Introduction to neutrino mass models

Lecture 1: Weyl, Majorana, Dirac, SM

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Content of the lecture set

1. Basics: Weyl vs Majorana vs Dirac fermions; various mass terms
2. Seesaw and radiative mass models
3. Tribimaximal mixing from A_4 symmetry group

Weyl fermions

Weyl spinors

Isotropy $\to$ laws are SO(3) symmetric $\to$ basic objects (fields) transform under $SO(3)$ irreducible representations: scalar $\phi(x)$, vector $A_i(x)$, tensor $T_{ij}(x)$, etc.

Relativistic theory: laws are Lorentz invariant $\to$ fields transform as irreducible representations of $SO(1,3)$. But since

$$so(1,3) \simeq su(2)_L \times su(2)_R \,,$$

we get more representations: scalar $\phi(x)$, left-handed Weyl spinor $\psi_a(x)$ $(a = 1,2)$, right-handed Weyl spinor $\psi^\dagger_{\dot a}$ $(\dot a = 1,2)$, 4-vector $A_\mu(x)$, tensors etc.

Chiral spinors exist because our space-time can accommodate them!

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Weyl spinors

Notation:

- left-handed = LH = "left"; right-handed = RH = "right";
- LH and RH spinors are different fields! They belong to different spaces; they transform differently under boosts and rotations.
- Do not think of them of two "polarizations" of a single field!

  Helicity and chirality are different notions!

Weyl spinors

Scalars fields can be real or complex. But Weyl spinors must be complex: $\psi^\dagger \neq \psi$. This is because the generators of $su(2)_L \times su(2)_R$ are complex.

They live in different spaces!

$$(\psi_a)^\dagger = (\psi^\dagger)_{\dot{a}}\,, \quad LH \xrightarrow{\text{conjugation}} RH\,,$$

because the generators of $su(2)_L \times su(2)_R$ are swapped under conjugation.

To avoid confusion, we adopt for now the following convention:

- Weyl spinors without $\dagger$ (e.g. ψ) are always LH and carry undotted indices a
- Weyl spinors with $\dagger$ (e.g. $\chi^\dagger$) are always RH and carry dotted indices $\dot{a}$

Later we will switch to the traditional notation with ν_L and ν_R.

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Lorentz invariant combinations

3D rotations: invariant symbol is δ^{ij}: $\delta^{ij} a_i b_j$ is $SO(3)$ invariant.

Lorentz symmetry: several invariant symbols possible.

- LH×LH: combining ψ_a and χ_b

$$\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \to \quad \epsilon^{ab}\psi_a\chi_b \text{ is Lorentz invariant.}$$

Conventional shorthand notation:

$$\psi\chi \equiv -\epsilon^{ab}\psi_a\chi_b = -\epsilon^{ab}(-\chi_b\psi_a) = -\epsilon^{ba}\chi_b\psi_a \equiv \chi\psi\,.$$

Majorana mass term for Weyl spinors contains $\nu\nu \equiv -\epsilon^{ab}\nu_a\nu_b$.

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Lorentz invariant combinations

- RH×RH: combining $\psi^\dagger_{\dot a}$ and $\chi^\dagger_{\dot b}$

$$\epsilon^{\dot a\dot b} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \to \quad \epsilon^{\dot a\dot b}\psi^\dagger_{\dot a}\chi^\dagger_{\dot b} \text{ is Lorentz invariant.}$$

Majorana mass term contains $\nu^\dagger\nu^\dagger \equiv +\epsilon^{\dot a\dot b}\nu^\dagger_{\dot a}\nu^\dagger_{\dot b}$.

This convention is consistent with the rule $(\psi\chi)^\dagger = \chi^\dagger\psi^\dagger$:

$$(\psi\chi)^\dagger \equiv (-\epsilon^{ab}\psi_a\chi_b)^\dagger = (-\epsilon^{ab})^*(\chi_b)^\dagger(\psi_a)^\dagger = -\epsilon^{\dot a\dot b}\chi^\dagger_{\dot b}\psi^\dagger_{\dot a} = +\epsilon^{\dot b\dot a}\chi^\dagger_{\dot b}\psi^\dagger_{\dot a} \equiv \chi^\dagger\psi^\dagger .$$

Full Majorana mass term contains $\nu\nu + \nu^\dagger\nu^\dagger$.

Lorentz invariant combinations

- RH×RH: combining $\psi^\dagger_{\dot a}$ and $\chi^\dagger_{\dot b}$

$$\epsilon^{\dot a \dot b} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \to \quad \epsilon^{\dot a \dot b}\psi^\dagger_{\dot a}\chi^\dagger_{\dot b} \text{ is Lorentz invariant.}$$

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Lorentz invariant combinations

Equivalent notation via raising and lowering indices:

$$\psi^a \equiv \epsilon^{ab}\psi_b\,, \quad \psi_a \equiv \epsilon_{ab}\psi^b\,, \quad \epsilon_{ab} = -\epsilon^{ab}\,,$$

and the same for RH spinors.

