Fine-Tuning in Multifield Inflation

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Introduction

- A universe without inflation requires at least two kinds of fine-tuning:
  - Fine-tuned primordial curvature $|\Omega - 1| < 10^{-62}$ (*flatness problem*)
  - Fine-tuned primordial temperature profile (*horizon problem*)
- Inflation solves the flatness and horizon problems, but at a **price**
- Inflationary potential and initial conditions for scalar fields must be carefully chosen to temper production of primordial perturbations (*fine-tuning problem of inflation*)
- In *multifield inflation*, presence of additional scalar degrees of freedom introduces another kind of fine-tuning; *fine-tuning of boundary conditions*

Outline

- Inflationary trajectories
- Boundary conditions
- Stability of trajectories
- Two-field examples
- Conclusion

Inflationary trajectories

- Action for scalar-curvature theories of inflation in Einstein frame:

$$S = \int d^4x \left[-\frac{M_P^2 R}{2} + \frac{G_{AB}}{2} (\nabla_\mu \varphi^A)(\nabla^\mu \varphi^B) - U(\varphi) \right]$$

- For n homogeneous fields $\varphi^A = \varphi^A(t)$ and FRW background metric:
  - Scalar fields φ^A take on role of coordinates parametrizing *field space*
  - Non-canonical kinetic term G_{AB} takes on role of *field space metric*

- Can define field space Christoffel symbols, Riemann tensor, etc.

$$\Gamma^A_{BC} \equiv \frac{G^{AD}}{2} \left(\partial_B G_{CD} + \partial_C G_{DB} - \partial_D G_{BC} \right),$$
$$R^A{}_{BMN} \equiv \partial_M \Gamma^A_{NB} - \partial_N \Gamma^A_{MB} + \Gamma^A_{ML} \Gamma^L_{NB} - \Gamma^A_{NL} \Gamma^L_{MB}$$

Inflationary trajectories

- Covariant scalar-curvature equations of motion

$$\ddot{\varphi}^A + \Gamma^A_{BC}\dot{\varphi}^B\dot{\varphi}^C \; + \; 3H\dot{\varphi}^A \; + \; G^{AB}U_{,B} \; = \; 0,$$
$$H^2 = \frac{1}{3}\left[\frac{1}{2}G_{AB}\,\dot{\varphi}^A\dot{\varphi}^B \; + \; U\right]$$

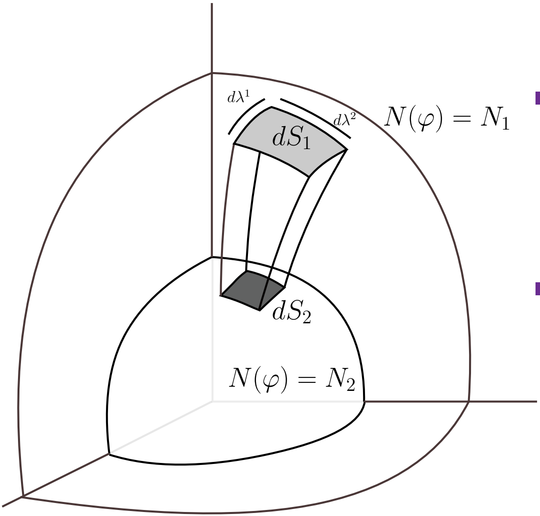
- *End-of-inflation condition* $\epsilon_H(t_0) \; = \; 1$ ($\epsilon_H \equiv -\dot{H}/H^2$) defines:
  - end-of-inflation *isochrone surface* $\varphi^A = \varphi^A(t_0)$
  - number of e-folds $N(t) \; = \; -\int_{t_0}^{t} dt'\, H(t')$

- For slow-roll inflation with $3H\dot{\varphi}^A \; = \; -G^{AB}U_{,B}$, trajectory is fully described by field values; may write isochrone surfaces as $N(\varphi) \; = \; N$

Boundary conditions

- Each point on some $(n-1)$-dimensional *boundary surface* corresponds to an inflationary trajectory $\varphi^A_\lambda(N)$
- Reject trajectories that do not agree with observational constraints
    - Small shift in boundary conditions $\rightarrow$ large change in observables
      fine-tuned/unnatural model
    - Large shift in boundary conditions $\rightarrow$ small change in observables
      robust/natural model
- How to quantify the degree of fine-tuning required?

