The Rise ☺ and Fall ☹ of the Cheng-Sher Ansatz

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2HDM and FCNC

The Standard Model has a single Higgs doublet, Φ, which acquires a vacuum expectation value

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

where v ≈ 246 GeV. The most general Yukawa coupling is given by

$$\mathcal{L}_Y = y_{ij}\bar{f}_{Li} f_{Rj}\Phi + \mathrm{h.\ c.}$$

When expanding about the vacuum, one gets the mass matrix:

$$M_{ij} = y_{ij} v/\sqrt{2}$$

So, when the mass matrix is diagonalized, the Yukawa coupling matrix is automatically diagonalized, so the Higgs only couples in a flavor-diagonal way. But with 2 doublets:

The most general Yukawa couplings are:

$$\mathcal{L}_Y = y^1_{ij}\bar{\psi}_i\psi_j\Phi_1 + y^2_{ij}\bar{\psi}_i\psi_j\Phi_2$$

where i and j are generation indices. This gives

$$M_{ij} = y^1_{ij}\frac{v_1}{\sqrt{2}} + y^2_{ij}\frac{v_2}{\sqrt{2}}$$

Since y^1 and y^2 are, in general, not simultaneously diagonalizable, this will lead to tree level FCNC

These are very problematic-- the $\bar{d}sH$ coupling will lead to very large K - $\tilde{\bar{K}}$ mixing, unless the coupling is very small or the H is very heavy.

One way to eliminate tree level FCNC is a discrete symmetry. Paschos-Glashow-Weinberg theorem, applied to a model with doublets and singlets, states that this can only be done if all fermions of a given charge couple to only one Higgs doublet.

Paschos, Phys. Rev. D15, 1966 (1977)
Glashow, Weinberg, Phys. Rev. D 15, 1958 (1977)

Type I: All fermions couple to one doublet, Φ_2

Type II: The Q=2/3 quarks couple to Φ_2, the Q=-1/3 quarks and leptons couple to Φ_1

Early 80s

The introduction of an ad hoc Z_2 symmetry seemed epicyclic.
How necessary was it?

Experimenter in 1980 measuring $K_L \rightarrow \mu\, e$ looked at the bound assuming Higgs exchange and claimed "if the flavor-changing coupling is O(1), we find a lower bound on the Higgs mass of 60 TeV -- this is higher than the energy of the SSC!!"
Of course, this ignored mixing, the difference between the two Higgs, etc.....

A more realistic assumption made by Shankar (1980) and by McWilliams and Li (1981). Assume that the flavor-changing coupling was the heaviest fermion of that particular charge times a mixing angle. Since the angle is unknown, assume it is O(1). That still gave a bound of a few TeV from $K_L \rightarrow \mu\, e$ and an even higher bound of 100 TeV from Δm_K (although there are greater uncertainties in that).

Partly for these reasons (and the rise of SUSY which gave the type II structure), FCNC at tree level was generally ignored for most of the decade.

In the early 80's, CKM matrix elements weren't well-known, and there was great interest in Fritzsch type matrices.

$$\begin{pmatrix} 0 & A \\ A & B \end{pmatrix}$$

If A << B, then the eigenvalues are A^2/B and B, so the off-diagonal term is the geometrical mean of the eigenvalues. If this is the down quark mass matrix, this leads to the numerically correct result that

$$\sin\theta_c = \sqrt{m_d/m_s}$$

Leads to the suggestion that the FCNC couplings should be the geometric mean of the individual Yukawa couplings. How general is this?

In ‘86, I moved to Washington Univ. and Ta-Pei Cheng from Missouri, St. Louis was a few miles away. Cheng and Li had just been published and I had questions about matrix elements.

We looked at 3x3 Fritzsch matrices and found precisely the same pattern – the FCNC couplings were the geometric mean of the individual couplings. Then Ta-Pei realized it was even more general – if you just require that there be no precise cancellations in getting the eigenvalues, it followed.

The ansatz was then written as

$$y_{ij} = \lambda_{ij} \frac{\sqrt{m_i m_j}}{v/\sqrt{2}}$$

where the λ_{ij} are O(1). This is order of magnitude – one expects mixing angles, etc.

