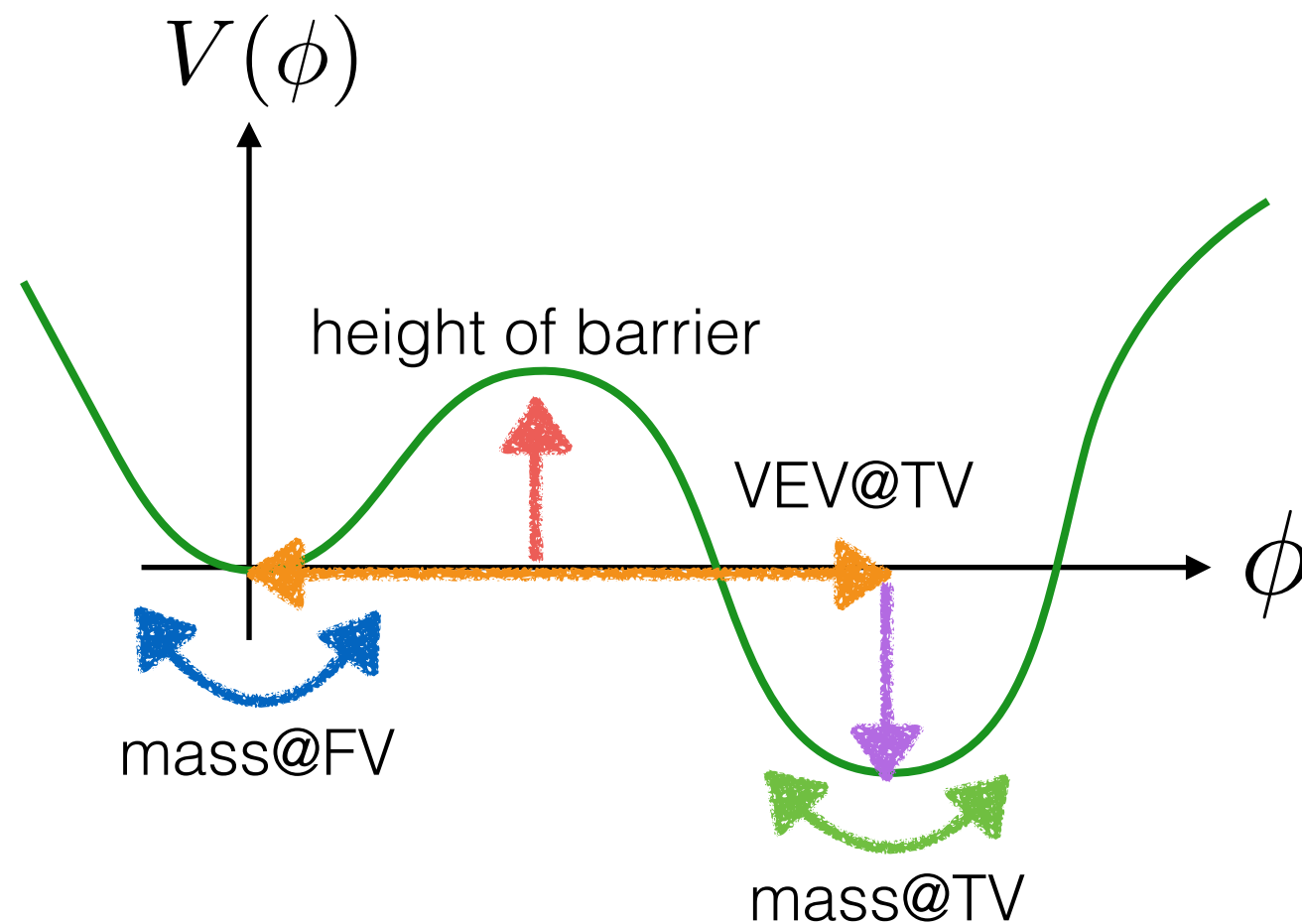
Renormalization scale



OK, use a "typical" scale!

But,... I don't know what is the best scale.

Renormalization scale



But,... I don't know what is the best scale.

OK, use a "typical" scale!

Does the decay rate change so much?

How large is the scale dependence?

