

Alternative vevs in the NMSSM

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May 25 2017 | PLANCK 2017 Warsaw

January 20, 2017



January 20, 2009



Desired = constructed

$$V = (m_{H_u}^2 + |\mu|^2)|H_u|^2 + (m_{H_d}^2 + |\mu|^2)|H_d|^2 - 2 \operatorname{Re}(B_\mu H_u \cdot H_d) \\ + \frac{g_1^2 + g_2^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2} |H_d^\dagger H_u|^2$$

with

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix},$$

and $v_u^2 + v_d^2 = (174 \text{ GeV})^2$, $v_u/v_d = \tan \beta$.

How to?

$$\left. \frac{\partial V}{\partial h_u^0} \right|_{h_{u,d} \rightarrow v_{u,d}} = 2(\mathbf{m}_{H_u}^2 + |\mu|^2)v_u - 2 \operatorname{Re} B_\mu v_d + \frac{g_1^2 + g_2^2}{4} (v_u^2 - v_d^2)v_u$$

$$\left. \frac{\partial V}{\partial h_d^0} \right|_{h_{u,d} \rightarrow v_{u,d}} = 2(\mathbf{m}_{H_d}^2 + |\mu|^2)v_d - 2 \operatorname{Re} B_\mu v_u - \frac{g_1^2 + g_2^2}{4} (v_u^2 - v_d^2)v_d$$

Colored scalars

$$V_{\text{MSSM}} = V_F + V_{\text{soft}} + V_D$$

with (only 3rd generation squarks and Higgses; intro to MSSM omitted)

$$\begin{aligned} V_{\text{soft}} = & m_{H_d}^2 |h_d|^2 + m_{H_u}^2 |h_u|^2 - (B_\mu h_d \cdot h_u + \text{h. c.}) \\ & + \tilde{t}_L^* \tilde{m}_Q^2 \tilde{t}_L + \tilde{t}_R^* \tilde{m}_t^2 \tilde{t}_R + \tilde{b}_L^* \tilde{m}_Q^2 \tilde{b}_L + \tilde{b}_R^* \tilde{m}_b \tilde{b}_R \\ & + \left(A_t h_u \tilde{t}_L^* \tilde{t}_R + A_b h_d \tilde{b}_L^* \tilde{b}_R + \text{h. c.} \right) \end{aligned}$$

$$V_F \supset -\mu y_t h_d \tilde{t}_R^* \tilde{t}_L - \mu y_b h_u \tilde{b}_R^* \tilde{b}_L$$

Existing analytic constraints

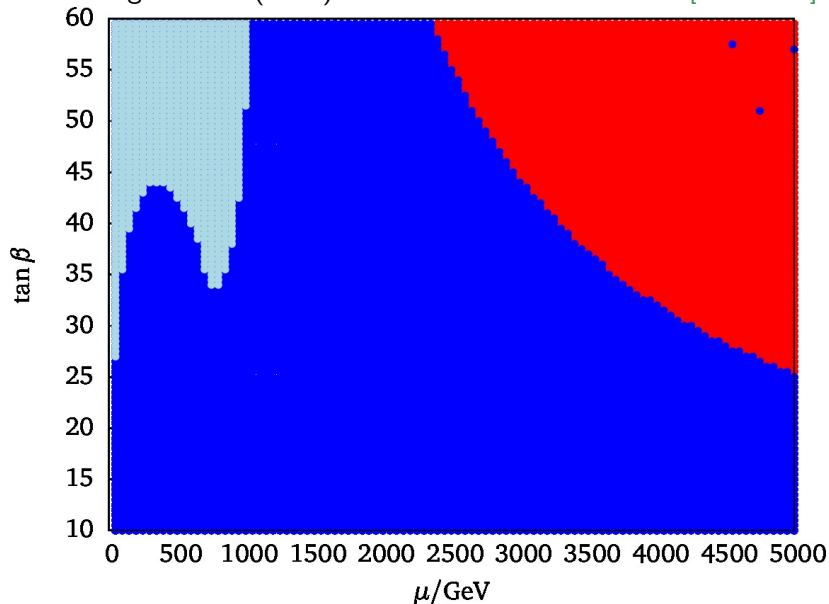
- define certain directions in field space: great simplification
- e.g. D -terms absent: $|\tilde{Q}_L| = |\tilde{t}_R| = |h_2|$ (possibly miss sth.)

[Frère et al. '83, Gunion et al. '88, Casas et al. '96]

$$A_t^2 < 3(m_{H_u}^2 + |\mu|^2 + \tilde{m}_Q^2 + \tilde{m}_t^2)$$

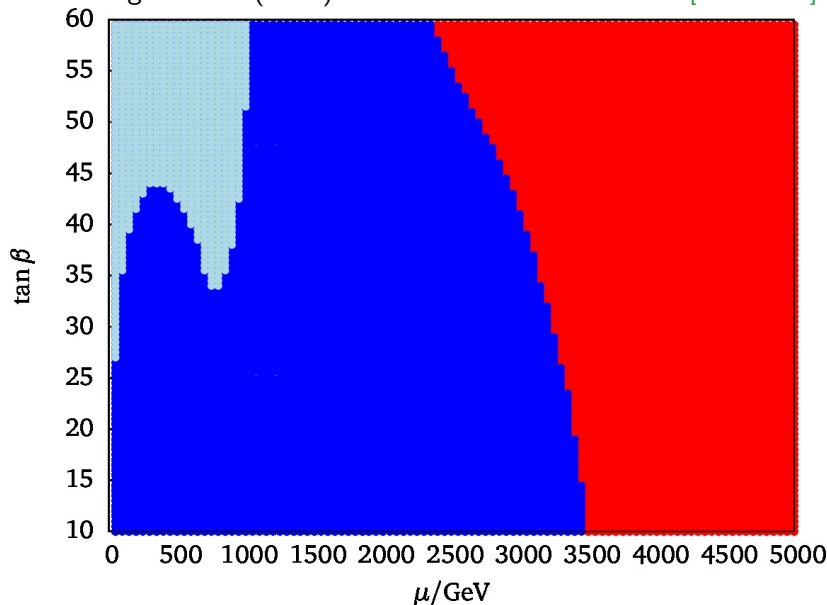
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[WGH 2016].