The convenient index-free notation is

$$\psi\chi \equiv -\epsilon^{ab}\psi_a\chi_b = \psi^b\chi_b = -\psi_b\chi^b\,.$$

In short, there are many ways to write the same expression. Just remember that the Lorentz invariant quantity is

$$\psi\chi = \psi_2\chi_1 - \psi_1\chi_2\,.$$

Lorentz invariant combinations

- LH×RH: ψ_a and $\chi^\dagger_{\dot b}$ $\to$ do not match $\to$ we need something extra!

  Correct combination: LH×vector×RH using $\bar\sigma^\mu = (\mathbb{I}, -\sigma^k)$

$$\chi^\dagger \bar\sigma^\mu \psi V_\mu \equiv \chi^\dagger_{\dot a} (\bar\sigma^\mu)^{\dot a b} \psi_b V_\mu \text{ is Lorentz invariant.}$$

  Again, intuitive rule works:

$$(\chi^\dagger \bar\sigma^\mu \psi)^\dagger = \psi^\dagger \bar\sigma^\mu \chi\,.$$

In addition to up-index tensors ϵ^{ab}, $\epsilon^{\dot a \dot b}$, $(\bar\sigma^\mu)^{\dot a b}$, we define lower-indices tensors ϵ_{ab}, $\epsilon_{\dot a \dot b}$, $(\sigma^\mu)_{a\dot b}$, which follow several consistency rules:

$$(\bar\sigma^\mu)^{\dot a a} = \epsilon^{ab}\epsilon^{\dot a \dot b}(\sigma^\mu)_{b\dot b}\,, \quad (\sigma^\mu)_{a\dot a}(\bar\sigma_\mu)_{b\dot b} = -2\epsilon_{ab}\epsilon_{\dot a \dot b}\,, \quad \text{etc.}$$

Dirac vs Majorana

Lagrangian

For real scalar field $\phi(x)$, we construct

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi\,\phi\,,$$

We need something similar for the Weyl LH field $\psi_a(x)$.

- mass term:
$$\psi\psi + \psi^\dagger\psi^\dagger \equiv -\epsilon^{ab}\psi_a\psi_b + \epsilon^{\dot a\dot b}\psi^\dagger_{\dot a}\psi^\dagger_{\dot b}\,.$$
- kinetic term $\partial_\mu\psi\,\partial^\mu\psi$ is bad because it is not positively definite.
- But single-derivative term $i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi$ is OK.

$$\left[i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi\right]^\dagger = -i(\partial_\mu\psi)^\dagger(\bar\sigma^\mu)\psi = -i\,\underbrace{\partial_\mu\left[\psi^\dagger(\bar\sigma^\mu)\psi\right]}_{=0} + i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi\,.$$

Lagrangian

$$\mathcal{L} = i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi - \frac{1}{2}\left(m\psi\psi + m^*\psi^\dagger\psi^\dagger\right)\,.$$

Mass parameter m can be complex $m = |m|e^{i\alpha}$, but the phase is not a physical parameter and can be removed via a global field redefinition $e^{i\alpha/2}\psi(x) = \psi_{\rm new}(x)$:

$$\boxed{\mathcal{L} = i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi - \frac{1}{2}m(\psi\psi + \psi^\dagger\psi^\dagger)\,.}$$

After that, we are no longer allowed to rephase ψ! We can only flip its sign.

- massless case: symmetry group is $U(1)$ $\to$ fermion can be equipped with a charge q: $\psi \to \psi\exp(iq\alpha)$ leaves kinetic term invariant.
- massive case: symmetry group is $\mathbb{Z}_2$ $\to$ fermion cannot have any conserved charge; it can only have some parity.

Equations of motion

Real scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi\,\phi\,, \quad S \equiv \int d^4x\mathcal{L}\,,$$

which leads to the equation of motion:

$$\frac{\delta S}{\delta\phi} = 0 \quad \to \quad \partial_\mu\partial^\mu\phi + m^2\phi = 0\,.$$

But for the Weyl fermion, the lagrangian

$$\mathcal{L} = i\psi^\dagger(\bar\sigma^\mu)\partial_\mu\psi - \frac{1}{2}m(\psi\psi + \psi^\dagger\psi^\dagger)$$

links together ψ and $\psi^\dagger$ $\to$ we will have a pair of coupled equations.