At the time, the strongest bound on the λ_{ij} came from Δm_K, and gave (for $\lambda_{ij} = 1$) a lower bound on the exchanged scalar (pseudoscalar) mass of 300 GeV (1 TeV). It ignores contributions from charged Higgs, and any mixing angles.

RISE

The CS ansatz received very little attention for a few years. Then the top turned out to be heavy, and the B-factories (BELLE/BABAR) began. The ansatz gave experimenters a target (give bounds in terms of λ_{ij} instead of a generic coupling whose value was arbitrary). It also meant that B decays and mixings would have a huge increase in precision, and thus $\lambda_{ij} =1$ was in reach. It received roughly 25 citations per year for the next 25 years. Alas, Nature is having the last word.

FALL

Over the years, bounds have become much more precise. The best and most recent analysis is Babu and Jana, arxiv: 1812.11943.

Strongest bounds are still from meson-meson mixing, but now we also have D, B and B_s mixing

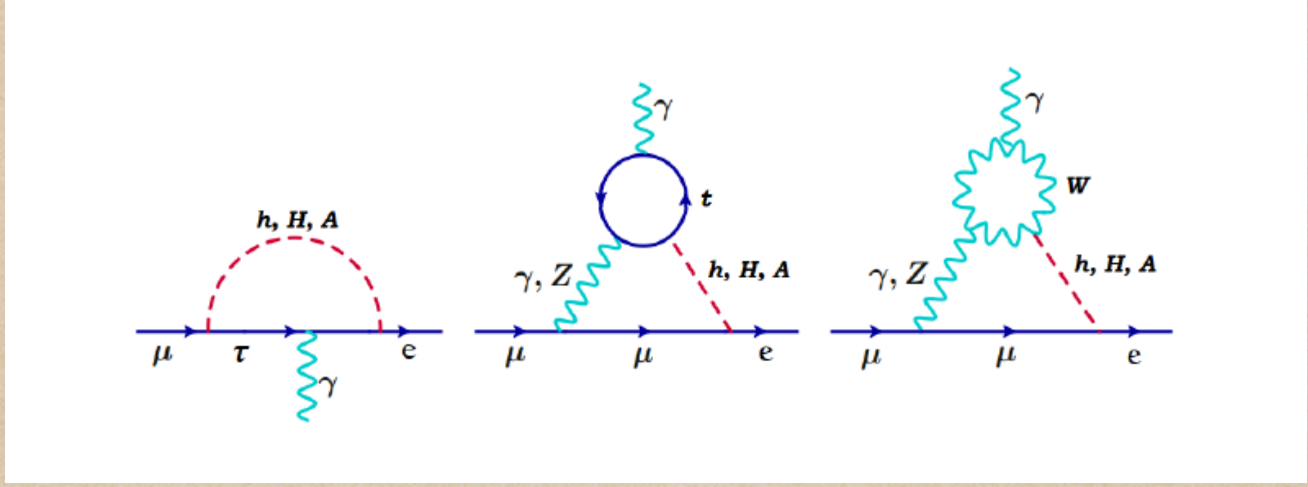
Table from Babu and Jana

arxiv:1812.11943.