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Beta functions

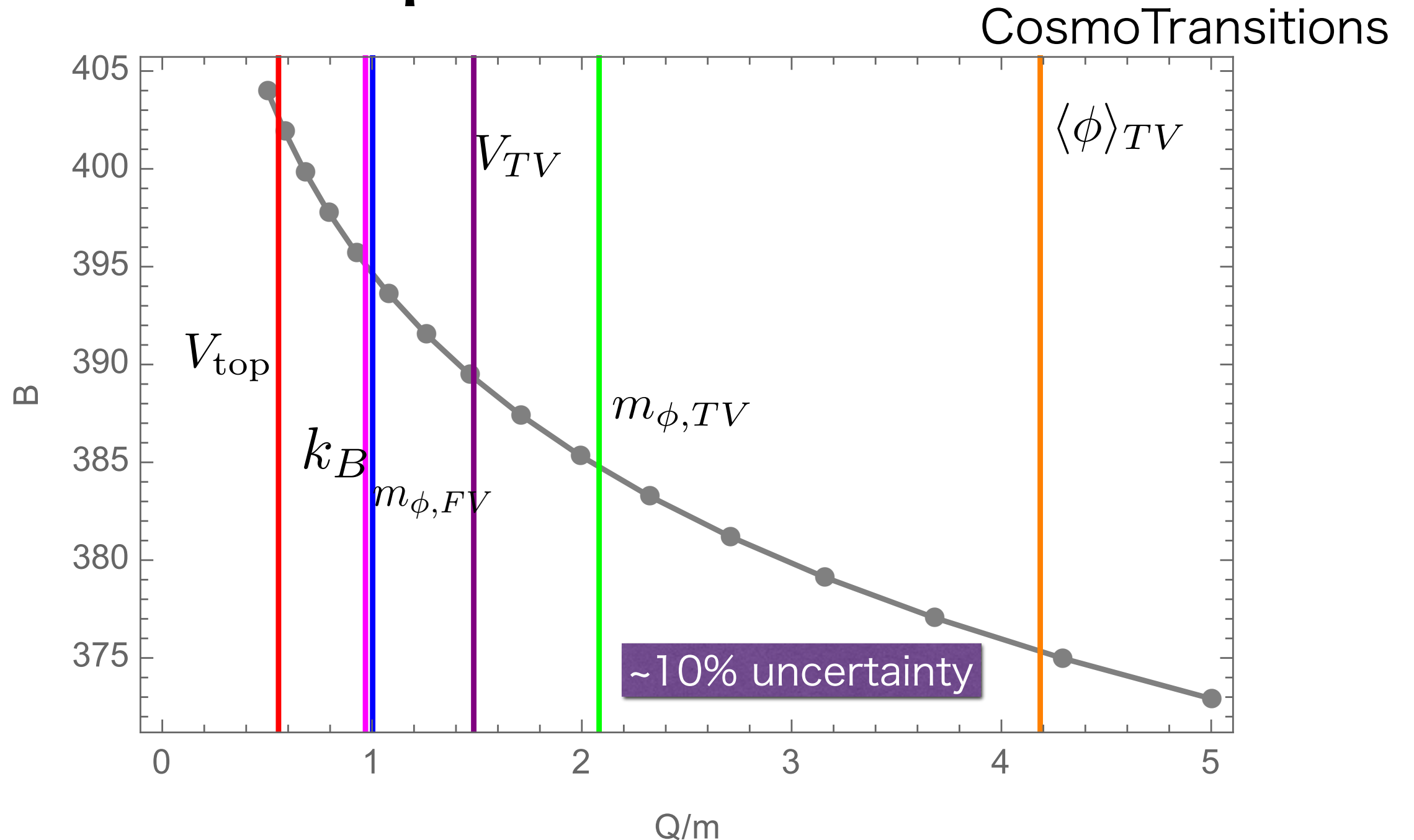
$$\begin{aligned}\beta_t &= \frac{3Am^2}{16\pi^2} & \beta_{m^2} &= \frac{3}{16\pi^2}(\alpha m^2 + 3A^2) \\ \beta_A &= \frac{9\alpha A}{16\pi^2} & \beta_\alpha &= \frac{9\alpha^2}{16\pi^2}\end{aligned}$$

Renormalization conditions

@ $Q = m$

$$\bar{m}^2(m) = m^2, \quad \bar{A}(m) = m, \quad \bar{t}(m) = 0, \quad \bar{\alpha}(m) = 0.6$$

How large is the scale dependence?

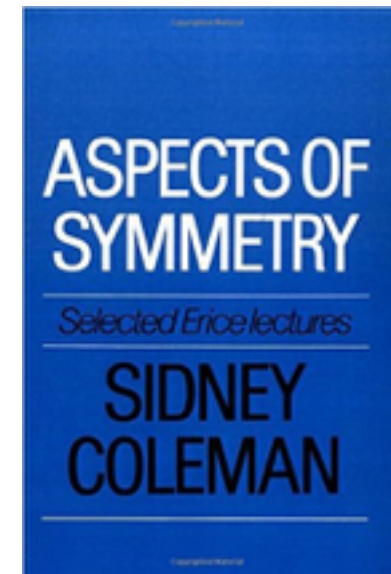


can be much larger in a realistic model (top loop)

Pre-exponential factor

$$\gamma = Ae^{-B}$$

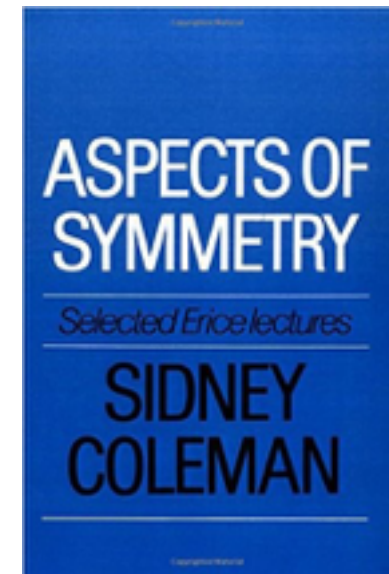
$$A = \frac{B^2}{4\pi^2} \left(\frac{\det' S''|_{\text{Bounce}}}{\det S''|_{\text{False}}} \right)^{-1/2}$$