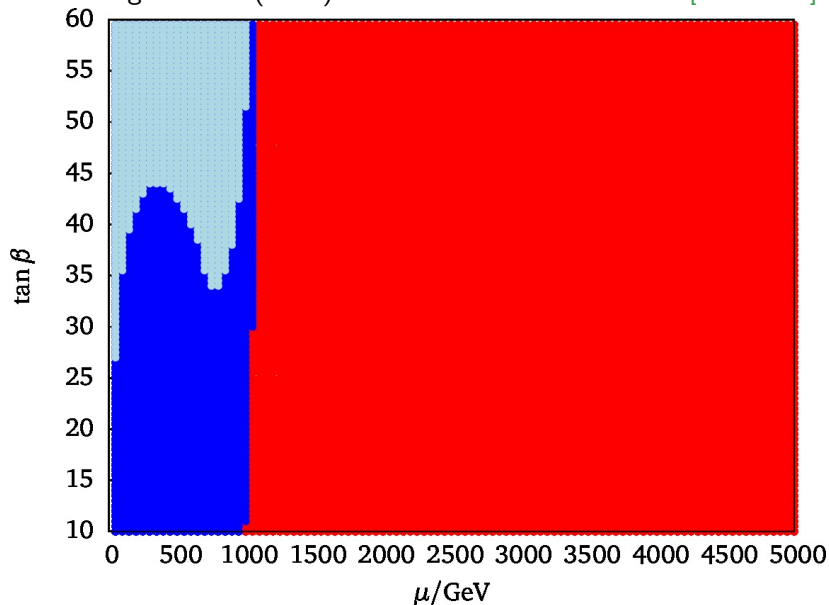
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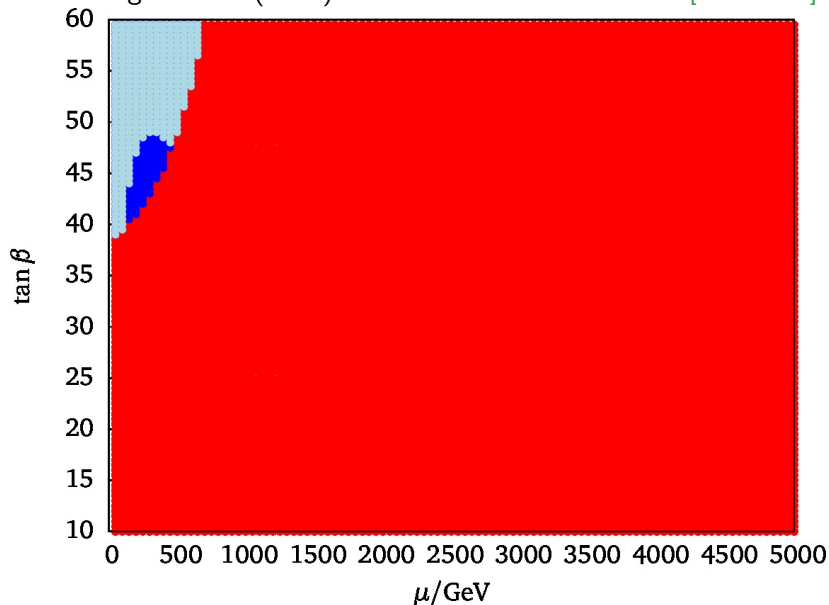
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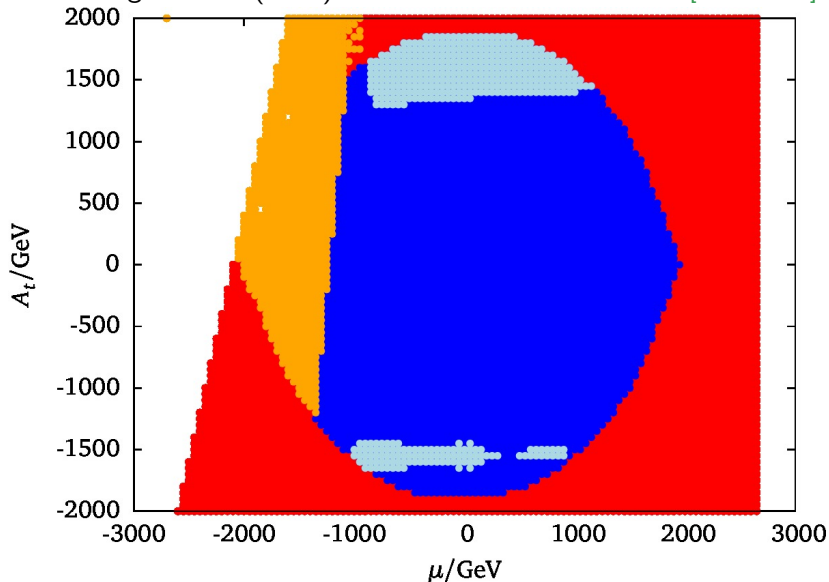
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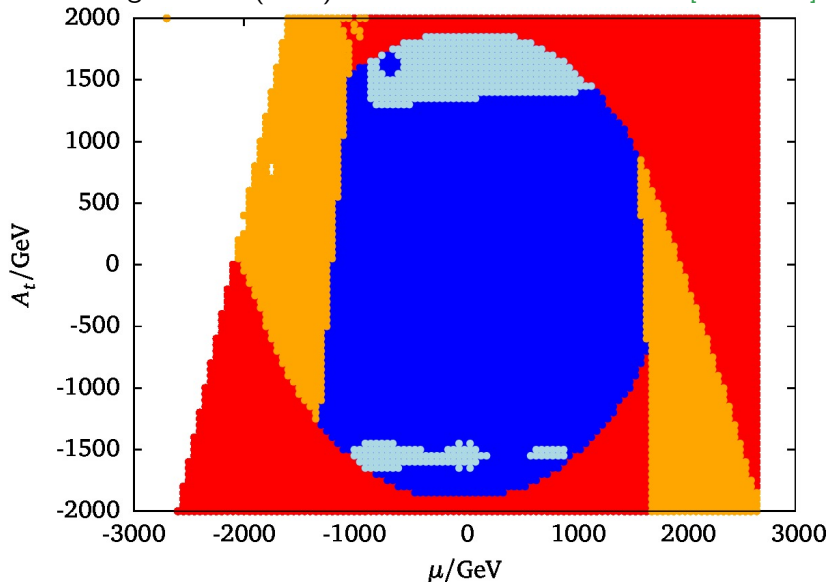
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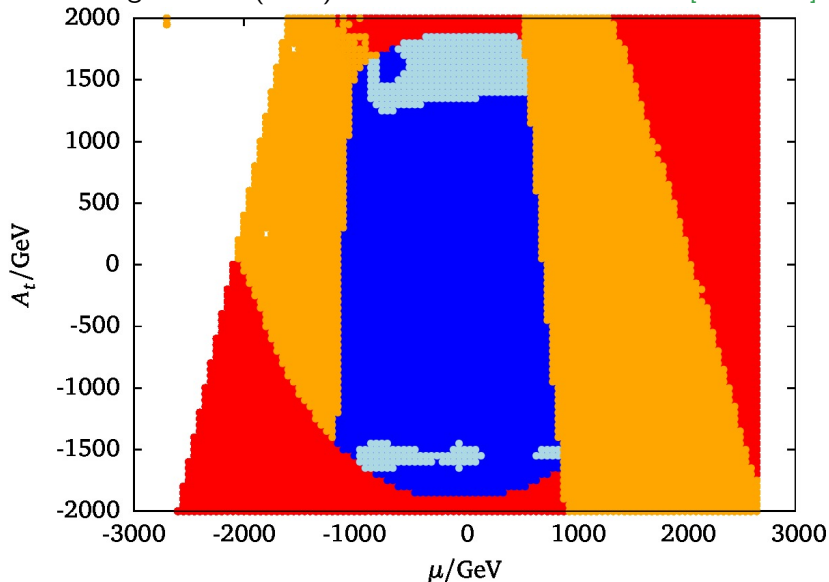
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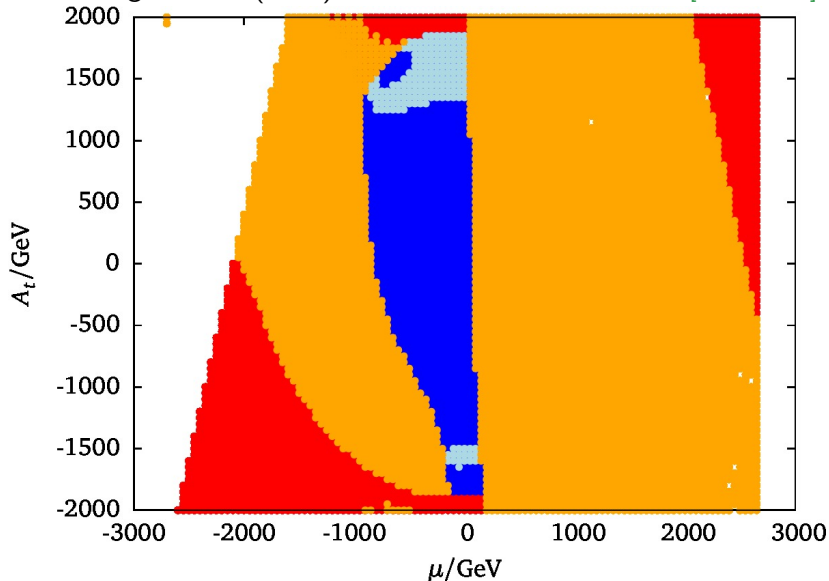
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Why next?

Why next?