Equations of motion

Omitting indices, we get:

$$-i\bar\sigma^\mu\partial_\mu\psi + m\psi^\dagger = 0$$
$$-i\sigma^\mu\partial_\mu\psi^\dagger + m\psi = 0$$

[Reminder: $\sigma^\mu = (\mathbb{I}_2, \sigma^k)$, $\bar\sigma^\mu = (\mathbb{I}_2, -\sigma^k)$.] In the matrix form:

$$\begin{pmatrix} m\cdot\mathbb{I}_2 & -i\sigma^\mu\partial_\mu \\ -i\bar\sigma^\mu\partial_\mu & m\cdot\mathbb{I}_2 \end{pmatrix}\begin{pmatrix}\psi \\ \psi^\dagger\end{pmatrix} = 0$$

We can make it compact by introducing 4×4 matrices

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0\end{pmatrix}\,, \quad \text{which satisfy} \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\,,$$

and obtain the Dirac equation:

$$(-i\gamma^\mu\partial_\mu + m\cdot\mathbb{I}_4)\Psi = 0\,, \qquad \Psi \equiv \begin{pmatrix}\psi_a \\ \psi^{\dagger\,\dot a}\end{pmatrix}\,.$$

Majorana fermion

$$(-i\gamma^\mu\partial_\mu + m)\Psi = 0\,, \qquad \Psi \equiv \begin{pmatrix} \psi_a \\ \psi^{\dagger\,\dot a} \end{pmatrix}.$$

We obtained the Dirac equation for Majorana fermion.

- Majorana fermion has two degrees of freedom ψ_a, $a = 1,2$. The most natural way to describe it is via 2-component Weyl spinors ψ_a.
- The 4-component bispinor Ψ is an artificial construction for Majorana fermion: up and down components are not independent.
- There is simply no way to perform transformation $\Psi \to e^{i\alpha}\Psi$ on Majorana bispinor!

Majorana fermion

In summary:

- Majorana fermion is not a "fancy fermion", it is not an exotic object.
- It appears naturally as the most basic form of fermion field.
- But the massive Majorana fermion can only get mass via Majorana mass term
$$\frac{m}{2}(\psi\psi + \psi^\dagger\psi^\dagger)\,,$$
which kills any possibility of rephasing apart from sign flip.

Majorana fermion cannot possess additive charges.

Dirac fermion

Dirac fermion $= 2 \times$ Majorana fermions with equal mass.

Take two Weyl fermions $(\psi_1)_a$ and $(\psi_2)_a$ with equal masses:

$$\mathcal{L} = \sum_i i\psi_i^\dagger \bar{\sigma}^\mu \partial_\mu \psi_i - \frac{m}{2}\sum_i \left(\psi_i\psi_i + \psi_i^\dagger\psi_i^\dagger\right) .$$

We still cannot rephase them, but can perform $SO(2)$ rotation:

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \to \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

and this leaves $\mathcal{L}$ invariant.

This is the origin of charges for Dirac fermions.

Dirac fermion

$$\mathcal{L} = \sum_i i\psi_i^\dagger \bar\sigma^\mu \partial_\mu \psi_i - \frac{m}{2}\sum_i \left(\psi_i\psi_i + \psi_i^\dagger\psi_i^\dagger\right) .$$

Define

$$\chi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)\,, \quad \xi = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)\,.$$

Then kinetic term stays the same, but mass terms change form:

$$\mathcal{L} = i\chi^\dagger\bar\sigma^\mu\partial_\mu\chi + i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi - m(\chi\xi + \xi^\dagger\chi^\dagger)\,.$$

This lagrangian has the global $U(1)$ symmetry under

$$\chi \to e^{-i\alpha}\chi\,, \quad \xi \to e^{i\alpha}\xi\,.$$

So, the pair of Weyl fields χ and ξ can describe fermions with charges!

Dirac equation

$$\mathcal{L} = i\chi^\dagger \bar\sigma^\mu \partial_\mu \chi + i\xi^\dagger \bar\sigma^\mu \partial_\mu \xi - m(\chi\xi + \xi^\dagger\chi^\dagger)\,.$$

leads to the following pair of equations of motion:

$$\begin{aligned} -i\bar\sigma^\mu\partial_\mu\chi + m\xi^\dagger &= 0 \\ -i\sigma^\mu\partial_\mu\xi^\dagger + m\chi &= 0 \end{aligned}$$

which can be combined into Dirac equation but for a different bispinor Ψ:

$$(-i\gamma^\mu\partial_\mu + m\cdot\mathbb{I}_4)\Psi = 0\,, \qquad \Psi \equiv \begin{pmatrix} \chi_a \\ \xi^{\dagger\,\dot a} \end{pmatrix}.$$

The Dirac bispinor Ψ has 4 d.o.f. — all its components are independent!