Pre-exponential factor

$$\gamma = Ae^{-B}$$

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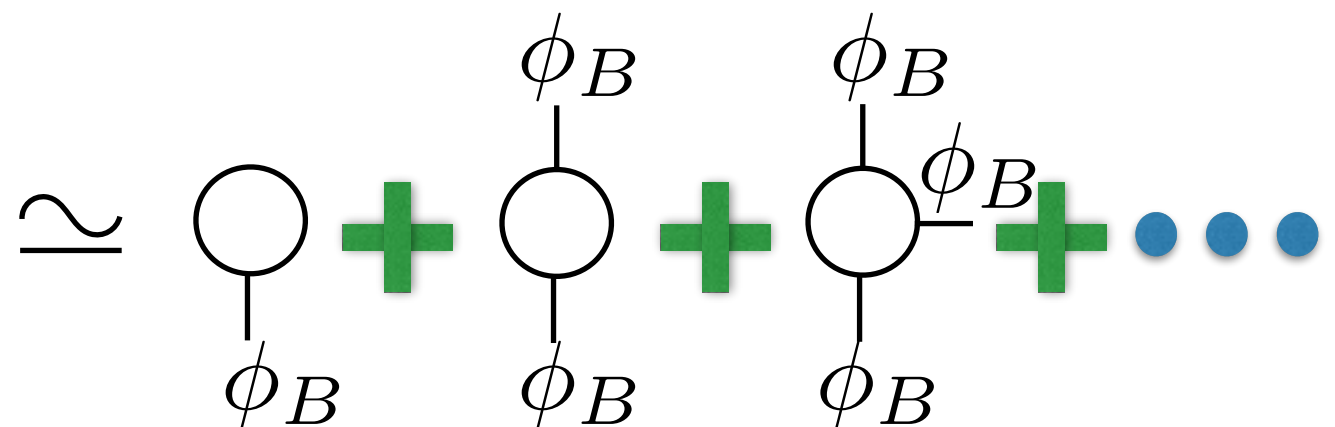
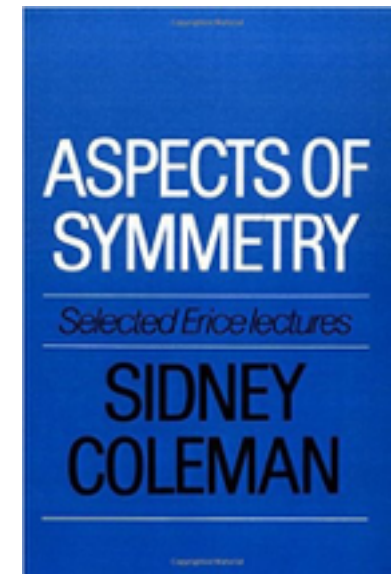
$$\simeq \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$

The diagram shows a series of Feynman diagrams representing a perturbative expansion. Each diagram consists of a circle with two external lines, both labeled ϕ_B . The first diagram has the lines at the bottom. The second diagram has the lines at the top. The third diagram has the lines at the top and bottom. The diagrams are separated by green plus signs, and the series ends with an ellipsis of three blue dots.

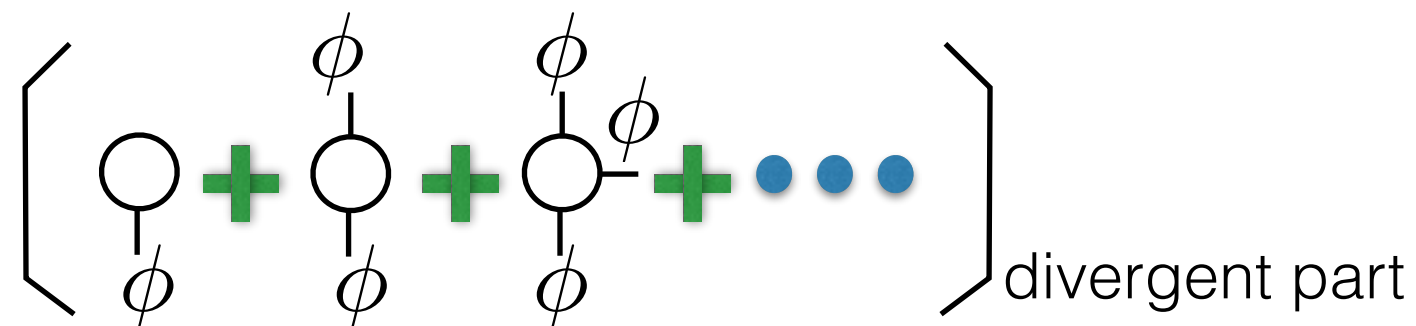
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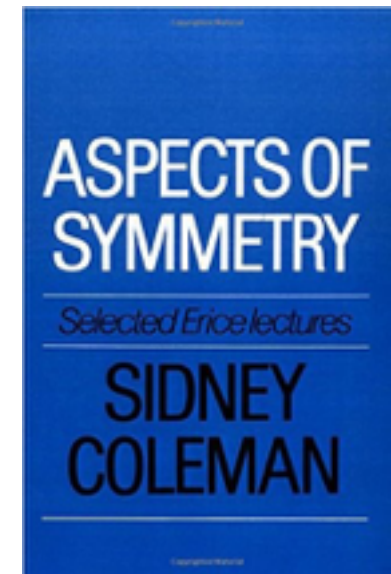
cf.) RGE is related to



