Off-shell Renormalization of Dimension-6 Operators in Higgs Effective Field Theories

Andrea Quadri
INFN Sez. di Milano

based on A.Q. Int.J.Mod.Phys. A32 (2017) no.16, 1750089

D.Binosi, A.Q., arXiv:1709.09937

Scalars 2017 - Warsaw, November 30-December 3, 2017

Probing BSM Physics:

Higgs Effective Field Theories

Operators of higher dimension are added to the SM Lagrangian without violating the symmetries of the theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{(7)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4} \mathcal{O}_i^{(8)} + \cdots$$

c are the Wilson coefficients, Λ is some large energy scale

UV Properties of HEFTs

HEFTs are renormalizable in the modern sense *à la* Gomis-Weinberg, i.e.:
- Power-counting renormalizability is lost
- Physical Unitarity (cancellation of ghost states)
guaranteed by BRST symmetry & Slavnov-Taylor identities
- Froissart bound usually not respected

In general all possible terms allowed by symmetry
must be included in an EFT approach

J.Gomis, S.Weinberg, Are nonrenormalizable gauge theories renormalizable?
Nucl.Phys. B469 (1996) 473-487

One-loop Anomalous Dimensions in the HEFTs

However a *tour de force* computation
of one-loop anomalous dimensions in general HEFTs
involving dim. six operators
has revealed surprising cancellations.

R.Alonso, E.Jenkins, A.Manohar, M.Trott
arXiv:1308.2627 , arXiv:1310.4838 , arXiv:1312.2014 , arXiv:1409.0868

Not all mixings in principle allowed by the symmetries
do indeed arise at one loop level.

Holomorphy

Basic idea: holomorphic operators do not mix
with anti-holomorphic and non-holomorphic operators.

True at the one-loop level
(up to some breaking proportional to Yukawa couplings)
on the S-matrix elements.

C.Cheung and C.Shen, arXiv: 1505.01844

Off-shell UV Patterns in HEFTs of $\Phi^\dagger\Phi$

The subclass of HEFT generated by higher-dimensional operators involving powers of $\Phi^\dagger\Phi$ and ordinary derivatives thereof only has some peculiar UV properties.

Use $\Phi^\dagger\Phi$ (after spontaneous symmetry breaking) as a new dynamical variable.

Some additional symmetries become apparent.

Extra Fields and the Scalar Constraint

$$\Gamma_{\mathrm{SSB}} = \int \Bigg[D_\mu \Phi^\dagger D^\mu \Phi - \frac{M^2 - m^2}{2} X_2^2 - \frac{m^2}{2v^2} \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 - \bar{c} \left(\Box + m^2 \right) c$$

$$+ \frac{1}{v} \left(X_1 + X_2 \right) \left(\Box + m^2 \right) \left(\Phi^\dagger \Phi - \frac{v^2}{2} - v X_2 \right) + \bar{c}^* \left(\Phi^\dagger \Phi - \frac{v^2}{2} - v X_2 \right) + V(X_2) \Bigg],$$

$\Phi = \frac{1}{\sqrt{2}} \binom{i\phi_1 + \phi_2}{\sigma + v - i\phi_3}$ SU(2) doublet $\quad X_2$ SU(2) singlet

A suitable additional BRST symmetry ensures that the physical degrees of freedom are unchanged.

Solving the constraint

The X1-e.o.m. is classically satisfied by the constraint

$$X_2 = \frac{1}{2v}\sigma^2 + \sigma + \frac{1}{2v}\phi_a^2$$

$$= \Phi^\dagger \Phi - \frac{v^2}{2}$$

One gets back the usual SM potential

$$V_{\mathrm{SM}} = \frac{M^2}{2v^2}\left(\Phi^\dagger \Phi - \frac{v^2}{2}\right)^2$$

right sign of the quartic
potential needed to ensure stability
from the sign of the mass term,
in turn fixed by the requirement
of the absence of tachyons

Dangereous Interactions for Renormalizability

The model contains derivative interactions of the schematic form

$$\chi \Box \chi^2$$

i.e. an operator of dimension 5.

Renormalizability?