Dirac fermion

The lagrangian

$$\mathcal{L} = i\chi^\dagger\bar\sigma^\mu\partial_\mu\chi + i\xi^\dagger\bar\sigma^\mu\partial_\mu\xi - m(\chi\xi + \xi^\dagger\chi^\dagger)\,.$$

can also be written via 4-component bispinor Ψ.

$$\Psi = \begin{pmatrix}\chi \\ \xi^\dagger\end{pmatrix}, \quad \Psi^\dagger = (\chi^\dagger,\, \xi), \quad \overline{\Psi} \equiv \Psi^\dagger\beta = \overline{\Psi} \equiv \Psi^\dagger \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0\end{pmatrix} = (\xi,\, \chi^\dagger)\,.$$

Then, the lagrangian is just

$$\mathcal{L} = i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m\overline{\Psi}\Psi\,.$$

Notice the evident global symmetry $\Psi \to e^{-i\alpha}\Psi$.

The Dirac equation $(-i\gamma^\mu\partial_\mu + m\cdot\mathbb{I}_4)\Psi = 0$ can be derived right from this $\mathcal{L}$ without further constraints because all components of Ψ are independent.

Dirac fermion

In summary:

- Dirac fermion $= 2\times$ Majorana fermion. It is a more complicated construction but we must make this doubling if we want to describe fermions with charges.
- The most natural way to define Dirac fermion is via the 4-component Dirac bispinor Ψ. The lagrangian is very compact, and the Dirac equation directly follows from it.

Charge conjugation

Extra symmetry

Extra sign flips

- For 1 Majorana fermion, the mass term $\psi\psi + \psi^\dagger\psi^\dagger$ had one residual symmetry: $\psi \to -\psi$. It was rather useless.
- For 2 Majorana fermions $\psi_i\psi_i + \psi_i^\dagger\psi_i^\dagger$ we can independently flip signs of ψ_1 and ψ_2. The relative sign flip enlarges the symmetry group from $SO(2)$ to $O(2)$: rotations and reflections.
- In terms of χ and ξ, it is the symmetry under $\chi \leftrightarrow \xi$:

$$\chi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)\,, \quad \xi = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)\,.$$

  Since it exchanges Weyl spinors with opposite charges, we call this transformation charge conjugation C.

Notice: $C \neq$ complex conjugation because ψ_i are themselves complex!

Extra symmetry

At the level of bispinors, the definition is

$$\Psi = \begin{pmatrix} \chi_a \\ \xi^{\dagger\dot a} \end{pmatrix} \xrightarrow{\text{charge conjugation}} \Psi^c \equiv \begin{pmatrix} \xi_a \\ \chi^{\dagger\dot a} \end{pmatrix} .$$

The question is how to produce Ψ^c from Ψ via spinor manipulations.

$$\Psi = \begin{pmatrix} \chi_a \\ \xi^{\dagger\dot a} \end{pmatrix}, \quad \overline{\Psi} = (\xi^a, \chi^\dagger_{\dot a}), \quad (\overline{\Psi})^T = \begin{pmatrix} \xi^a \\ \chi^\dagger_{\dot a} \end{pmatrix},$$

To arrive to Ψ^c, we need one more step:

$$\Psi^c = \begin{pmatrix} \xi_a \\ \chi^{\dagger\dot a} \end{pmatrix} = \mathcal{C}(\overline{\Psi})^T, \quad \text{where} \quad \mathcal{C} = \begin{pmatrix} \epsilon_{ab} & 0 \\ 0 & \epsilon^{\dot a\dot b} \end{pmatrix} .$$

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Operator $\mathcal{C}$

Some useful properties:

$$\mathcal{C} = \begin{pmatrix} \epsilon_{ab} & 0 \\ 0 & \epsilon^{\dot{a}\dot{b}} \end{pmatrix} = -\begin{pmatrix} \epsilon^{ab} & 0 \\ 0 & \epsilon_{\dot{a}\dot{b}} \end{pmatrix},$$

$$\mathcal{C}^{-1} = \mathcal{C}^T = \mathcal{C}^\dagger = -\mathcal{C}\,, \qquad \mathcal{C}^{-1}\gamma^\mu\mathcal{C} = -(\gamma^\mu)^T\,.$$

Then, since $\Psi^c = \mathcal{C}(\overline{\Psi})^T$, we get $\mathcal{C}^{-1}\Psi^c = (\overline{\Psi})^T$ and

$$\overline{\Psi} = [\mathcal{C}^{-1}\Psi^c]^T = (\Psi^c)^T(\mathcal{C}^{-1})^T = (\Psi^c)^T\mathcal{C}\,.$$