Pre-exponential factor

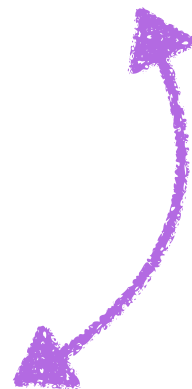
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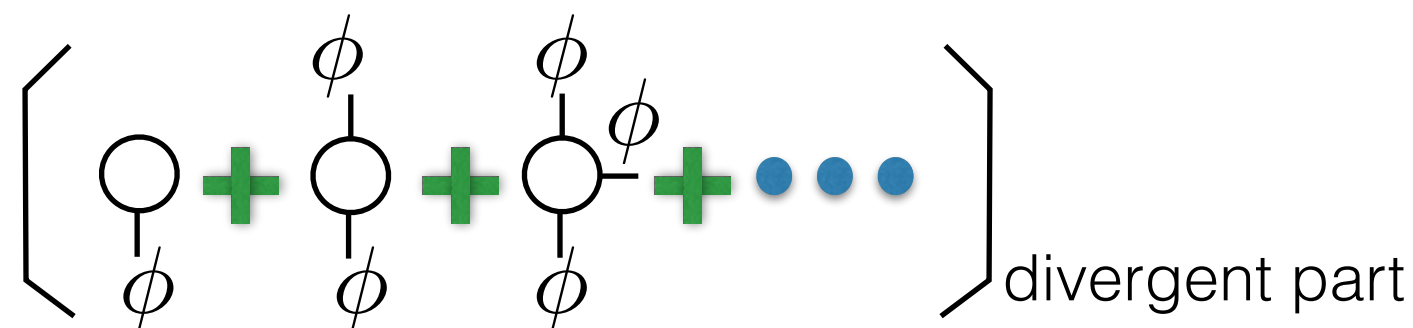
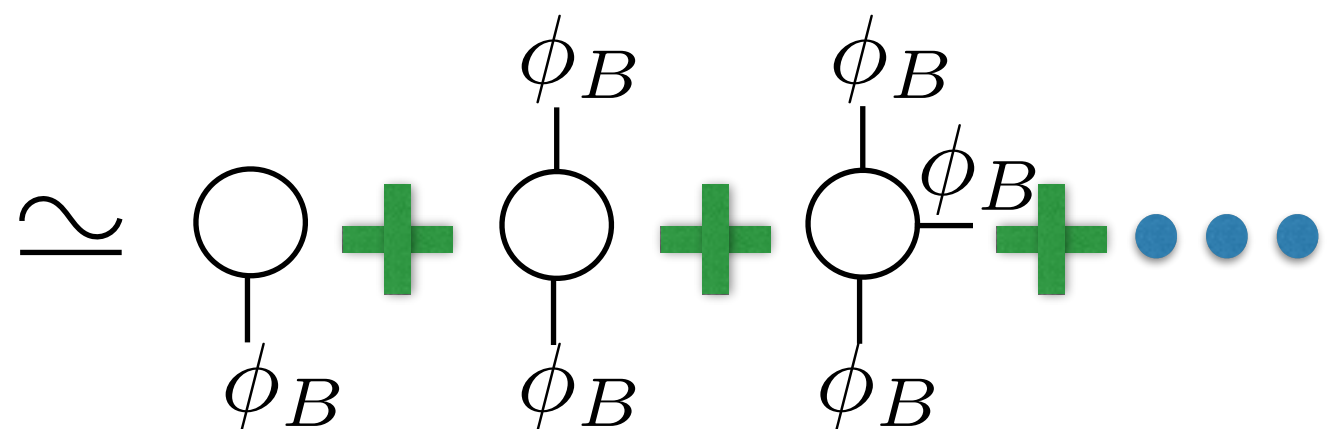


Expectation

cancellation of
the scale dependence
@1-loop



cf.) RGE is related to

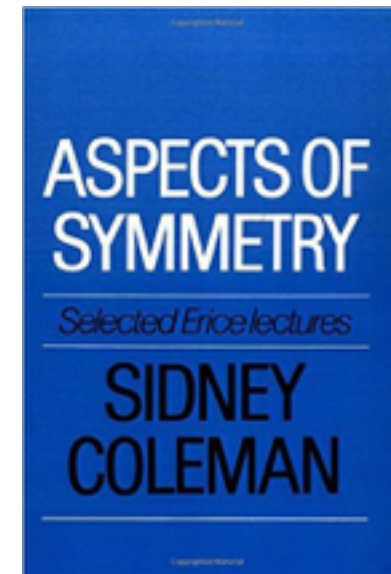


Pre-exponential factor

A way to calculate

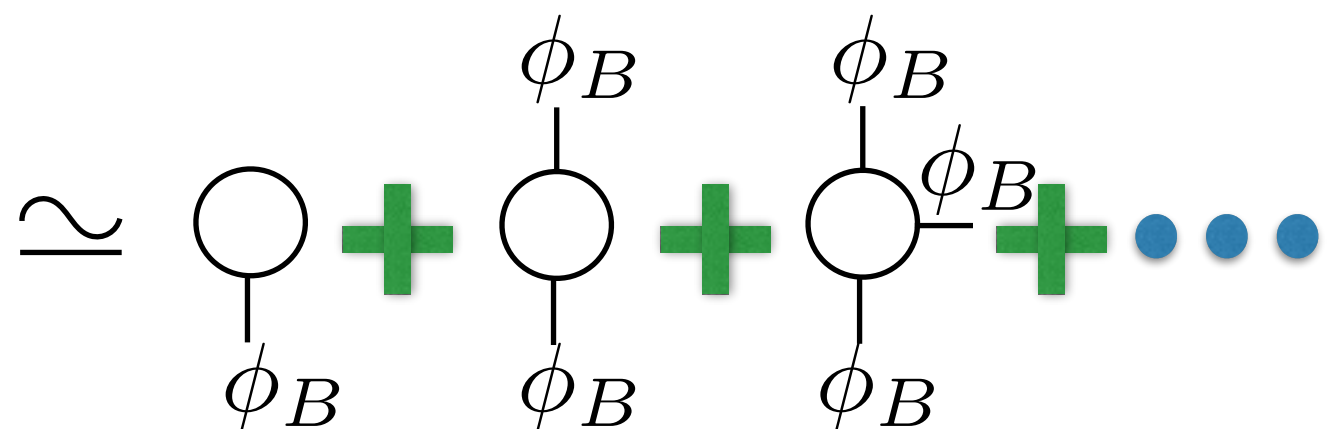
$$\gamma = Ae^{-B}$$

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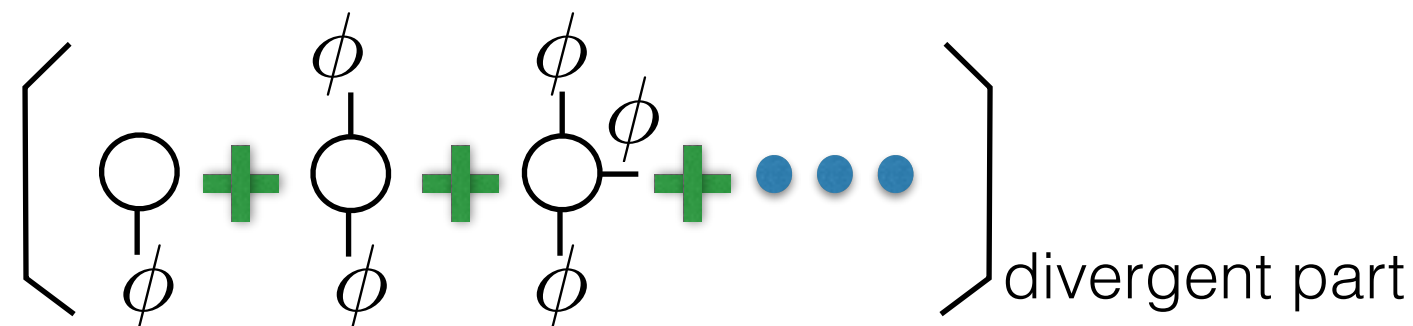