Propagators

The quadratic part is diagonalized by $\sigma = \sigma' + X_1 + X_2$

$$\Delta_{\sigma'\sigma'} = \frac{i}{p^2}\,, \quad \Delta_{\phi_a\phi_b} = \frac{i\delta_{ab}}{p^2}\,, \quad \Delta_{\bar{c}c} = \frac{i}{p^2}$$

$$\Delta_{X_1X_1} = -\frac{i}{p^2}\,, \quad \Delta_{X_2X_2} = \frac{i}{p^2 - M^2}\,.$$

The derivative interaction only depends on $X = X_1 + X_2$ whose propagator has an improved UV behaviour

$$\Delta_{XX} = \frac{iM^2}{p^2(p^2 - M^2)}$$

Thus the derivative interaction is harmless and p.c. renormalizability still holds

Mapping between the $X_{1,2}$-theory and the Standard formalism

By going on-shell with the $X_{1,2}$-fields
we obtain the 1-PI amplitudes of the standard formalism
(let us call the latter the "target" theory).

For $V(X_2)=0$ one recovers the SM.

In the $X_{1,2}$-formalism (a class of) BSM operators
admits a reformulation in terms of suitable external sources coupled
to a tower of X_2-dependent operators, with a better UV behaviour
than those of the quantized fields.

Mapping on the HEFT

X2-theory

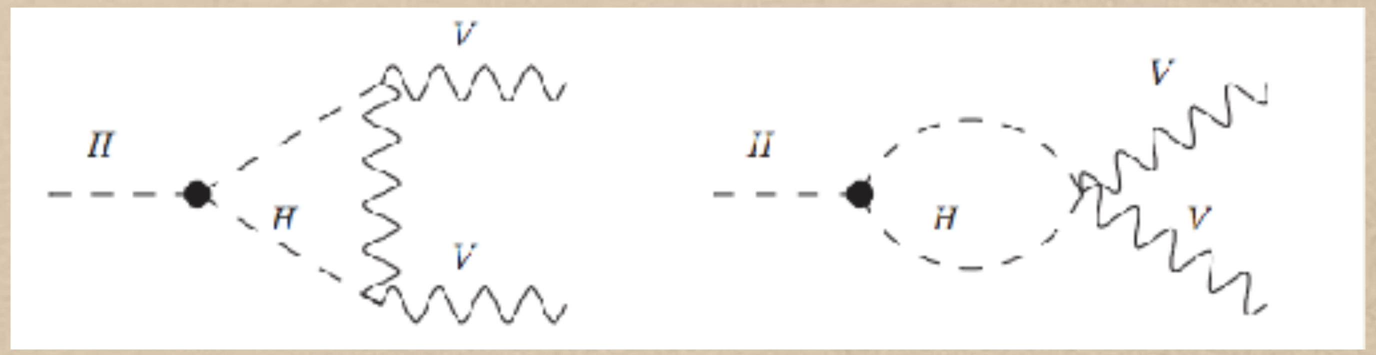
F.eqs. governing amplitudes involving the new dynamical variables in terms of ext. sources

Diagrammatic isolation of BSM operators

Transition function

HEFT

Do we generate derivative dim.6 ops if we add the third power of $\Phi^\dagger\Phi$?



What is the off-shell pattern of ops. mixing?

Cubic BSM potential

In the presence of a cubic BSM potential

$$V(X_2) = g_6 v X_2^3$$

a single additional external source R is needed in order
to control the composite operator X_2^2,
arising from the derivative of the action w.r.t. X_2.

Equations of motion for the Auxiliary Fields in the presence of a Cubic Potential

$$
\begin{aligned}
\Gamma_{X_1} &= \frac{1}{v}\left(\square + m^2\right)\Gamma_{\bar{c}^*}\,, \\
\Gamma_{X_2} &= \frac{1}{v}\left(\square + m^2\right)\Gamma_{\bar{c}^*} + 3g_6 v\Gamma_R - \left(\square + m^2\right)X_1 \\
&\quad - \left(\square + M^2\right)X_2 + 2RX_2 - v\bar{c}^*.
\end{aligned}
$$

Recovering the dependence on $X_{1,2}$

The equations of motion imply that the all-order dependence of the vertex functional on the auxiliary fields is encoded into the combinations

$$\mathcal{R} = R + 3g_6 v X_2; \qquad \bar{\mathcal{C}}^* = \bar{c}^* + \frac{1}{v}\left(\Box + m^2\right)(X_1 + X_2)$$

Hence one can limit oneself to the study of 1-PI amplitudes involving the external sources and the field σ

Moving to the target theory

We impose the equations of motion of the auxiliary fields.