- Why not?
- Add a SM singlet superfield.
- Richer phenomenology (Higgs and neutralino sector)

The NMSSM solves the “ μ -problem”

$$\mathcal{W}_{\text{MSSM}} = \mu H_u \cdot H_d + \text{Yukawa}$$

only dimensionful parameter μ has to be $\sim$ electroweak scale

$$\mathcal{W}_{\text{NMSSM}} \supset \lambda S H_u \cdot H_d + \frac{\kappa}{3} S^3$$

dynamical μ -term: $\lambda \langle S \rangle = \mu_{\text{eff}}$

$\mathbb{Z}_3$ symmetry forbids dimensionful couplings (bilinear, tadpole terms)

Trilinear terms in the Higgs sector

$$V_{\text{soft}} = m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 + m_S^2 |s|^2 \\ + \left(A_\lambda \lambda s h_u \cdot h_d + \frac{1}{3} A_\kappa \kappa s^3 + \text{h.c.} \right)$$

V_{Higgs} is a *multivariate* polynomial to order 4

$$V_{\text{Higgs}} \subset \{h_u^2, h_d^2, s^2, sh_u h_d, s^2, s^2 h_u h_d, s^2 h_u^2, s^2 h_d^2, h_u^2 h_d^2, h_u^4, h_d^4, s^4\}$$

Not only one minimum! V_{ew} not necessarily the global minimum!

Minimisation conditions are in general misleading!

$$\left. \frac{\partial V}{\partial h_u} \right|_{\text{vev}} = 2m_{H_u}^2 v_u + \dots$$

$$\left. \frac{\partial V}{\partial h_d} \right|_{\text{vev}} = 2m_{H_d}^2 v_d + \dots$$

$$\left. \frac{\partial V}{\partial h_u} \right|_{\text{vev}} = 2m_S^2 v_s + \dots$$

linear equations for soft SUSY breaking masses $m_{H_u}^2, m_{H_d}^2, m_S^2$, can be solved uniquely; determine numerical values for those

Simplification: h_u^0, h_d^0, s^0 only (three fields, many vacua),
real fields and parameters

$$\begin{aligned}
 V_{\text{Higgs}} = & \mathbf{m}_{H_u}^2 h_u^2 + \mathbf{m}_{H_d}^2 h_d^2 + \mathbf{m}_S^2 s^2 \\
 & + \frac{2}{3} \kappa A_\kappa s^3 + 2\lambda A_\lambda s h_u h_d \\
 & + (\kappa s^2 - \lambda h_u h_d)^2 + \lambda^2 s^2 (h_u^2 + h_d^2) \\
 & + \frac{g_1^2 + g_2^2}{8} (h_u^2 - h_d^2)^2
 \end{aligned}$$

However

Solutions for minimization equations with $\langle h_u \rangle \neq v_u$, $\langle h_d \rangle \neq v_d$ and $\langle s \rangle \neq \mu_{\text{eff}}/\lambda$ possible, viable, existing *and* leading to a true vacuum.

Potential value at the minimum to be compared with

$$\begin{aligned}
 V_{\text{min}}^{\text{des}} = & -\frac{g_1^2 + g_2^2}{8} v^4 \cos^2(2\beta) - \frac{\lambda^2}{4} v^4 \sin^2(2\beta) - \frac{\kappa^2}{\lambda^4} \mu_{\text{eff}}^4 \\
 & - v^2 \mu_{\text{eff}}^2 \left(1 - \frac{\kappa^2}{\lambda^2} \sin(2\beta) \right) - \frac{1}{3} \frac{\kappa A_\kappa}{\lambda^3} \mu_{\text{eff}}^3 + \frac{v^2}{2} A_\lambda \mu_{\text{eff}} \sin(2\beta)
 \end{aligned}$$

Constraint on A_κ

$$A_\kappa^2 > 9m_S^2$$

$A_\kappa^2 < 8m_S^2$: no $\langle s \rangle \neq 0$. [Derendinger, Savoy '84; Ellwanger et al. '97]

“Tachyonic” Higgs masses

- “problem” of tachyonic masses well known
- one mass eigenvalue of $\mathcal{M}_S^2$, $\mathcal{M}_P^2$ or charged Higgs mass $m_{H^\pm}^2$ negative
- tachyonic mass = negative curvature = alternative vev (!)

However

Careful analysis shows that tachyonic masses are not enough!

[see e.g. Kanehata, Kobayashi, Konishi, Seto, Shimomura '11]

A_λ from $m_{H^\pm}^2$

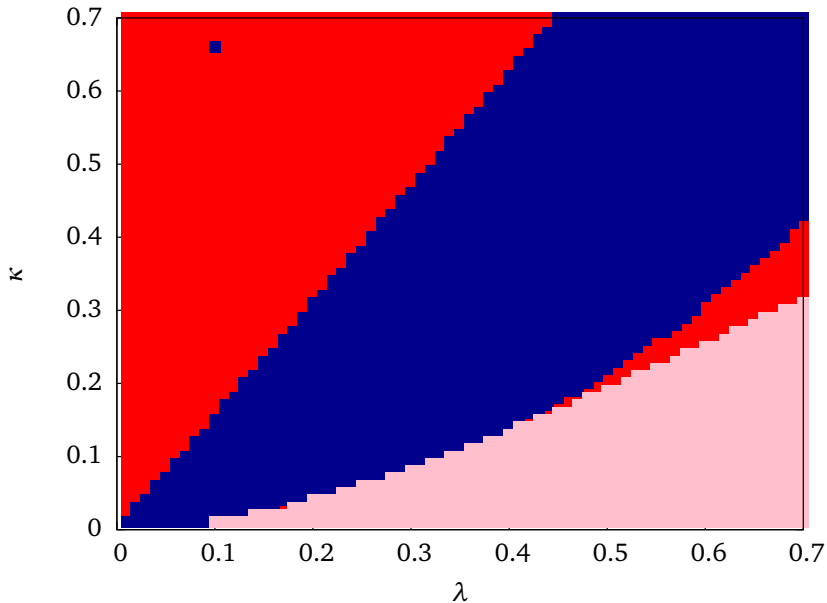
$$m_{H^\pm}^2 = \frac{2\mu_{\text{eff}}}{\sin 2\beta} \left(A_\lambda + \mu_{\text{eff}} \frac{\kappa}{\lambda} \right) + v^2 \left(\frac{g_2^2}{2} - \lambda^2 \right)$$

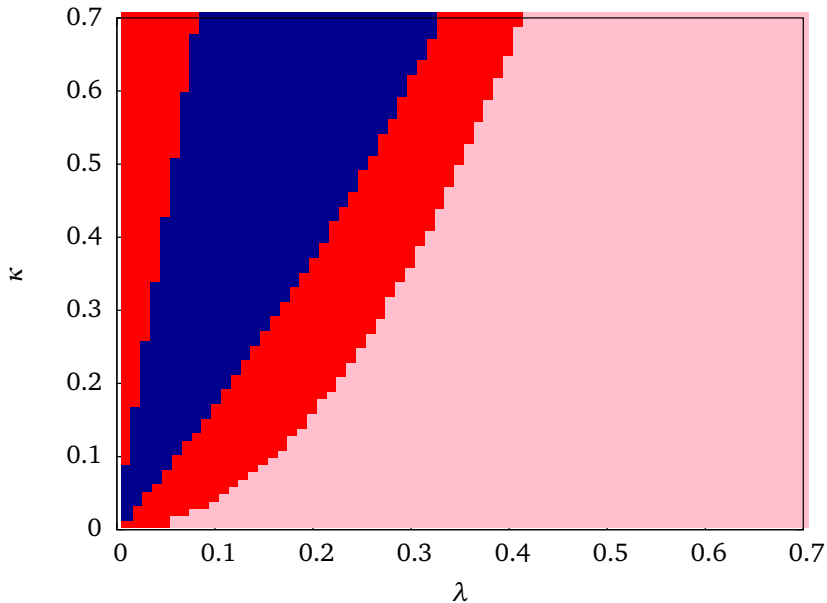
- invert: $A_\lambda = A_\lambda(m_{H^\pm}^2)$
- advantage: no tachyonic charged Higgs by construction
- select benchmark points:
“small” (300 GeV) and “large” (800 GeV) $m_{H^\pm}$