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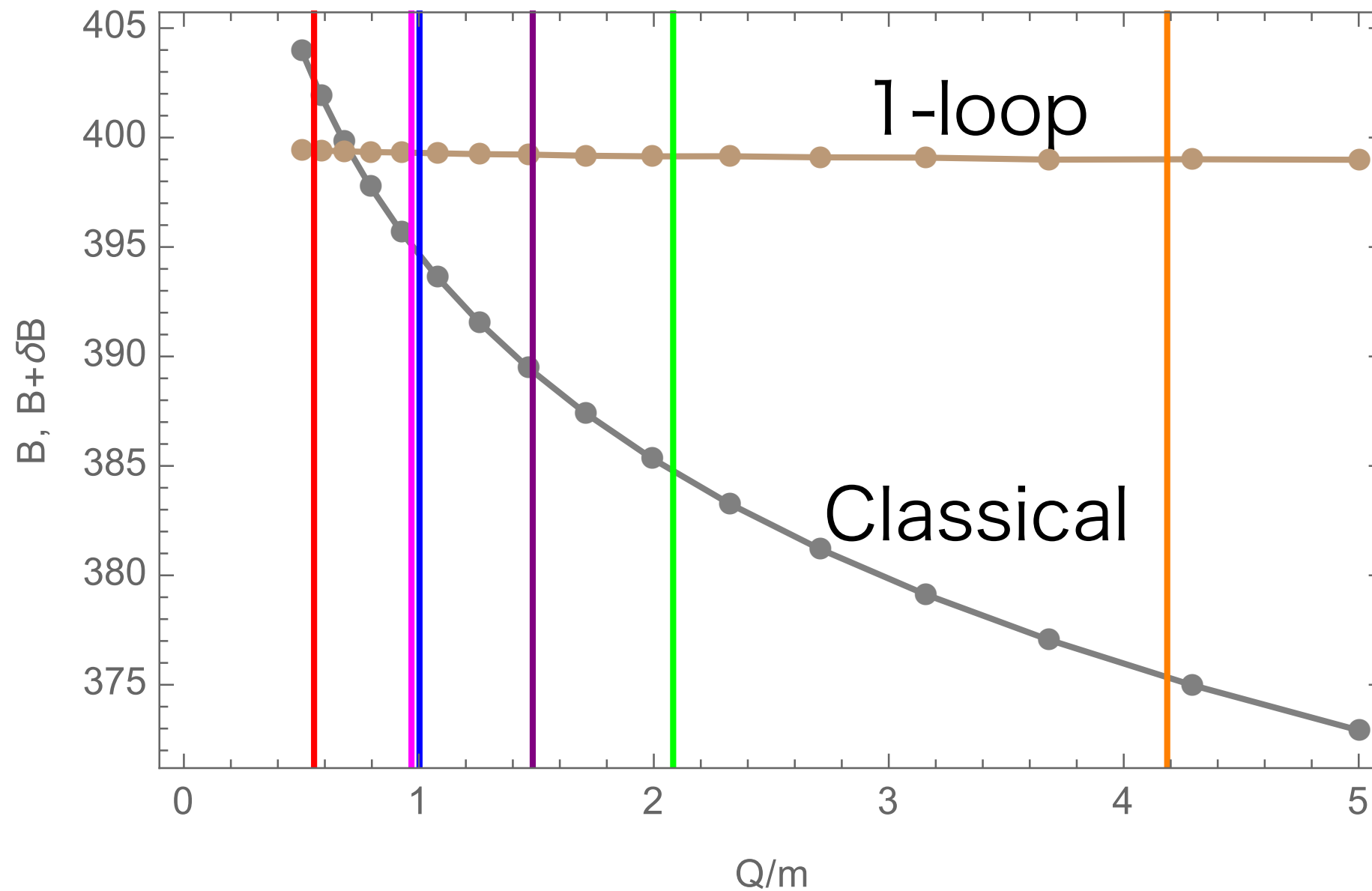


cf.) RGE is related to



Result

$$\gamma = Ae^{-B} \equiv m^4 e^{-B-\delta B}$$



SM + stau system

Light stau

Stau can be light

$$m_{\tilde{\tau}} > 103.5 \text{ GeV} \quad (\text{LEP})$$

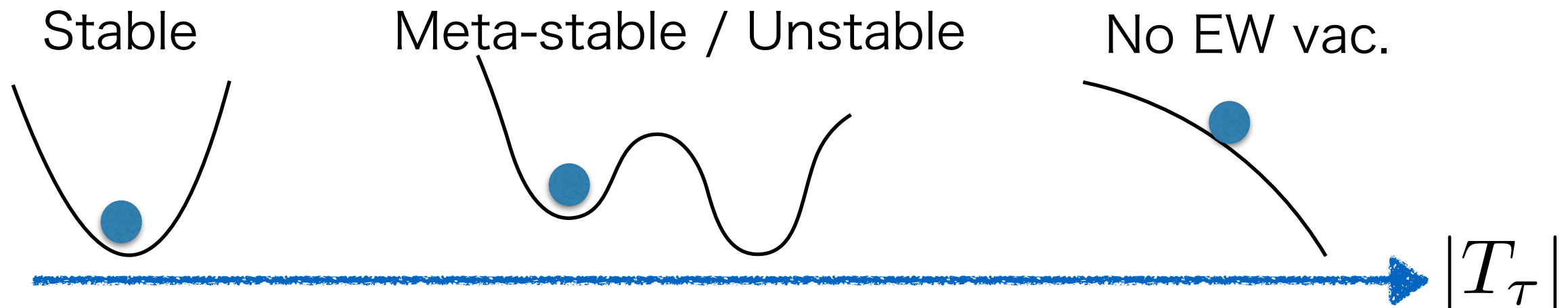
$h \gamma \gamma$ coupling, co-annihilation with bino, ...