From the X_1 equation (at the classical level):

$$X_2 = \frac{1}{v}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right) = \sigma + \frac{1}{2}\frac{\sigma^2}{v} + \frac{1}{2}\frac{\phi_a^2}{v},$$

From the X_2 equation (at the classical level):

$$\left(\Box + m^2\right)\left(X_1 + X_2\right) = -\left(M^2 - m^2\right)X_2 + 3g_6 v X_2^2.$$

The mapping (1-loop approximation)

Eventually one gets the mapping in the following form:

$$\mathcal{R} \to 3g_6 \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right),$$

$$\bar{\mathcal{C}}^* \to -\frac{1}{v^2} \left(M^2 - m^2 \right) \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right) + \frac{3g_6}{v^2} \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

The one-point amplitude

$X_{1,2}$-theory	Target theory
$\Gamma_R^{(1)}$	$\int \Gamma_{R_x}^{(1)} R_x \to \int \Gamma_{R_x}^{(1)} \mathcal{R}_x \underset{\sigma\ \text{term}}{\to} 3g_6 v \int \Gamma_{R_x}^{(1)} \sigma_x$
$\Gamma_{\bar{c}^*}^{(1)}$	$\int \Gamma_{\bar{c}_x^*}^{(1)} \bar{c}_x^* \to \int \Gamma_{\bar{c}_x^*}^{(1)} \bar{\mathcal{c}}_x^* \underset{\sigma\ \text{term}}{\to} -\frac{1}{v}\left(M^2 - m^2\right) \int \Gamma_{\bar{c}_x^*}^{(1)} \sigma_x$

$$\widetilde{\Gamma}_\sigma^{(1)} = 3g_6 v \Gamma_R^{(1)} - \frac{1}{v}\left(M^2 - m^2\right) \Gamma_{\bar{c}^*}^{(1)}$$

The two-point amplitude

$X_{1,2}$-theory	Target theory
$\Gamma^{(1)}_{R}$	$\int \Gamma^{(1)}_{R_x} R_x \underset{\sigma^2 \text{ term}}{\rightarrow} \frac{3}{2} g_6 \int \Gamma^{(1)}_{R_x} \sigma_x \sigma_x$
$\Gamma^{(1)}_{\bar{c}^*}$	$\int \Gamma^{(1)}_{\bar{c}^*_x} \bar{c}^*_x \underset{\sigma^2 \text{ term}}{\rightarrow} -\frac{1}{2v^2}(M^2 - m^2 - 6g_6 v^2) \int \Gamma^{(1)}_{\bar{c}^*_x} \sigma_x \sigma_x$