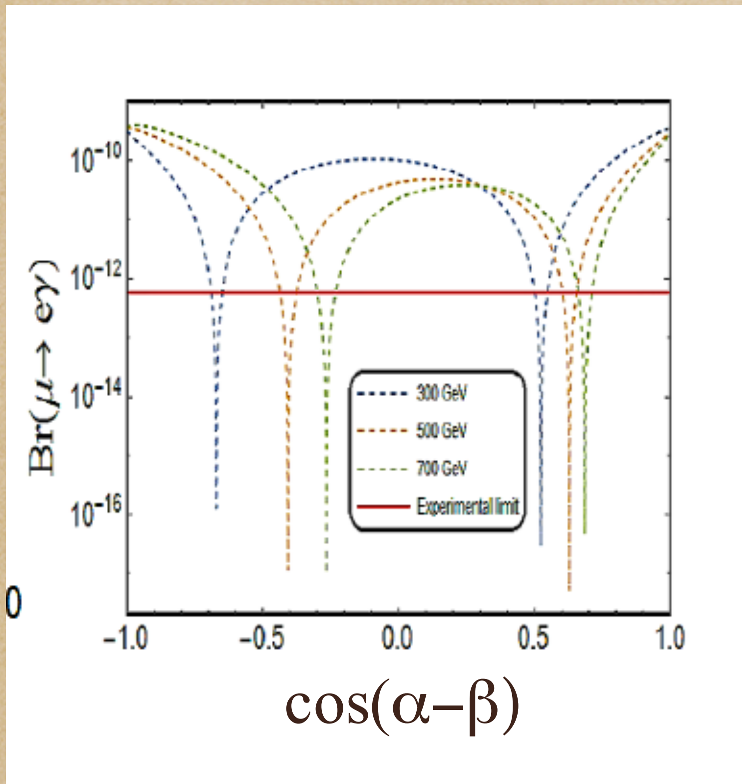
Upper bound on C_{ij}	Cheng-Sher Ansatz
$K^0 - \overline{K^0}$ mixing constraint	0.26
$B_s^0 - \overline{B_s^0}$ mixing constraint	0.436
$B_d^0 - \overline{B_d^0}$ mixing constraint	0.379
$D^0 - \overline{D^0}$ mixing constraint	0.222

Bounds on λ_{ij} obtained from meson mixing, assuming a pseudoscalar mass of 500 GeV (bound scales approximately linearly). The bound from scalar exchange is a factor of 3 or so weaker. This assumes real couplings. If there is a CP-violating phase bigger than .005, then the bounds become even worse

Radiative muon decay



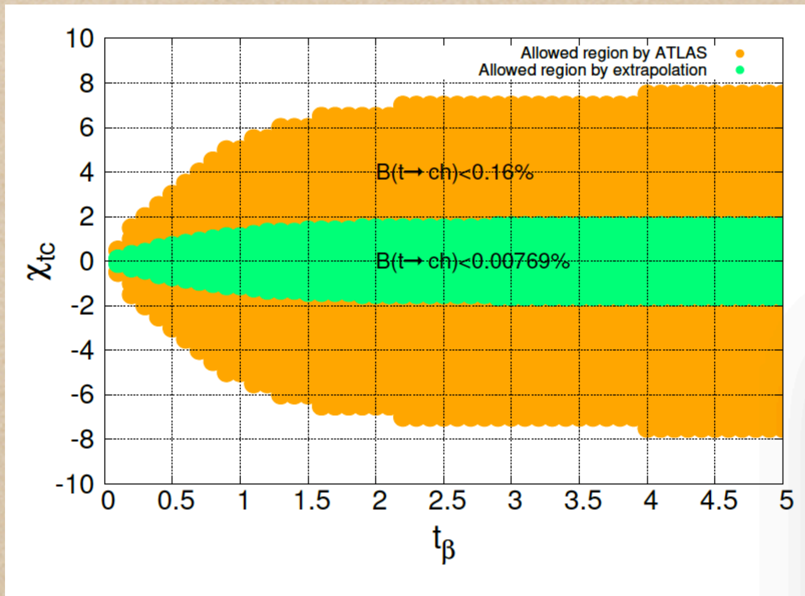
The two-loop diagrams give a bigger contribution (Barr-Zee). The result does depend on the mixing angle, sin(α−β). Assuming the λ_{ij} are all equal to one:



Taking $\cos(\alpha-\beta) = 0.4$ and a mass of H to be 500 GeV, one finds $\lambda_{\mu e}$ must be less than 0.12 to satisfy current bounds.

What about Higgs decays? Note: t $\rightarrow$ hc does currently gives a bound of $\lambda_{tc}\cos(\alpha-\beta) < 2$, which is quite weak. Better bounds will be available in a couple of decades, but $\lambda_{tc} = 1$ is beyond reach for a long time.