But, the potential may become unstable towards the stau direction

$$V = T_{\tau} (H^{\dagger} \tilde{\ell}_L \tilde{\tau}_R^{*} + h.c.) + m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 + \dots$$

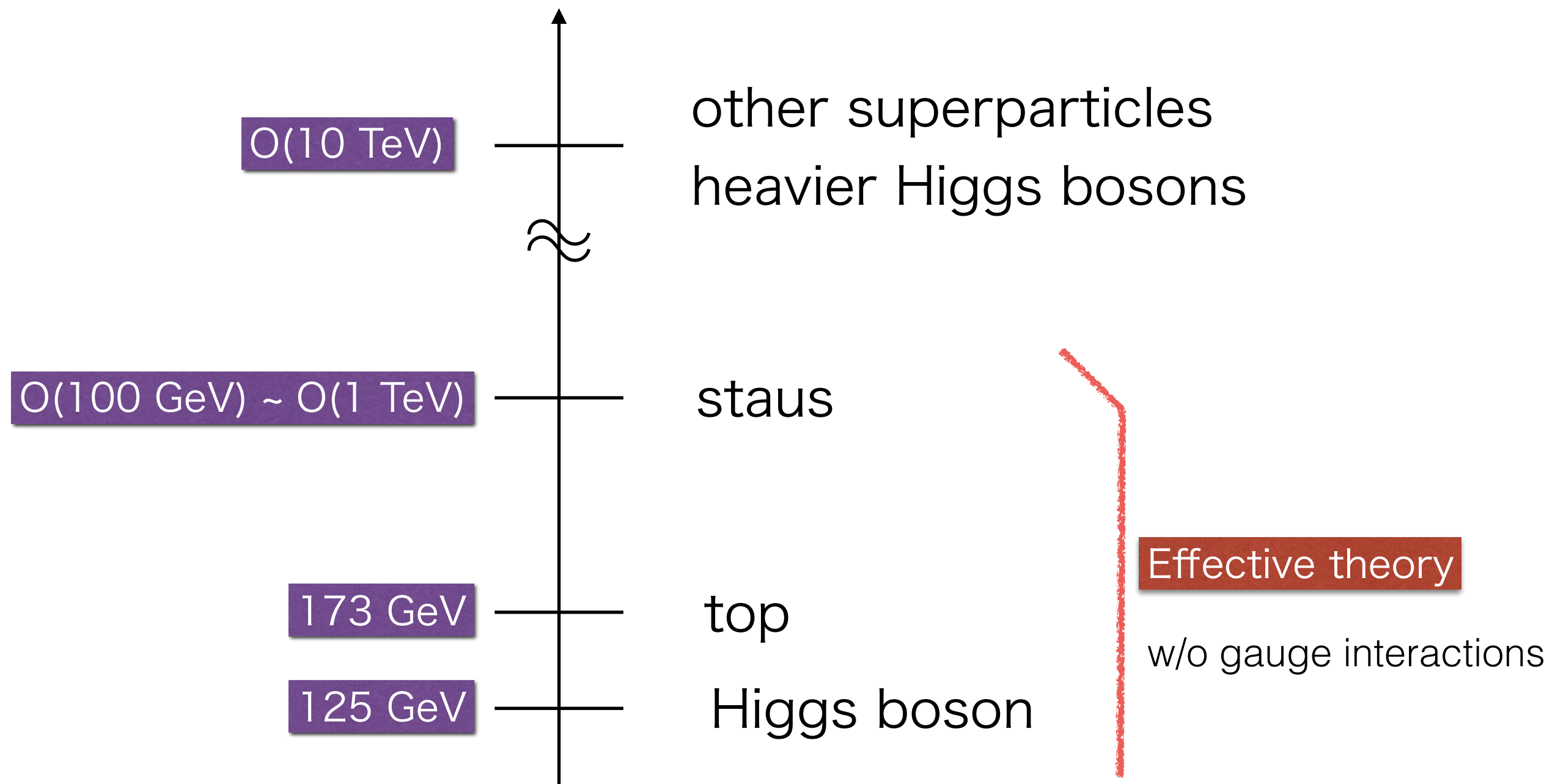
$$T_{\tau} = y_{\tau} (A_{\tau} - \mu \tan \beta)$$

$$\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$$



Spectrum

For simplicity,
we assume only the staus are light



Effective theory

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{kin}} - y_t(H q_L t_R^c + \text{h.c.}) - m_H^2 |H|^2 - \frac{1}{4} \lambda_H |H|^4 \\ & - m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 - T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + \text{h.c.}) - \frac{1}{4} \kappa^{(1)} |\tilde{\ell}_L|^4 - \frac{1}{4} \kappa^{(2)} |\tilde{\tau}_R|^4 \\ & - \frac{1}{4} \lambda^{(1)} |H|^2 |\tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(2)} |H^\dagger \tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(3)} |H|^2 |\tilde{\tau}_R|^2 - \frac{1}{4} \kappa^{(3)} |\tilde{\ell}_L|^2 |\tilde{\tau}_R|^2,\end{aligned}$$