The two-point amplitude

$X_{1,2}$-theory	Target theory
$\Gamma^{(1)}_{RR}$	$\iint \frac{1}{2}\Gamma^{(1)}_{R_xR_y} R_x R_y \underset{\sigma^2 \ \text{term}}{\longrightarrow} \frac{9}{2} g_6^2 v^2 \iint \Gamma^{(1)}_{R_xR_y} \sigma_x \sigma_y$
$\Gamma^{(1)}_{R\sigma}$	$\iint \Gamma^{(1)}_{R_x\sigma_y} R_x \sigma_y \underset{\sigma^2 \ \text{term}}{\longrightarrow} 3 g_6 v \iint \Gamma^{(1)}_{R_x\sigma_y} \sigma_x \sigma_y$
$\Gamma^{(1)}_{\bar{c}^*\bar{c}^*}$	$\iint \frac{1}{2}\Gamma^{(1)}_{\bar{c}^*_x\bar{c}^*_y} \bar{c}^*_x \bar{c}^*_y \underset{\sigma^2 \ \text{term}}{\longrightarrow} \frac{1}{2v^2}(M^2 - m^2)^2 \iint \Gamma^{(1)}_{\bar{c}^*_x\bar{c}^*_y} \sigma_x \sigma_y$
$\Gamma^{(1)}_{\bar{c}^*\sigma}$	$\iint \Gamma^{(1)}_{\bar{c}^*_x\sigma_y} \bar{c}^*_x \sigma_y \underset{\sigma^2 \ \text{term}}{\longrightarrow} -\frac{1}{v}(M^2 - m^2) \iint \Gamma^{(1)}_{\bar{c}^*_x\sigma_y} \sigma_x \sigma_y$
$\Gamma^{(1)}_{R\bar{c}^*}$	$\iint \Gamma^{(1)}_{R_x\bar{c}^*_y} R_x \bar{c}^*_y \underset{\sigma^2 \ \text{term}}{\longrightarrow} -3 g_6 (M^2 - m^2) \iint \Gamma^{(1)}_{R_x\bar{c}^*_y} \sigma_x \sigma_y$

The two-point amplitude

$$\widetilde{\Gamma}^{(1)}_{\sigma\sigma} = {\color{red}\Gamma^{(1)}_{\sigma\sigma}} + 3g_6 \left(\Gamma^{(1)}_{R} + 2\Gamma^{(1)}_{\bar{c}^*} + 2v\Gamma^{(1)}_{R\sigma} + 3g_6 v^2 \Gamma^{(1)}_{RR} \right)$$
$$- \frac{1}{v^2}\left(M^2 - m^2\right)\left[\Gamma^{(1)}_{\bar{c}^*} + 2v\Gamma^{(1)}_{\bar{c}^*\sigma} + 6g_6 v^2 \Gamma^{(1)}_{R\bar{c}^*}\left(M^2 - m^2\right)\Gamma^{(1)}_{\bar{c}^*\bar{c}^*}\right].$$

The g6-dependence originates from the mapping only

(true at the one loop order.
At higher orders the cubic interaction vertex in X_2 inside loops introduces
a further source of g6-dependence)

The UV divergence proportional to the momentum squared
arises from (a subset [in red] of) the SM amplitude at g_6=0

The wave-function renormalization constant is the same as in the SM

Power-counting

Dangerous diagrams at non-vanishing g6 arises from propagators

$$\Delta_{X_2 X}\,, \Delta_{X_2 \sigma}$$

	R-independent sector	R and/or $\bar{c}^*$ only	R, c*, σ	R and σ
UV degree	$\dim\,\bar{c}^* = 2\,, \dim\,\sigma = 1$	$\dim\,R = \dim\,\bar{c}^* = 2$	$\dim\,\bar{c}^* = 2\,,$ $\dim\,R = \dim\,\sigma = 1$	$\dim\,R = 0\,,$ $\dim\,\sigma = 1$
UV div. amps.	$\Gamma^{(1)}_{\sigma},\ \Gamma^{(1)}_{\sigma\sigma},\ \Gamma^{(1)}_{\sigma\sigma\sigma},\ \Gamma^{(1)}_{\sigma^4}$ $\Gamma^{(1)}_{\bar{c}^*},\ \Gamma^{(1)}_{\bar{c}^*\sigma},\ \Gamma^{(1)}_{\bar{c}^*\sigma\sigma},\ \Gamma^{(1)}_{\bar{c}^*\bar{c}^*}$	$\Gamma^{(1)}_{R},\ \Gamma^{(1)}_{R\bar{c}^*},\ \Gamma^{(1)}_{RR}$	$\Gamma^{(1)}_{R\bar{c}^*\sigma}, \Gamma^{(1)}_{R\bar{c}^*\sigma\sigma}$ log. div.	$\Gamma_{R\sigma^{\leq 4}}\,,\quad \Gamma_{RR\sigma^{\leq 4}}$

$$\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$$

This operator could be generated in the target theory:

A) by amplitudes in the $X_{1,2}$ theory with external sigma legs

B) by amplitudes involving external sources via the mapping

Type A-amplitudes do no give rise to such operator since they do not depend on g6 and at g6=0 the theory is power-counting renormalizable

The mapping does not involve derivatives (at one loop), so type B-amplitudes must contain a UV divergence proportional to the momentum squared if they are to contribute.