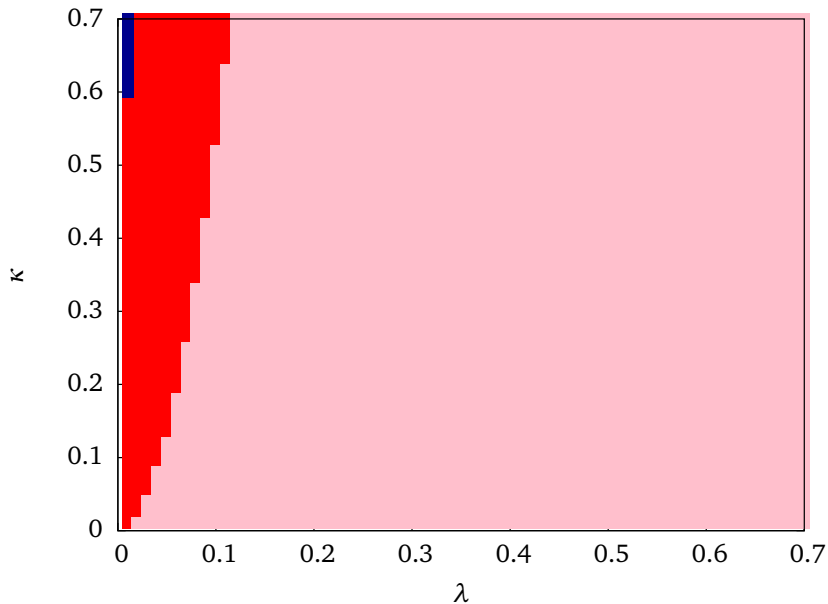
free parameters

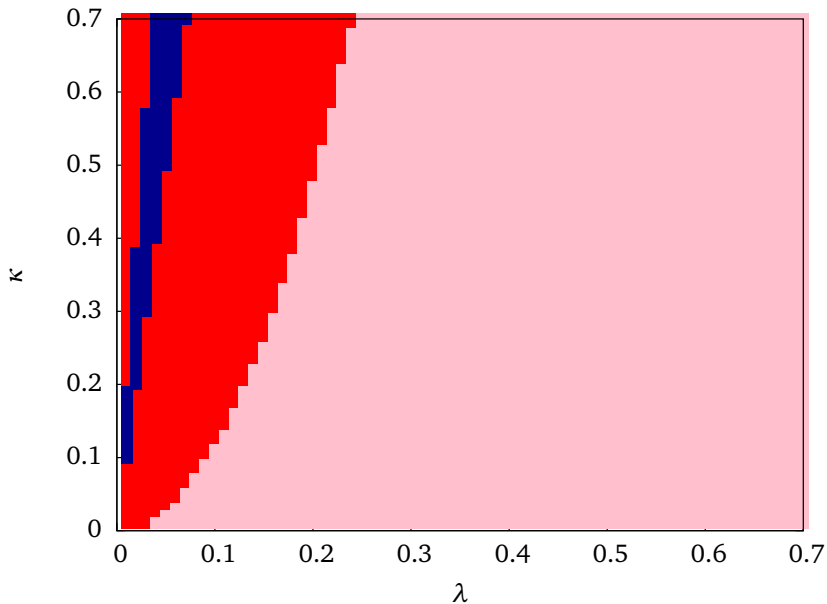
$$\lambda, \kappa, \tan \beta, \mu_{\text{eff}}, A_\kappa, m_{H^\pm}$$

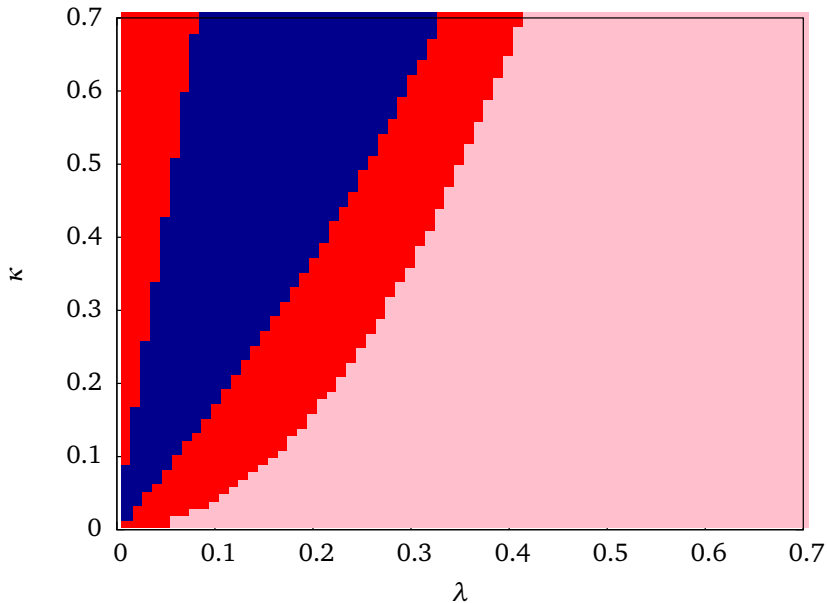
- $\tan \beta$ smallish: $\tan \beta = 3$ (fixed for all cases)
- strong constraints on λ (not too large), similar for μ_{eff} and $A_\kappa(\mu_{\text{eff}})$