So, yet another form of the Dirac mass term:

$$\overline{\Psi}\Psi = (\Psi^c)^T\mathcal{C}\Psi = (\xi_a,\, \chi^{\dagger\dot{a}})\,\mathcal{C}\begin{pmatrix} \chi_b \\ \xi^{\dagger\dot{b}} \end{pmatrix} = \xi\chi + \chi^\dagger\xi^\dagger\,,$$

Operator $\mathcal{C}$

Operations $\overline{(\cdot)}$ and $(\cdot)^c$ sort of compensate each other. Applying them together:

$$\overline{\Psi^c} = (\Psi)^T \mathcal{C} \,.$$

And yet another form of the Dirac mass term:

$$\overline{\Psi^c}\Psi^c = (\Psi)^T \underbrace{\mathcal{C}\cdot\mathcal{C}}_{=-1} (\overline{\Psi})^T = -(\Psi)^T (\overline{\Psi})^T = +\overline{\Psi}\Psi \,.$$

The last step:

$$-(\chi_a, \xi^{\dagger\dot{a}}) \begin{pmatrix} \xi^a \\ \chi^\dagger_{\dot{a}} \end{pmatrix} = -\left(\chi_a \xi^a + \xi^{\dagger\dot{a}}\chi^\dagger_{\dot{a}}\right) = +\left(\xi^a\chi_a + \chi^\dagger_{\dot{a}}\xi^{\dagger\dot{a}}\right) = \xi\chi + \chi^\dagger\xi^\dagger$$

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The last step:

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Several fields

If you have several fields Ψ_i, $i = 1, \dots, n$, then be careful at the last step!

$$\overline{\Psi_i^c}\Psi_j^c = \overline{\Psi_j}\Psi_i \,.$$

So, the (complex) Dirac mass matrix can be written in different ways:

$$\overline{\Psi_i} m_{ij} \Psi_j + h.c. = \overline{\Psi_i^c} m_{ij}^T \Psi_j^c + h.c. = \frac{1}{2}\left(\overline{\Psi_i} m_{ij} \Psi_j + \overline{\Psi_i^c} m_{ij}^T \Psi_j^c\right) + h.c.$$

This is how m_D and m_D^T will appear later in the seesaw mechanism.

C for Majorana fermion

For Majorana field, instead of different χ and ξ we have a single ψ:

$$\Psi = \begin{pmatrix} \psi \\ \psi^\dagger \end{pmatrix} .$$

- At the level of Weyl spinors, there is simply no such transformation as "charge conjugation".
- At the level of Majorana bispinor Ψ, we can define it via the same bispinor manipulation as before:

  $$\Psi^c \equiv \mathcal{C}(\overline{\Psi})^T .$$

  Then, we get the Majorana condition:

  $$\Psi^c = \Psi .$$

  This is almost a tautology: Ψ is artificially constructed from two copies of ψ.

C for Majorana fermion

So, for Majorana fermion, the lagrangian can be written as

$$\mathcal{L} = \frac{i}{2}\Psi^T \mathcal{C}\gamma^\mu \partial_\mu \Psi - \frac{m}{2}\Psi^T \mathcal{C}\Psi\,.$$

In this way, it is written only in terms of Ψ, without $\overline{\Psi}$, and leads to the Dirac equation.

LH and RH components

and link to traditional notation

LH/RH projectors

In the chiral basis, bispinors have LH and RH components:

$$\text{Dirac:} \quad \Psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix} .$$

You can extract LH or RH parts using chiral projectors:

$$\gamma_5 \equiv \begin{pmatrix} -\mathbb{I}_2 & 0 \\ 0 & \mathbb{I}_2 \end{pmatrix} , \quad P_L = \frac{1-\gamma_5}{2} , \quad P_R = \frac{1+\gamma_5}{2} .$$

Then, for Dirac fermion

$$\Psi_L = P_L \Psi = \begin{pmatrix} \chi \\ 0 \end{pmatrix} , \quad \Psi_R = P_R \Psi = \begin{pmatrix} 0 \\ \xi^\dagger \end{pmatrix} ,$$

are independent fields.

LH/RH projectors

$$\text{Majorana:} \quad \Psi = \begin{pmatrix} \psi \\ \psi^\dagger \end{pmatrix}.$$

Acting with projectors P_L and P_R gives

$$P_L\Psi = \begin{pmatrix} \psi \\ 0 \end{pmatrix} \equiv \Psi_L\,, \quad P_R\Psi = \begin{pmatrix} 0 \\ \psi^\dagger \end{pmatrix} = (\Psi_L)^c\,.$$

$P_R\Psi$ is not a new field! We cannot label it as Ψ_R. We must label it as $(\Psi_L)^c$.