$\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$

There is just one candidate of type B:

$$\int_x\int_y \Gamma^{(1)}_{R_x\sigma_y}\mathscr{R}_x\sigma_y \underset{\text{UV div}}{=} \int_x \mathscr{R}_x(c_0^{(1)} + c_1^{(1)}\Box)\sigma_x$$
$$\rightarrow 3g_6\int_x\left[c_0^{(1)}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 - c_1^{(1)}\partial_\mu\left(\Phi^\dagger\Phi\right)\partial^\mu\left(\Phi^\dagger\Phi\right)\right]$$

$\Box X\sigma^2$ is the vertex that dimensionally could contribute to c_1

However the differential op. does not act on the sigma leg and its effect is to remove one internal propagator

At one loop order c_1 is zero.

More general potentials

The analysis can be generalized to an arbitrary derivative-independent potential

$$V(X_2) = \sum_{j=3}^{N} g_{2j} v^{4-j} X_2^j$$

More external sources are needed in order to derive the X_2-equation

$$\Gamma_{X_2} = \frac{1}{v}\left(\Box + m^2\right)\Gamma_{\bar{c}^*} - \left(\Box + m^2\right) X_1 - \left(\Box + M^2\right) X_2 + \sum_{j=3}^{N}\left[j g_{2j} v^{4-j}\Gamma_{R_{j-1}} + (j-1)R_{j-1}\Gamma_{R_{j-2}}\right] - v\bar{c}^*,$$

More general potentials

The recursive iteration is

$$\mathcal{R}_{j-1} = R_j + j\left[v^{4-j} g_{2j} + (1-\delta_{j,N}) R_j\right] X_2$$

The solution

$$\mathcal{R}_j = R_j - \sum_{k=1}^{N-j} (-1)^k \frac{(j+1)(j+2)\dots(j+k)}{k!}$$
$$\times \left[v^{4-(j+k)} g_{2(j+k)} + (1-\delta_{j+k,N}) \mathcal{R}_{j+k}\right] X_2^k; \quad j = 2, \dots, N-1.$$

An example

$$\mathcal{R}_3 = R_3 + 4 g_8 X_2, \mathcal{R}_2 = R_2 + 3(v g_6 + \mathcal{R}_3) X_2 - 6 g_8 X_2^2,$$

The solution of the X_1 eom fixing X_2 changes order by order in the loop expansion

BSM Extensions: derivative dependent dim.6 operators

The X_2 equation is not the most general functional symmetry holding true for the vertex functional.

The breaking term on the R.H.S. of the shift symmetry stays linear in the quantum fields even if one adds a kinetic term for the scalar singlet

$$\int d^4x \, \frac{z}{2} \partial^\mu X_2 \partial_\mu X_2$$

Upon integration over the auxiliary field this is equivalent to the addition of the dimension-six operator

$$\int d^4x \ \frac{z}{v^2} \partial_\mu \Phi^\dagger \Phi \partial^\mu \Phi^\dagger \Phi$$

Outlook

- HEFTs based on powers of $\Phi^\dagger\Phi$ and ordinary derivatives thereof have some nice UV properties rooted in some functional identities which become transparent if one uses the field X_2
- Some applications: off-shell operator mixing, consistent set of higher dimensional operators, resummation

Back-up slides

BRST implementation of the on-shell constraint

Off-shell there is one more scalar field X_1.
What about this field? Physical or unphysical?

BRST symmetry (it does not originate from gauge invariance)

$$sX_1 = vc\,, \qquad sc = 0\,, \qquad s\sigma = s\phi_a = sX_2 = 0\,,$$
$$s\bar{c} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\,.$$

Ghost action

$$S_{ghost} = -\int d^4x\, \bar{c}\Box c\,.$$

Invariance under the nilpotent BRST symmetry formally associated with a $U(1)_{constr}$ group

A.Q., Phys.Rev. D73 (2006) 065024