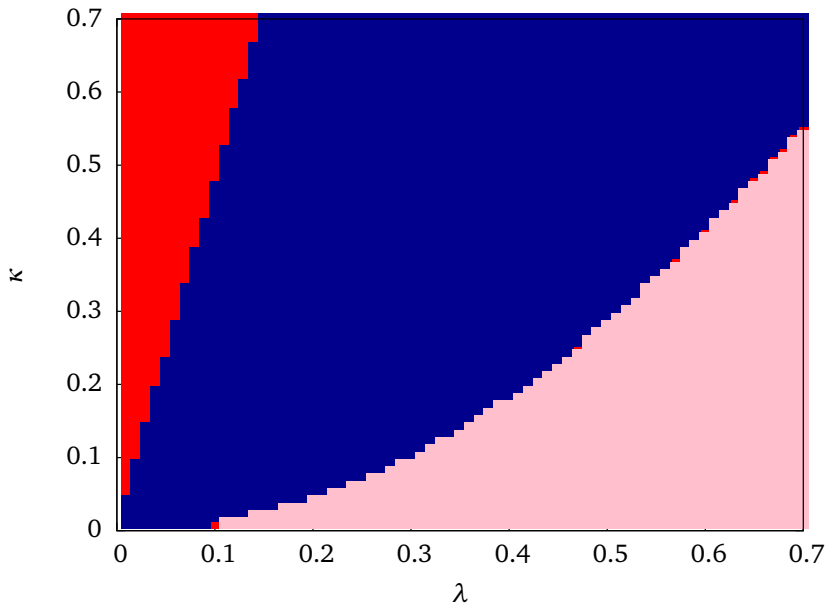


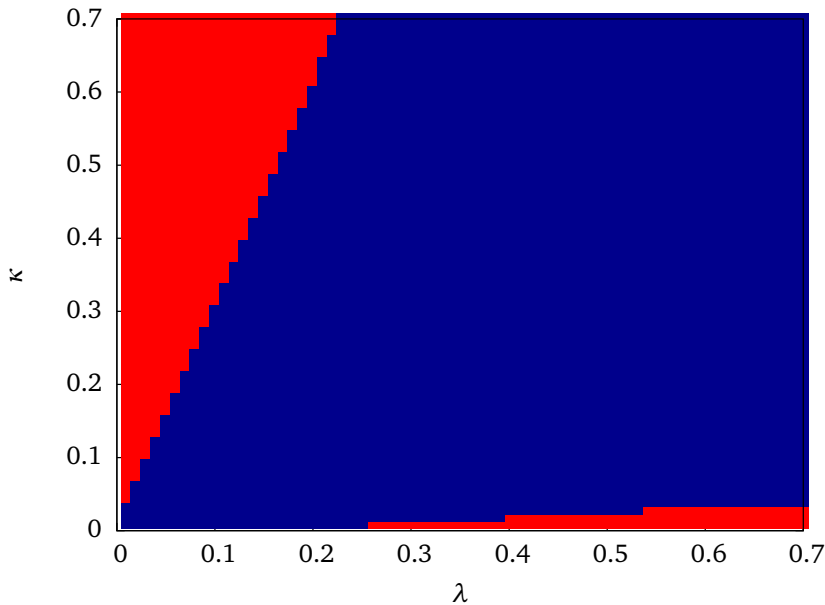


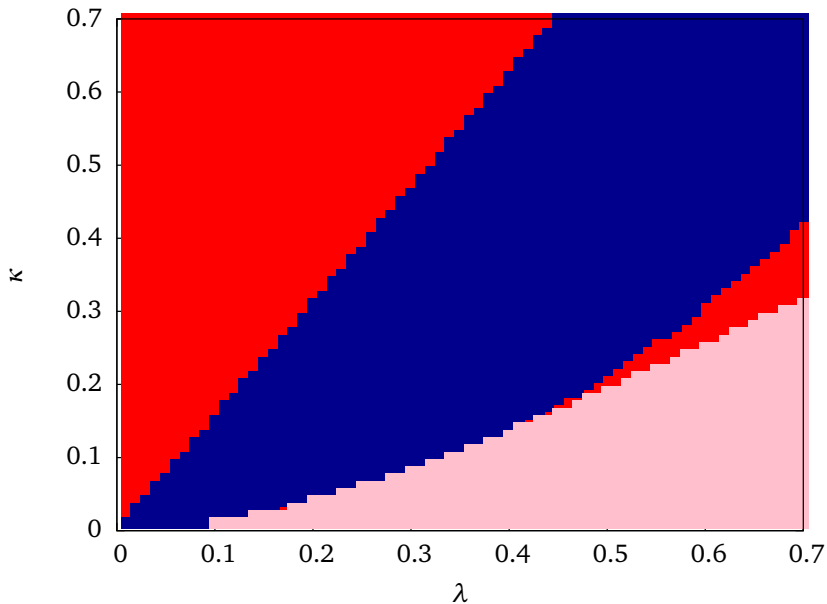


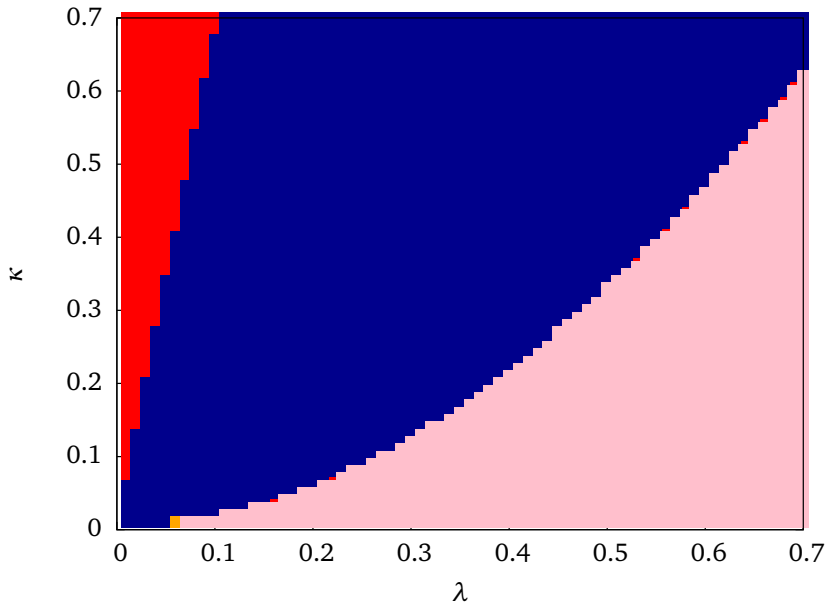


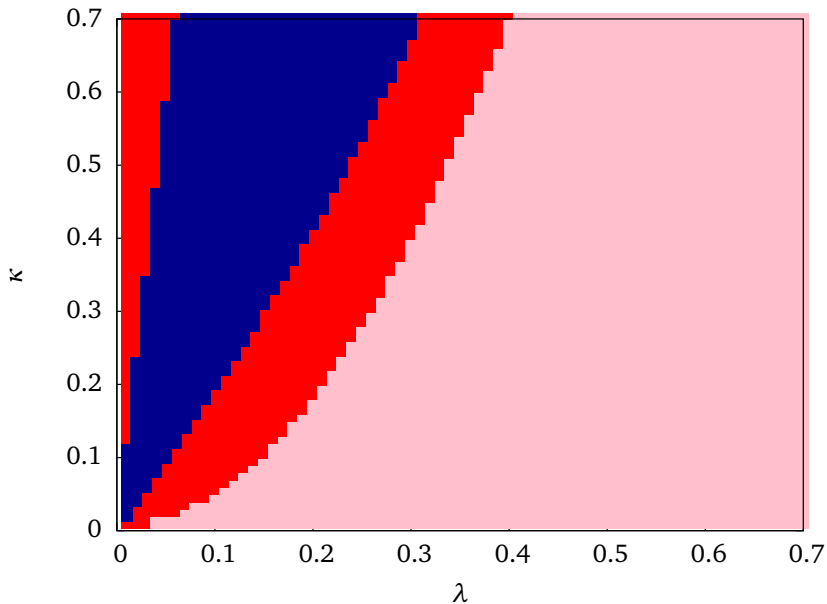


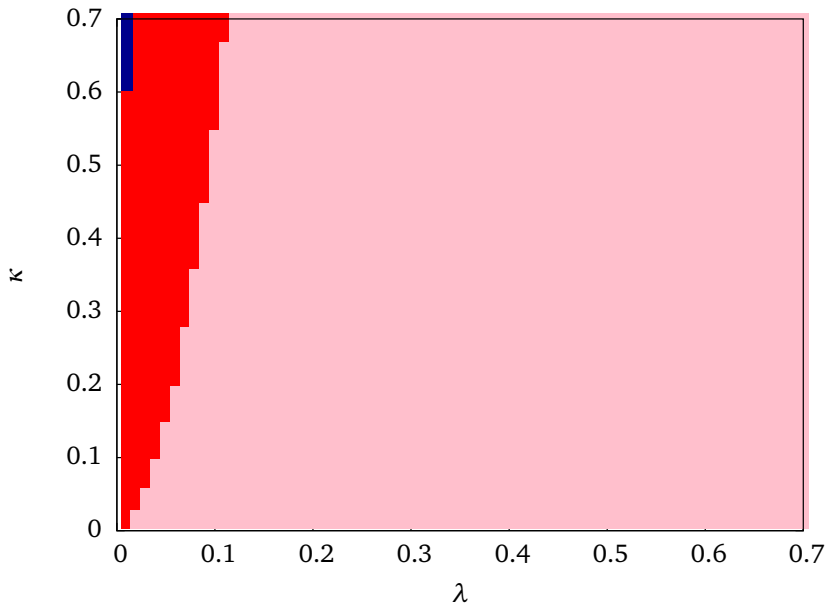


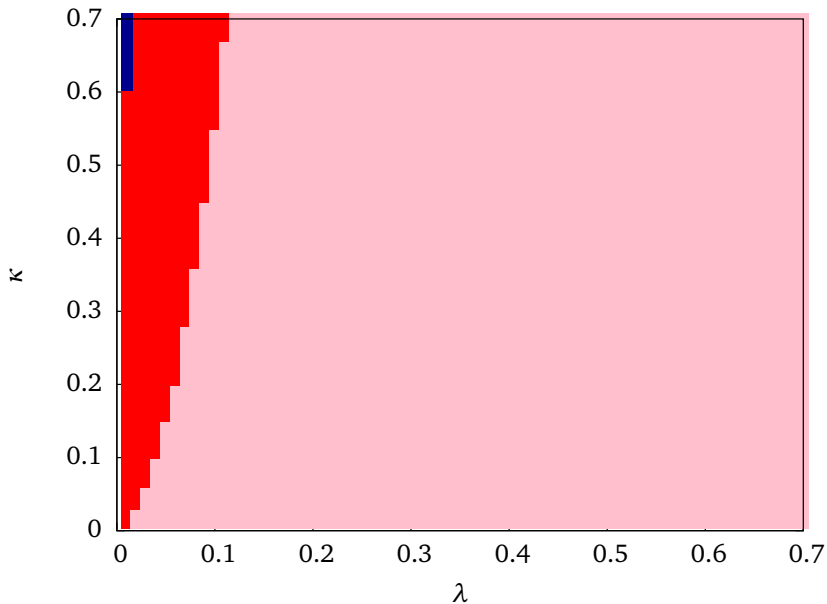


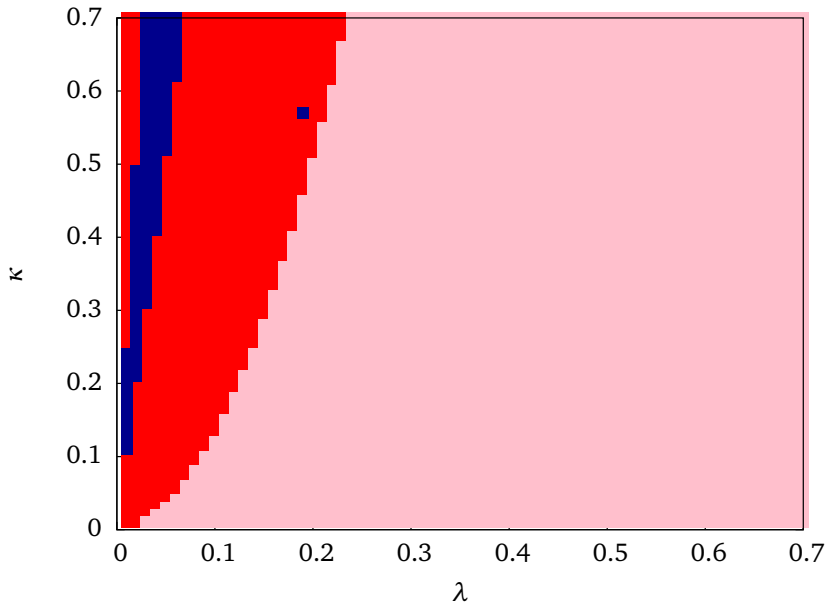


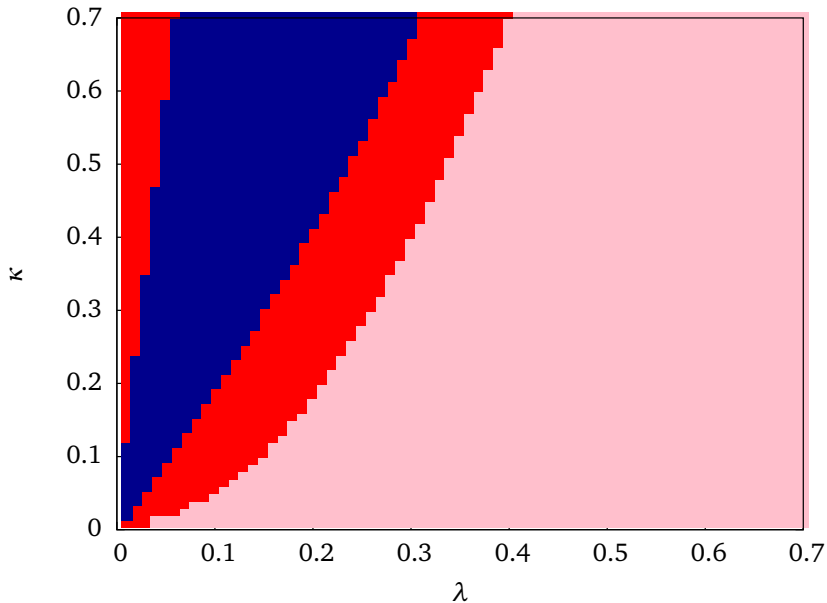


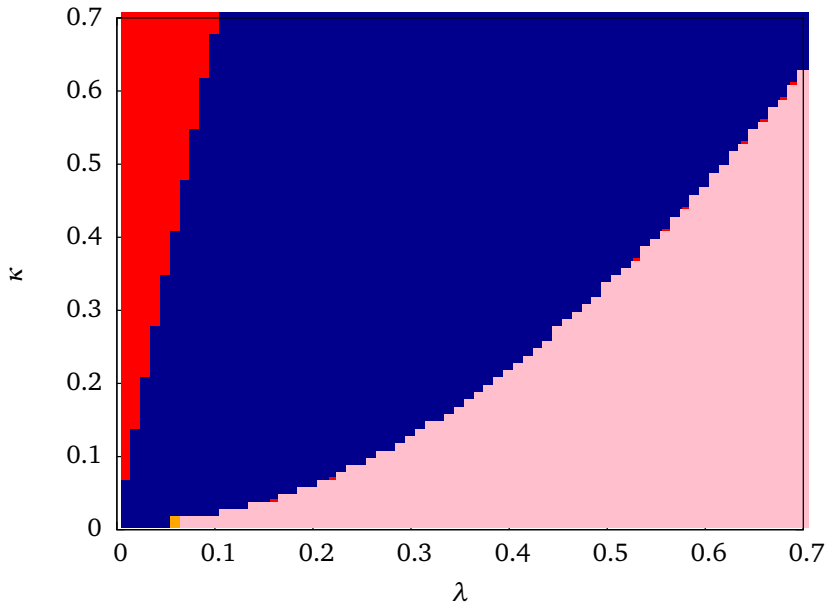