Boundary conditions

EW scale

$$y_t = \frac{M_t}{v},$$

$$m_H^2(M_t) = -\frac{1}{2} M_h^2,$$

$$\lambda_H(M_h) = \frac{M_h^2}{2v^2},$$

stau mass

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250 \text{ GeV},$$

$$T_\tau = 300 \text{ GeV}.$$

SUSY scale (10TeV)

$$\lambda^{(1)}(M_{\text{SUSY}}) = (g^2 + g'^2) \cos 2\beta,$$

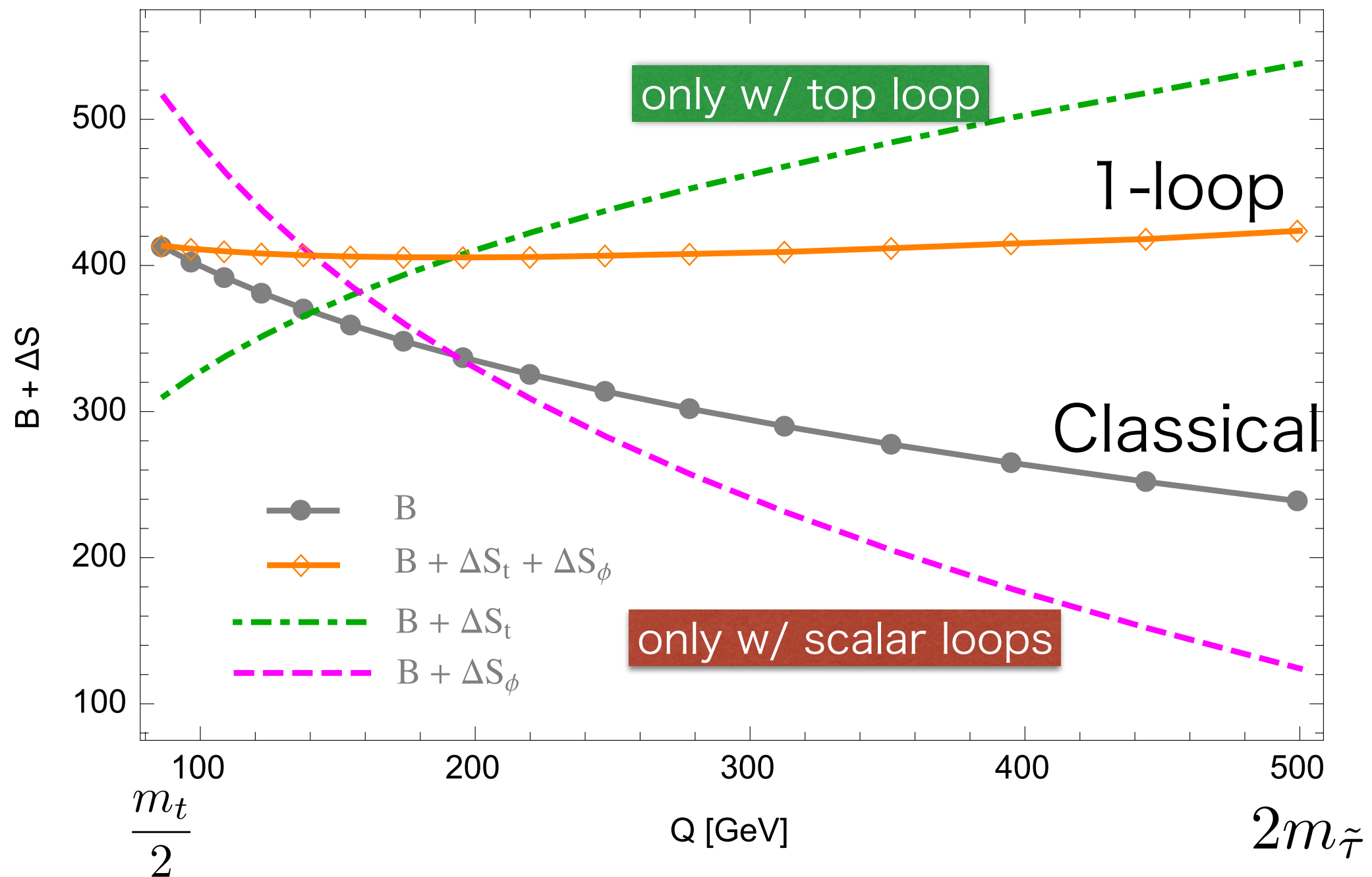
$$\lambda^{(2)}(M_{\text{SUSY}}) = 4y_\tau^2 - 2g^2 \cos 2\beta,$$

$$\lambda^{(3)}(M_{\text{SUSY}}) = 4y_\tau^2 - 2g'^2 \cos 2\beta,$$

$$\kappa^{(1)}(M_{\text{SUSY}}) = \frac{1}{2}(g^2 + g'^2),$$

$$\kappa^{(2)}(M_{\text{SUSY}}) = -\kappa^{(3)}(M_{\text{SUSY}}) = 2g'^2,$$

Result



Summary

- The bubble nucleation rate has often been estimated without calculating the pre-exponential factor.
- This estimate involves uncertainty in the renormalization scale, which, we showed, results in $O(10\%)$ uncertainty in the exponent of the bubble nucleation rate.
- To reduce the uncertainty, we explicitly calculated the pre-exponential factor and showed that it is greatly reduced.
- Scalars and fermions have already been implemented, but the gauge bosons are now ongoing.