Also notice: $(\Psi_L)^c$ is different from Ψ_L, even if the initial Ψ is Majorana fermion! It is their sum which is C-invariant:

$$\Psi_L + (\Psi_L)^c \xrightarrow{C} \Psi_L + (\Psi_L)^c\,.$$

LH/RH projectors

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$P_R\Psi$ is not a new field! We cannot label it as Ψ_R. We must label it as $(\Psi_L)^c$.

Also notice: $(\Psi_L)^c$ is different from Ψ_L, even if the initial Ψ is Majorana fermion! It is their sum which is C-invariant:

$$\Psi_L + (\Psi_L)^c \xrightarrow{C} \Psi_L + (\Psi_L)^c\,.$$

LH/RH neutrinos

Traditional notation in neutrino phenomenology:

$$\text{Dirac } \Psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \equiv \nu_L + \nu_R \,,$$

Under charge conjugation:

$$\nu_L \xrightarrow{C} (\nu_L)^c = \begin{pmatrix} 0 \\ (\psi_L)^\dagger \end{pmatrix} \text{ is RH}\,, \quad \nu_R \xrightarrow{C} (\nu_R)^c = \begin{pmatrix} (\psi_R)^\dagger \\ 0 \end{pmatrix} \text{ is LH}\,.$$

Conjugated spinors:

$$\overline{\nu_L} = (0, (\psi_L)^\dagger) \text{ is RH}, \qquad \overline{\nu_R} = ((\psi_R)^\dagger, 0) \text{ is LH},$$

$$\overline{(\nu_L)^c} = (\psi_L, 0) \text{ is LH}, \qquad \overline{(\nu_R)^c} = (0, \psi_R) \text{ is RH},$$

Remember: $\overline{\Psi^c} = \Psi^T \mathcal{C}$.

Be careful! When you see $\bar{\nu}_L$, read it as $\overline{(\nu_L)}$, not $(\bar{\nu})_L$.

When you see $\bar{\nu}_L^c$, read it as $\overline{(\nu_L)^c}$.

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$$\overline{\nu_L} = (0, (\psi_L)^\dagger) \text{ is RH}, \qquad \overline{\nu_R} = ((\psi_R)^\dagger, 0) \text{ is LH},$$
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Remember: $\overline{\Psi^c} = \Psi^T \mathcal{C}$.

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LH/RH neutrinos

Same for LH Majorana fermion:

$$\text{Majorana } \Psi = \begin{pmatrix} \psi_L \\ (\psi_L)^\dagger \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ (\psi_L)^\dagger \end{pmatrix} = \nu_L + (\nu_L)^c\,,$$

Conjugated spinors:

$$\overline{\nu_L} = (0, (\psi_L)^\dagger)\,, \quad \overline{(\nu_L)^c} = (\psi_L, 0)\,.$$

And for RH Majorana fermion:

$$\Psi = \begin{pmatrix} (\psi_R)^\dagger \\ \psi_R \end{pmatrix} = \begin{pmatrix} (\psi_R)^\dagger \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = (\nu_R)^c + \nu_R\,,$$

Conjugated spinors:

$$\overline{\nu_R} = ((\psi_R)^\dagger, 0)\,, \quad \overline{(\nu_R)^c} = (0, \psi_R)\,.$$

LH/RH neutrinos

Same for LH Majorana fermion:

$$\text{Majorana } \Psi = \begin{pmatrix} \psi_L \\ (\psi_L)^\dagger \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ (\psi_L)^\dagger \end{pmatrix} = \nu_L + (\nu_L)^c \,,$$

Conjugated spinors:

$$\overline{\nu_L} = (0, (\psi_L)^\dagger)\,, \quad \overline{(\nu_L)^c} = (\psi_L, 0)\,.$$

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$$\Psi = \begin{pmatrix} (\psi_R)^\dagger \\ \psi_R \end{pmatrix} = \begin{pmatrix} (\psi_R)^\dagger \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} = (\nu_R)^c + \nu_R \,,$$

Conjugated spinors:

$$\overline{\nu_R} = ((\psi_R)^\dagger, 0)\,, \quad \overline{(\nu_R)^c} = (0, \psi_R)\,.$$

LH/RH neutrinos

NB: terminology "LH/RH Majorana fermion" is a slight abuse of notation.

$$\text{Majorana } \Psi = \begin{pmatrix} \psi_L \\ (\psi_L)^\dagger \end{pmatrix} = \begin{pmatrix} (\psi_R)^\dagger \\ \psi_R \end{pmatrix}.$$

LH vs. RH simply denotes which component we label as the "primary" and which is the conjugated.

You can use either notation; they are equivalent.