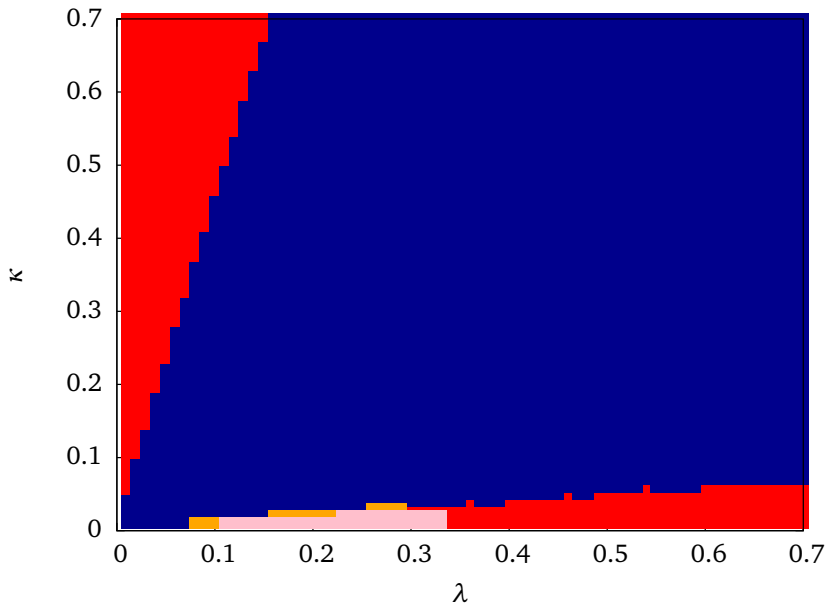


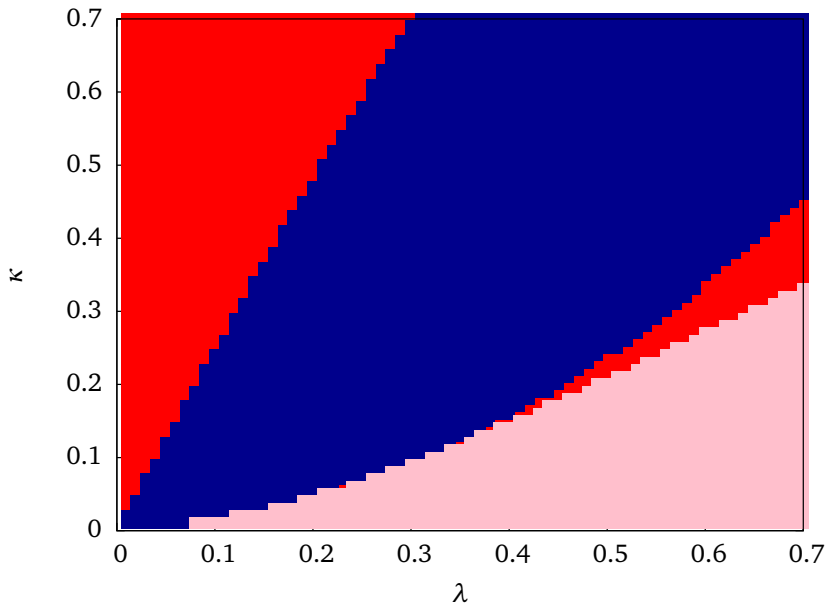












Problems

- minimization of multivariate potentials in general tedious, delicate and demanding
- numerical approaches
 - get stuck in wrong local minimum
 - may not find all minima
 - time consuming

Available tools

- try to define analytic boundaries (difficult to impossible)
- rely on some machinery [Vevacious]
- still not without problems. . .