Stability of trajectories



- Define relative measure on boundary surface: *density of trajectories*

$$n \equiv \frac{1}{S} \frac{d^{n-1} S}{d^{n-1} \lambda}$$

- *Sensitivity* of trajectories in neighbourhood of some trajectory: ratio of densities

$$Q(N_2, N_1) \equiv \frac{n_2}{n_1}$$

Stability of trajectories

- Explicit form of measure on isochrone surface $\varphi^A = \varphi_N^A$ embedded in field space is defined through determinant of *induced metric*

$$[\Gamma_{IJ}]_N = G_{AB} \frac{\partial \varphi_\lambda^A(N)}{\partial \lambda^I} \frac{\partial \varphi_\lambda^B(N)}{\partial \lambda^J}$$

- *Sensitivity parameter* $Q_* \equiv Q(N_*, N_0)$ at horizon crossing:

$$Q_* = \frac{\sqrt{\det[\Gamma_{IJ}]_*}/S_*}{\sqrt{\det[\Gamma_{IJ}]_0}/S_0}$$

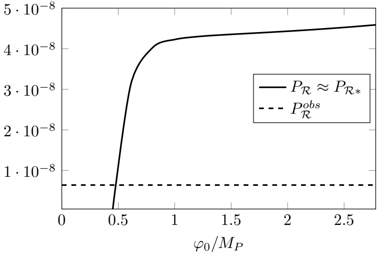
- Value of Q_* related to amount of fine-tuning required at $N = N_0$ to match model to observations
  - $Q_* > 1$: unstable trajectory – large degree of fine-tuning
  - $Q_* < 1$: stable trajectory – small degree of fine-tuning

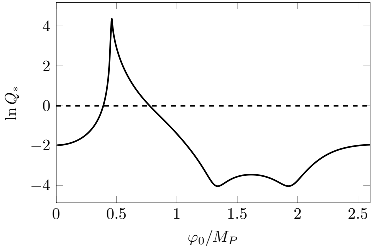
Two-field models

- Minimal two-field model

$$\mathcal{L} = -\frac{M_P^2 R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda\varphi^4}{4} - \frac{m^2\chi^2}{2}$$

- Boundary condition taken on end-of-inflation curve parametrized by φ_0
- Model is very fine-tuned to observations: $Q_*(\varphi_0/M_P \approx 0.5) \sim e^4$



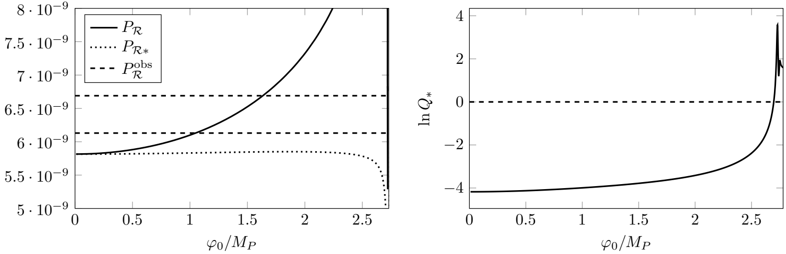


Two-field models

- Non-minimal two-field model

$$\mathcal{L} = -\frac{(M_P^2+\xi\varphi^2)R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda(\varphi^2-v^2)^2}{4} - \frac{m^2\chi^2}{2}$$

- Entropy transfer needed to normalize to power spectrum
- For most boundary conditions, $Q_* \ll 1$; robust model



Conclusion

- A kind of fine-tuning arises in multifield inflation; subtly distinct from the initial condition problem
- Many models robust, but not all; need a way to discriminate between natural and fine-tuned models
- Assign measure to parameter space $\rightarrow$ sensitivity parameter Q_* which quantifies degree of fine-tuning required at a boundary to match with observations
- Measure approach: potential way to quantify effectiveness of solutions to the initial conditions problem of inflation