Backup

Effective theory

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{kin}} - y_t(H q_L t_R^c + \text{h.c.}) - m_H^2 |H|^2 - \frac{1}{4} \lambda_H |H|^4 \\ & - m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 - T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + \text{h.c.}) - \frac{1}{4} \kappa^{(1)} |\tilde{\ell}_L|^4 - \frac{1}{4} \kappa^{(2)} |\tilde{\tau}_R|^4 \\ & - \frac{1}{4} \lambda^{(1)} |H|^2 |\tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(2)} |H^\dagger \tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(3)} |H|^2 |\tilde{\tau}_R|^2 - \frac{1}{4} \kappa^{(3)} |\tilde{\ell}_L|^2 |\tilde{\tau}_R|^2,\end{aligned}$$

Beta functions (leading in y_t and T_τ)

$$\frac{d\lambda_H}{d\ln Q} = \frac{3y_t^2}{4\pi^2} \lambda_H - \frac{3}{8\pi^2} y_t^4,$$

$$\frac{dT_\tau}{d\ln Q} = \frac{3y_t^2}{16\pi^2} T_\tau,$$

$$\frac{d\lambda^{(I)}}{d\ln Q} = \frac{3y_t^2}{8\pi^2} \lambda^{(I)},$$

$$\frac{d\kappa^{(I)}}{d\ln Q} = 0,$$

$$\frac{dm_H^2}{d\ln Q} = \frac{3y_t^2}{8\pi^2} m_H^2 + \frac{1}{8\pi^2} T_\tau^2,$$

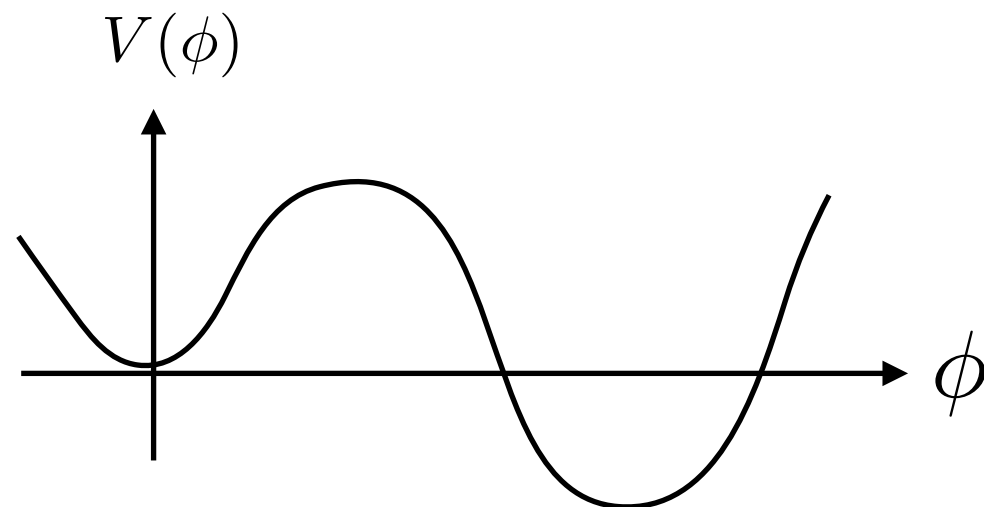
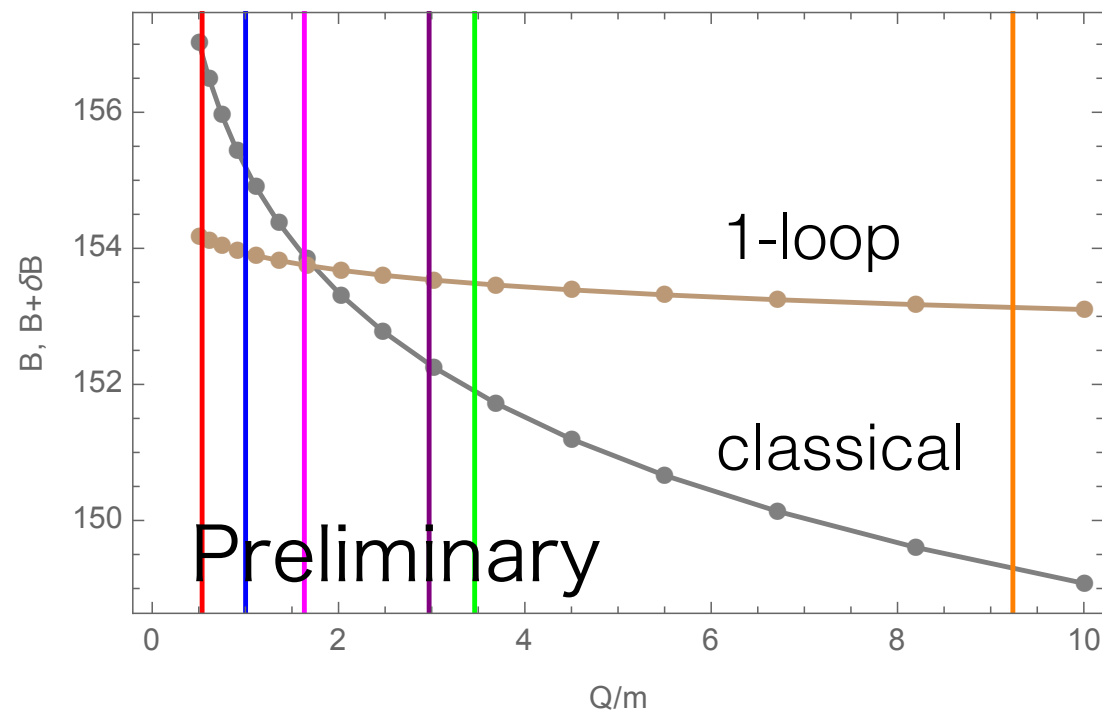
$$\frac{dm_{\tilde{\ell}_L}^2}{d\ln Q} = \frac{1}{8\pi^2} T_\tau^2,$$

$$\frac{dm_{\tilde{\tau}_R}^2}{d\ln Q} = \frac{1}{4\pi^2} T_\tau^2,$$

Results

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

$$\alpha = 0.3$$



$$\alpha = 0.9$$