An easy pedestrian way to check constraints

- 1 take NMSSM Higgs potential
- 2 determine $m_{H_u}^2$, $m_{H_d}^2$, m_S^2
- 3 reprocess potential: e.g. look for stationary points

An easy pedestrian way to check constraints

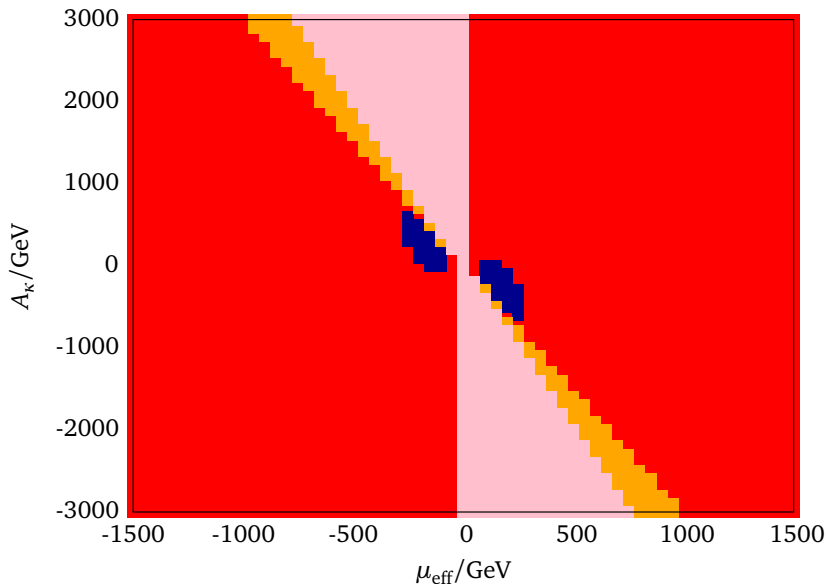
- 1 take NMSSM Higgs potential
- 2 determine $m_{H_u}^2, m_{H_d}^2, m_S^2$
- 3 reprocess potential: e.g. look for stationary points

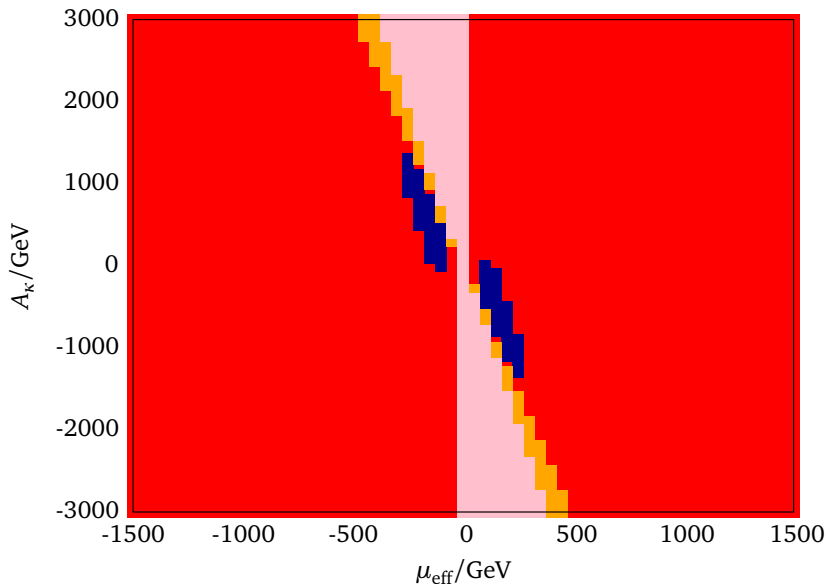
$$\begin{aligned}
 V_{\text{Higgs}} = & \textcolor{red}{m}_{H_u}^2 h_u^2 + \textcolor{red}{m}_{H_d}^2 h_d^2 + \textcolor{red}{m}_S^2 s^2 \\
 & + \frac{2}{3} \kappa A_\kappa s^3 + 2\lambda A_\lambda s h_u h_d \\
 & + (\kappa s^2 - \lambda h_u h_d)^2 + \lambda^2 s^2 (h_u^2 + h_d^2) \\
 & + \frac{g_1^2 + g_2^2}{8} (h_u^2 - h_d^2)^2
 \end{aligned}$$

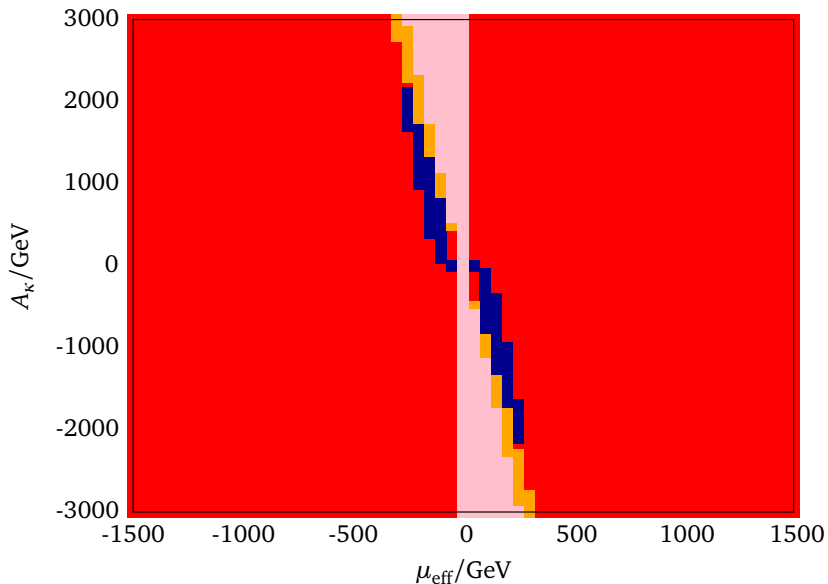
$$m_{H_u}^2 = -\mu_{\text{eff}}^2 + \frac{g_1^2 + g_2^2}{4} v^2 c_{2\beta} - \frac{\lambda^2 v^2}{4} (1 + c_{2\beta}) + \frac{\kappa}{\lambda} \mu_{\text{eff}}^2 / t_\beta + A_\lambda \mu_{\text{eff}} / t_\beta,$$