Mass terms

One Majorana fermion

LH Majorana fermion:

$$\Psi = \begin{pmatrix} \psi_L \\ (\psi_L)^\dagger \end{pmatrix} = \nu_L + (\nu_L)^c\,.$$

Majorana mass term:

$$\frac{M_L}{2}\left[\psi_L\psi_L + (\psi_L)^\dagger(\psi_L)^\dagger\right] = \boxed{\frac{M_L}{2}\left[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L\right]}$$

Similarly for RH Majorana fermion:

$$\frac{M_R}{2}\left[\psi_R\psi_R + (\psi_R)^\dagger(\psi_R)^\dagger\right] = \boxed{\frac{M_R}{2}\left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right]}$$

One Majorana fermion

Another form of the same term (e.g. RH):

$$\frac{M_R}{2}\left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right]$$
$$= \frac{M_R}{2}\left[((\nu_R)^c)^T\mathcal{C}(\nu_R)^c + (\nu_R)^T\mathcal{C}\nu_R\right] = \boxed{\frac{M_R}{2}(\nu_R)^T\mathcal{C}\nu_R + h.c.}$$

Reminder: $\mathcal{C} = \mathrm{diag}(\epsilon_{ab}, \epsilon^{\dot{a}\dot{b}})$; it just links two LH or two RH Weyl fields.

Sometimes, in abuse of notation, the last form is shortened to

$$\frac{M_R}{2}\nu_R\nu_R + h.c.$$

But you should always remember the hidden notation!

A memo on "$+h.c.$"

In terms of Weyl spinors, we have $\frac{M_R}{2}\left[\psi_R\psi_R + (\psi_R)^\dagger(\psi_R)^\dagger\right]$.
In terms of bispinors:

- if you use $\nu_R = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$, then you must write $+h.c.$:

$$\frac{M_R}{2}(\nu_R)^T\mathcal{C}\nu_R + h.c.$$

- if you work with Majorana bispinor $\Psi_R \equiv \begin{pmatrix} \psi_R^\dagger \\ \psi_R \end{pmatrix}$, then do not add $+h.c.$:

$$\frac{M_R}{2}(\Psi_R)^T\mathcal{C}\Psi_R$$

Dirac fermion with Dirac mass term

Dirac fermion:

$$\Psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \nu_L + \nu_R\,.$$

Dirac Mass term:

$$\overline{\Psi}\Psi = \xi\chi + \chi^\dagger\xi^\dagger = (\psi_R)^\dagger\psi_L + (\psi_L)^\dagger\psi_R = \overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\,.$$

In terms of bispinors, the structure is $\overline{RH}\times LH + \overline{LH}\times RH$. But we have two LH bispinors: ν_L and $(\nu_R)^c$. Let's write this mass terms as a 2×2 matrix:

$$m_D\overline{\Psi}\Psi = m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) = \frac{1}{2}\left[\overline{\nu_L},\, \overline{(\nu_R)^c}\right]\begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

Two equal mass eigenvalues is a hallmark feature of the Dirac fermion!

Dirac mass for several neutrino generations

Several Dirac neutrino generations: ν_{Li}, ν_{Ri}, $i = 1, \dots, n$.

Then m_D is a $n \times n$ matrix:

$$\overline{\nu_{Li}}(m_D)_{ij}\nu_{Rj} + h.c. = \frac{1}{2}\left[\overline{\nu_L}, \overline{(\nu_R)^c}\right] \begin{pmatrix} 0 & m_D \\ m_D^T & 0 \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

If ν_{Li} and ν_{Ri} are not assumed to be combined into Dirac fermions, then n_L and n_R can be different:

$$\frac{1}{2}\left[\overline{\nu_L}, \overline{(\nu_R)^c}\right] \begin{pmatrix} 0 & m_D \\ m_D^T & 0 \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

where m_D is a matrix $n_L \times n_R$.

Adding Majorana mass terms to Dirac fermion

Start with Dirac fermion = 2 × Majorana fermions

Then we can construct two more mass terms!

In terms of Weyl spinors:

$$m_D(\xi\chi + \chi^\dagger\xi^\dagger) + \frac{1}{2}M_L(\chi\chi + \chi^\dagger\chi^\dagger) + \frac{1}{2}M_R(\xi^\dagger\xi^\dagger + \xi\xi)\,.$$

In terms of bispinors:

$$m_D\left[\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right] + \frac{1}{2}M_L\left[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L\right] + \frac{1}{2}M_R\left[\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R\right]\,.$$

Adding Majorana mass terms to Dirac fermion

In terms of mass matrix between two RH and two LH fields:

$$\frac{1}{2}\left[\overline{\nu_L}, \overline{(\nu_R)^c}\right] \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

Eigenvalues are not equal $\Rightarrow$ Dirac fermion equipped with Majorana mass terms splits into two Majorana fermions.