From 1903.02718



$h \rightarrow \mu\tau$

The branching ratio is $0.0076\ \lambda_{\mu\tau}^2 \cos^2(\alpha-\beta)$. The current CMS experimental bound is 0.0025, or $\lambda_{\mu\tau} < .6/\cos(\alpha-\beta)$. This gives a weak bound, not yet lethal.

Sher, Thrasher (2016)
Hou, et al (2019)

SIDE NOTE: The branching ratio for $H \rightarrow \mu\tau$ is proportional to $\sin^2(\alpha-\beta)$, which is much larger. Same is true for other FNCN decays of H.

Any way to avoid these bounds without fine-tuning?

Some have suggested replacing v with the smaller vev

$$y_{ij} = \lambda_{ij} \frac{\sqrt{m_i m_j}}{v/\sqrt{2}}$$

effectively rescaling λ by a factor of $\cos\beta$

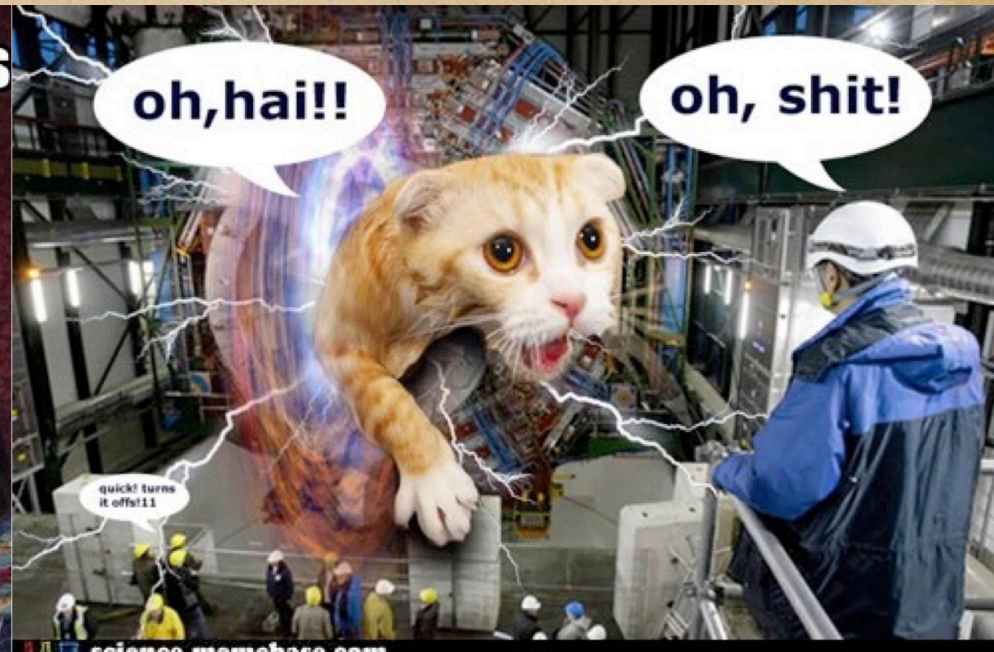
But while $\alpha-\beta$ is basis-independent, β is not. Davidson and Greiner (2010) chose a basis where one Higgs only couples to the tau, and then the λ is rescaled by $\cos\beta_\tau$, but this angle has nothing to do with the ratio of vevs and thus is completely arbitrary. Davidson and Haber (2005) showed that IF the scalar self-couplings satisfied a relation, then a basis can be chosen in which the Z_2 symmetry appears, and then $\tan\beta$ has its usual meaning, but that requires tuning.

Conclusions

The Cheng-Sher ansatz parametrizes tree-level flavor-changing neutral currents in terms of coefficients that, in the absence of fine-tuning, should be O(1).

Now, 30 years later, data has challenged this ansatz. Five of the nine off-diagonal coefficients must be substantially smaller then 1. It is possible that there might be some wiggle-room, but it appears that the ansatz is no longer viable. It may still be useful in parametrizing and comparing FCNC studies.


The lesser-known "Schrödinger's Tube" experiment.


oh,hai!!
oh, shit!
quick! turns it offs!11


i've located a wormhole


time travel cat...
...almost cause paradox.
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