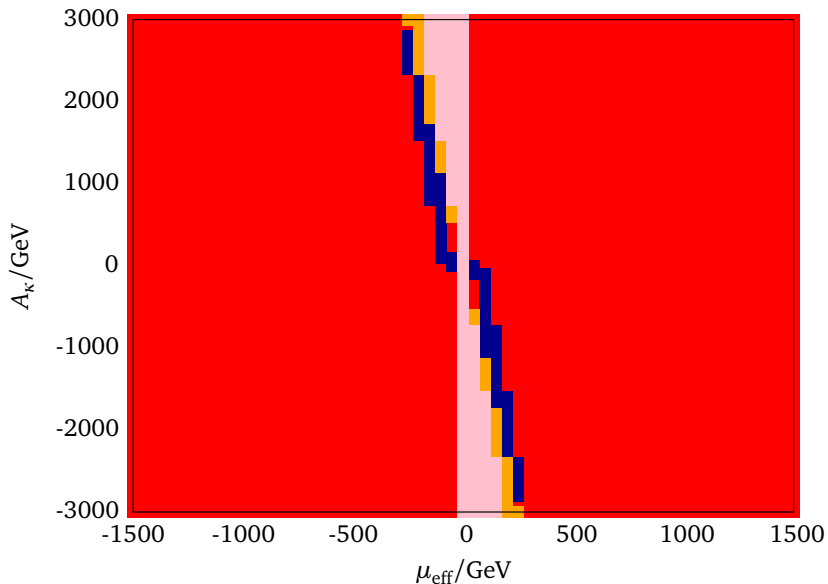
$$m_{H_d}^2 = -\mu_{\text{eff}}^2 - \frac{g_1^2 + g_2^2}{4} v^2 c_{2\beta} - \frac{\lambda^2 v^2}{4} (1 - c_{2\beta}) + \frac{\kappa}{\lambda} \mu_{\text{eff}}^2 t_\beta + A_\lambda \mu_{\text{eff}} t_\beta,$$

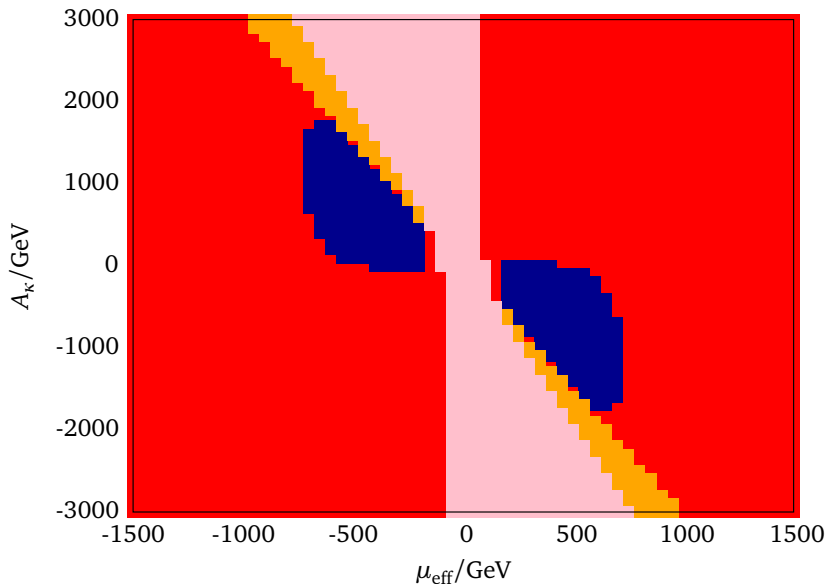
$$m_S^2 = -\lambda^2 v^2 - 2 \frac{\kappa^2}{\lambda^2} \mu_{\text{eff}}^2 + v^2 \kappa \lambda s_{2\beta} + A_\lambda \lambda^2 v^2 \frac{s_{2\beta}}{2\mu_{\text{eff}}} - \frac{\kappa}{\lambda} A_\kappa \mu_{\text{eff}}$$

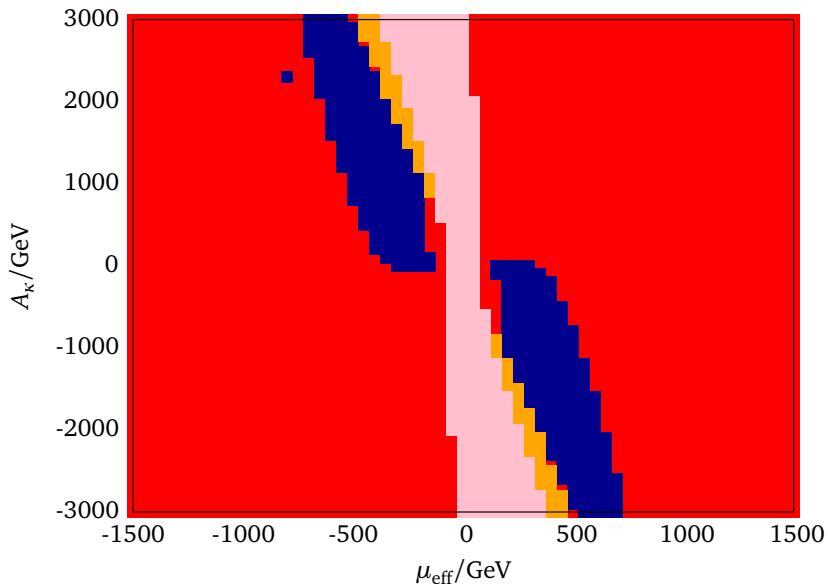


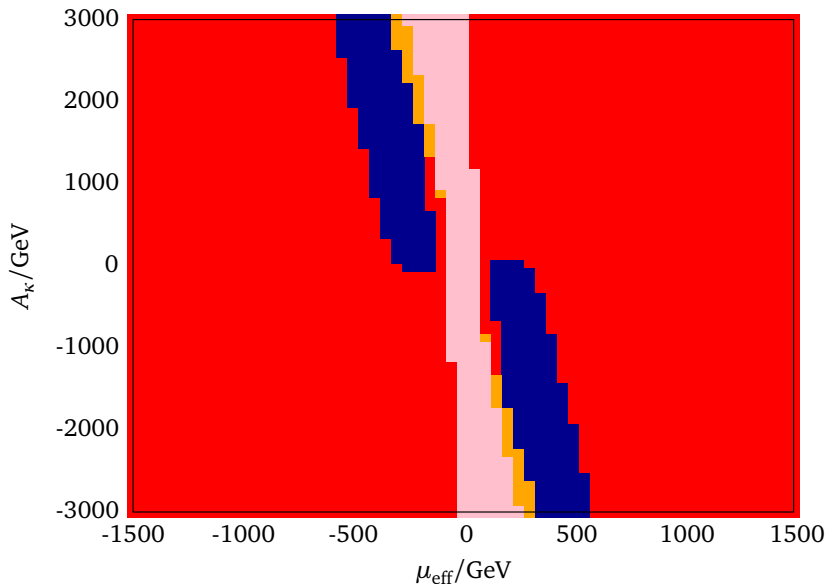


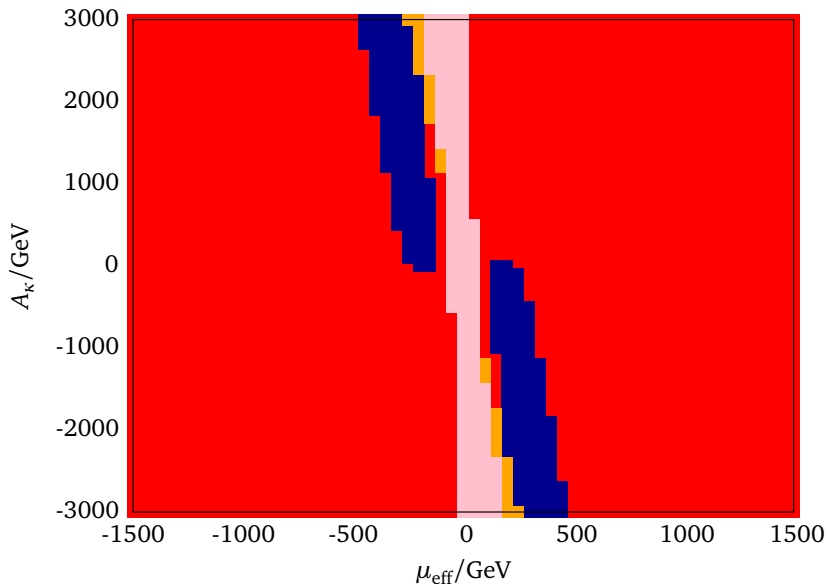












Conclusions

- mind alternative vevs: do not blindly rely on input
- non-standard Higgs vevs in the NMSSM
- global minimum: severe constraints on model parameters
- connect to realistic physics: confine to values for reasonable Higgs masses (one neutral: 125 GeV, charged not too heavy)
- pedestrian sanity check can be done very easily



Simon Riggs

(courtesy of Jens Hoff)