Majorana vs Dirac fermions

Looking at Weyl fermions is most convenient for counting fermionic d.o.f.

- 1 Weyl field (LH or RH) $\to$ Majorana fermion $\to$ only Majorana mass term allowed; no conserved charges possible.
- 2 Weyl fields (does not matter, LH+LH, RH+RH, LH+RH) with different masses $\to$ 2 Majorana fermions; no conserved charges possible.
- 2 Weyl fields with equal masses $\to$ Dirac fermion with $U(1)$ symmetry (conserved charge) $\Leftrightarrow$ 2 mass-degenerate Majorana fermions.
- Dirac field + Majorana mass terms $\to$ back to 2 Majorana fermions; $U(1)$ symmetry destroyed.

Neutrinos in the SM

Quark masses

One quark generation: EW doublet $Q_L = (d_L, u_L)$, EW singlets d_R, u_R.

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y(\Phi) = +1, \qquad \tilde{\Phi} = \epsilon^{ij}\Phi_j^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}, \quad Y(\tilde{\Phi}) = -1.$$

Yukawa interactions:

$$\boxed{y_d \overline{Q_L}\Phi d_R + y_u \overline{Q_L}\tilde{\Phi} u_R + h.c.} \quad \to \quad \frac{y_d v}{\sqrt{2}}(\overline{d_L} d_R + \overline{d_R} d_L) + \frac{y_u v}{\sqrt{2}}(\overline{u_L} u_R + \overline{u_R} u_L)$$
$$= \quad m_d \bar{d} d + m_u \bar{u} u\,.$$

Mass terms must be Dirac because d and u are electrically charged!

Same for charged leptons: $y_e \overline{L}\Phi e_R + h.c. \quad \to \quad y_e v(\overline{e_L} e_R + \overline{e_R} e_L)/\sqrt{2} = m_e \bar{e} e.$

Neutrinos in the minimal SM

For neutrinos, naively, several mass terms are possible:

$$\frac{1}{2}\left[\overline{\nu_L}, \overline{(\nu_R)^c}\right]\begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

But in the minimal SM

- only LH neutrinos exist: $L \equiv (\nu_L, e_L)$, e_R, but no ν_R,
- only renormalizable interactions included,
- the minimal Higgs sector.

Then, ν_L is strictly massless.

Removal of any of these requirements allows for non-zero neutrino masses.

What is allowed in the SM?

Keep only ν_L, keep minimal Higgs sector, but allow for non-renormalizable terms.

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad Y(L) = -1\,, \qquad \tilde{\Phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}, \quad Y(\tilde{\Phi}) = -1\,.$$

We can build the composite gauge-invariant operator:

$$(\overline{L}\tilde{\Phi}) = \overline{\nu_L}\phi^{0*} - \overline{e_L}\phi^-$$

It behaves under Lorentz transformations as a "RH fermion N_R".

We can then mimic the Majorana mass term: $\overline{N_R}(N_R)^c + \overline{(N_R)^c}N_R$!

Weinberg operator

It produces the unique dim-5 operator possible in the minimal SM:

$$Q_W = \left(\overline{L^c}\tilde{\Phi}^*\right)\left(\tilde{\Phi}^\dagger L\right) + \left(\overline{L}\tilde{\Phi}\right)\left(\tilde{\Phi}^T L^c\right)\,,$$

known as the Weinberg operator [Weinberg, 1979]. Explicitly expanding the doublets, we get the new term in the Lagrangian:

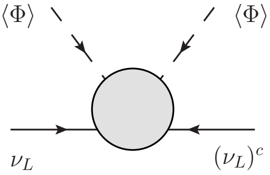
$$\frac{c}{\Lambda}\left[\left(\overline{\nu_L^c}\phi^0 - \overline{e_L^c}\phi^+\right)\left(\phi^0\nu_L - \phi^+ e_L\right) + h.c.\right]$$

$$= \quad \frac{c}{\Lambda}\left[(\nu_L^T\phi^0 - e_L^T\phi^+)\mathcal{C}(\phi^0\nu_L - \phi^+ e_L) + h.c.\right]$$

Here, Λ is the scale which must be introduced because $\dim(Q_W) = 5$.

For 3 fermion generations: same structure, $L \to L_i$, $c \to c_{ij}$.

Weinberg operator



After EW symmetry breaking, $\phi^0 \to v/\sqrt{2}$, we get LH Majorana mass term:

$$\boxed{\frac{cv^2}{2\Lambda}\left(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c\right)\,,}$$

with the Majorana mass

$$m_\nu = \frac{cv^2}{\Lambda}\,.$$

$c \sim 1$, $\Lambda \sim 10^{15}$ GeV (typical GUT scale) $\to$ $m_\nu \sim$ meV, correct scale!